

Adaptive On-Line Learning Algorithm for Robust Estimation of Parameters of Noisy Sinusoidal Signals

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Abstract. In many applications, very fast methods are required for estimating of parameters of harmonic signals distorted by noise. Most of the known digital algorithms are not fully parallel, so that the speed of processing is quite limited. In this paper new parallel algorithms are proposed, which can be implemented by analogue adaptive circuits employing some neural networks principles. Algorithms based on the least-squares (LS) and the total least-squares (TLS) criteria are developed and compared. Extensive computer simulations confirm the validity and performance of the proposed algorithms.

Keywords: adaptive algorithms, parameter estimation, neural networks, optimization problems.

1 Introduction

Estimation of parameters (amplitudes) of harmonic signals is important in electric power systems and power electronics due to increasing use of nonlinear dynamic loads. The harmonics produced have usually varying amplitudes due to dynamic nature of nonlinear loads. Fast estimation of the parameters is essential for the control and protection of electric power systems. It is also useful in modelling, measurements and compensation of higher harmonics [1, 2].

The purpose of this paper is to present novel on-line techniques for estimation of parameters of harmonic signals based on the least-squares (LS) and the total least-squares (TLS) criteria [3]. The problems are formulated as optimization problems and solved by using the steepest descent continuous-time optimization algorithm [4, 5, 6]. The solution of the optimization problem bases on some principles of neural networks technique given by Tank and Hopfield [7, 8]. The corresponding architectures of analogue neuron-like adaptive processors are also shown. The developed methods are more robust against random noise in comparison with other known algorithms.

2 Mathematical formulation of the problem

The following standard problem has been analysed and solved.
Let $y(t)$ denote a measured noisy signal

$$y(t) = \sum_{i=1}^n a_i \sin(i\omega t) + \sum_{i=1}^n b_i \cos(i\omega t) + r(t) \quad (1)$$

where

$\omega = 2\pi f$ is a known or rather approximately estimated angular frequency,
 $r(t)$ is unknown noise or error (residual),
 a_i, b_i are unknown amplitudes of harmonic signals.

On basis of values $y(t)$ it is necessary to find or estimate in real time the amplitudes a_i, b_i . Assuming that a continuous-time signal $y(t)$ is sampled and hold with a sampling interval T , the problem can be mathematically reformulated as the problem of solving a large overdetermined system of linear equations

$$Dx = y \quad (2)$$

where:

$$x = [a_1, b_1, a_2, b_2, \dots, a_n, b_n]^T \in \mathbb{R}^{2n} \quad \text{and}$$

$$y = [y(T), y(2T), \dots, y(mT)] \in \mathbb{R}^m$$

$$D = \begin{bmatrix} \sin(\omega T) & \cos(\omega T) & \dots & \sin(n\omega T) & \cos(n\omega T) \\ \sin(2\omega T) & \cos(2\omega T) & \dots & \sin(2n\omega T) & \cos(2n\omega T) \\ \vdots & \vdots & \dots & \vdots & \vdots \\ \sin(m\omega T) & \cos(m\omega T) & \dots & \sin(mn\omega T) & \cos(mn\omega T) \end{bmatrix}$$

3 Total least squares approach

The standard LS method assumes that the matrix D is exactly determined and only the vector y is contaminated by noise. In practice the matrix D is also perturbed by error. In fact, the frequency ω is not exactly known. Moreover, it can be slightly fluctuated during the measurement, and these fluctuations are unknown. Furthermore, the sampling period is sometimes not fixed but also fluctuates (i.e. the sampling of the signal is not ideally regular). For these reasons, in order to obtain more reliable and robust solution, the total least squares (TLS) approach was applied. The approach is known in the statistics literature as *orthogonal regression* and *errors-in-variable regression*. The TLS criterion assumes errors both in the matrix D and in the vector y . Therefore, whereas LS minimizes the prediction error, TLS minimizes the vector normal to the graph of the linear predictor. Applying the TLS criterion, the following linear matrix equation has been obtained

$$(\hat{D} + R)x_{\text{TLS}} = \hat{y} + r \quad (3)$$

where: $D = \hat{D} + R$ and $y = \hat{y} + r$

$\hat{D} \in \mathbb{R}^{m \times n}, \hat{y} \in \mathbb{R}^m$ are exact but unknown matrices,

$R \in \mathbb{R}^{m \times n}, r \in \mathbb{R}^m$ are corresponding errors.

In other words, the TLS problem can be formulated as the optimization problem: to find the vector x_{TLS}^* that minimizes

$$\|R\|_F^2 + \|r\|_F^2 \quad (4)$$

subject to the equality constraints (3), where $\|R\|_F$ denotes the Frobenius norm of R . The main numerical tool for solving the TLS problem is the singular value decomposition (SVD) of the extended matrix [3]

$$\tilde{D} = [D, y] = U \Sigma V^T \quad (5)$$

The TLS solution is computed as

$$x_{\text{TLS}}^* = \frac{1}{v_{2n+1,2n+1}} [v_{1,2n+1}, v_{2,2n+1}, \dots, v_{2n,2n+1}]^T \quad (6)$$

where $v_{2n+1} = [v_{1,2n+1}, v_{2,2n+1}, \dots, v_{2n,2n+1}]^T$ is the right singular vector associated to the smallest singular value σ_{n+1} of the extended matrix $[D, y]$.

As the singular value σ_{n+1} goes to zero, the LS and TLS approaches to each other. It is important to point out that the LS solution is based on the minimization of the sum of the squared errors, while the TLS solution is based on the minimization the sum of weighted squared errors [3]

$$e(x) = \frac{Dx - y}{[1 + x^T x]^{\frac{1}{2}}} \quad (7)$$

In other words, the TLS problem can be formulated as the minimization of the energy function

$$E_{\text{TLS}}(x) = \frac{\|Dx - y\|_2^2}{1 + x^T x} = \sum_{k=1}^m \frac{\left(\sum_{j=1}^{2n} d_{kj} x_j - y_k \right)^2}{1 + x^T x} \quad (8)$$

In comparison with the standard LS technique obtaining the solution of the TLS problem is generally quite burdensome and very time consuming. This probably because the TLS approach has not been as widely used as the usual LS approach, although the TLS approach was investigated in robust statistics long ago.

In order to simplify complexity of the algorithm, an instantaneous error was introduced, defined as

$$e(t) = S^T(t)(Dx - y) = S^T(t)e = \sum_{k=1}^m \left(\sum_{j=1}^{2n} d_{kj} x_j - y_k \right) S_k(t) = \sum_{j=1}^{2n} \tilde{d}_j x_j(t) - \tilde{y}(t) \quad (9)$$

where

$S(t) = [S_1(t), S_2(t), \dots, S_m(t)]^T$ is the vector of zero-mean independent identically distributed (i.i.d.) externally excitation signals (e.g. zero-mean noise sources),

$$\tilde{d}_j(t) \stackrel{\text{df}}{=} \sum_{k=1}^m d_{kj} S_k(t) \quad \tilde{y}(t) \stackrel{\text{df}}{=} \sum_{k=1}^m y_k S_k(t) \quad (10)$$

The TLS problem can be now reformulated as minimization of the following instantaneous energy function

$$E_{\text{TLS}}[x(t)] = \frac{1}{2} \frac{e^2(t)}{x^T x + 1} \quad (11)$$

Applying the gradient descent approach, the system of differential equations was obtained

$$\frac{dx_j(t)}{dt} = -\mu(t) \frac{\partial E_{\text{TLS}}[x(t)]}{\partial x_j} = -\mu(t) e(t) \frac{\tilde{d}_j(t)(x^T x + 1) - e(t)x_j(t)}{(x^T x + 1)} \quad (12)$$

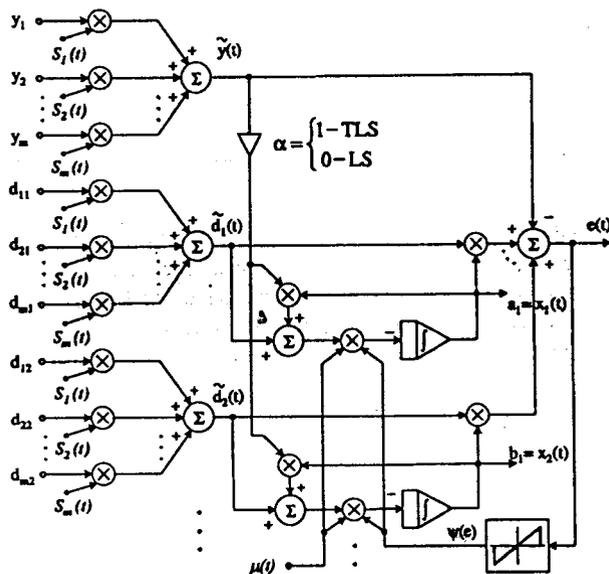
where $\mu(t) > 0$.

The above set of differential equations can be further simplified, after linearization, as

$$\frac{dx_j}{dt} = -\mu(t) e(t) [\tilde{d}_j(t) + \tilde{y}(t)x_j(t)] \quad (13)$$

Functional block diagram illustrating implementation of the algorithm (13) is shown in the Fig. 1. The block diagram can be considered as a single neuron with synapses x_j , learned (adjusted) according to eqn. (13).

a)



b)

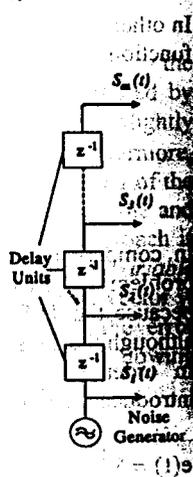


Fig. 1. a) Analog neural network for solving the estimation problem (see Eq.(13));
b) Exemplary method for generation signals S

4 Simulation experiments

Extensive computer simulation experiments have confirmed the validity and performance of the proposed algorithms. The associated networks were simulated on computer. Owing to limited place, we shall present only some illustrative results (Fig. 2). First, a signal

$$y(t) = \sin(\omega t) + r(t) \quad (14)$$

was simulated. The algorithms were investigated for the frequency of the basic component equal to 50 Hz. The investigations were carried out when the frequency of the simulated signal changes. The sampling window was $NT = 0.02; 0.03; 0.04$ and 0.06 s. The number of samples $N = 20+100$.

The influence of the random (white) noise harmonics and impulsive noise in the measured (simulated) signal on the estimation error was also investigated. The new LS and especially TLS methods show a great immunity against the noise. It can be stated, that the accuracy of the TLS method depends strongly on the sampling frequency and sampling window. In our investigations the most accurate results were obtained for the sampling frequency $f_s = 2500$ Hz and the sampling window $NT = 0,04$ s. The investigations were made for the learning rate $\mu(t) = 500$ and for $\mu(t) = 1500 \exp(-50t)$. In the second case the estimations errors were smaller. The nonlinear activation function $\psi(e)$ enables suppress large impulsive errors.

The level of the auxiliary noise $S_n(t)$ was about 1% of the amplitudes of the simulated signal.

When using the new LS algorithm the trajectories of the estimated parameter converge in less than 5 ms, and for TLS algorithm in less than 2 ms.

Conclusions

Adaptive analogue neural networks represent a very promising approach for high-speed estimation of parameters of signals. In this paper new algorithms and architectures of neuron-like adaptive circuits were developed, according to the LS and TLS optimization criteria, applying the gradient descent approach. They are more robust against noise in the measured signal than other known neural network algorithms. The network based on the TLS criterion optimize the estimation under the assumption, that the signal model can be also perturbed (frequency or sampling interval fluctuation and so forth). The TLS estimates are better and more reliable than the corresponding LS estimates, when applying a higher sampling frequency and a wider sampling window.

The TLS algorithm is a generalization of the well known LMS rule and could be in some applications superior to family of LMS algorithms. Extensive computer simulation experiments confirmed the validity and performance of the proposed algorithms.

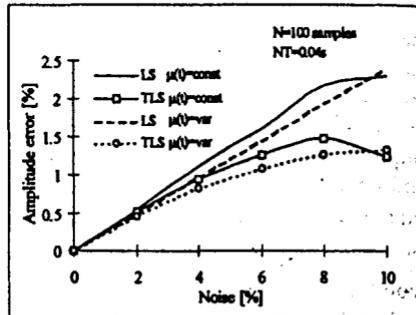
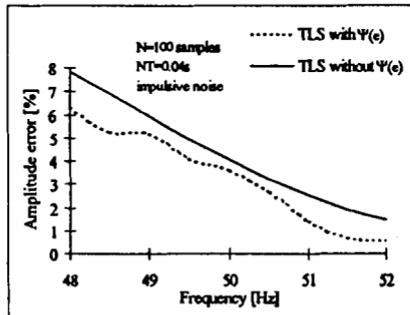
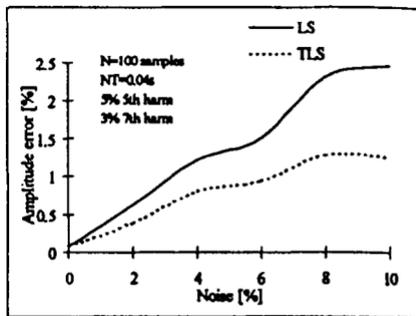
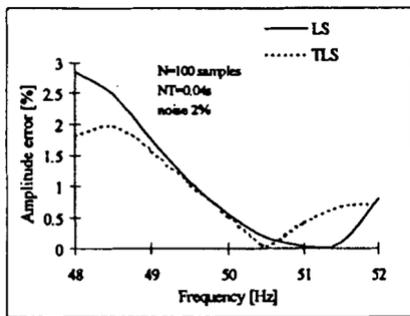


Fig. 2. Errors of the amplitude estimation

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