

NEURAL NETWORKS FOR REAL - TIME ESTIMATION OF PARAMETERS OF SIGNALS IN POWER SYSTEMS

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Abstract: Fast determination of parameters of the fundamental waveform of voltages and currents is essential for the control and protection of electrical power systems. Most of the known digital algorithms are not fully parallel, so that the speed of processing is quite limited. New parallel algorithms, which can be implemented by analogue adaptive circuits employing some neural networks principles, are proposed. The problem of estimation is formulated as an optimization problem and solved by using the gradient descent method. Algorithms based on the least absolute value, the minimax, the least-squares and the robust least-squares criteria are developed and compared. The networks process samples of observed noisy signals (voltages or currents) and give as a solution the desired parameters of signal components. Extensive computer simulations confirm the validity and performance of the proposed algorithms and neural network realizations. The proposed methods seem to be particularly useful for real-time, high-speed estimation of parameters of sinusoidal signals in electrical power systems.

Keywords: neural networks, estimation, signal processing, electrical power systems

1 Introduction

Real-time estimation of parameters of sinusoidal signals from noisy and distorted data has received considerable attention recently. Many sophisticated methods have been proposed including the Prony method, the Pisarenko harmonic decomposition and the Yule-Walker method. Many of these algorithms lead to a large computation burden and are rather numerically time consuming.

Fast determination of parameters of the fundamental waveform of voltages and currents is essential for the control and protection of electrical power systems. For this purpose various numerical algorithms have been developed, e.g. based on the Fourier and Kalman filtering [1, 2, 3]. Most of the algorithms are not fully parallel algorithms, so that the speed of processing is quite limited.

Recently, there has been a great interest in parallel algorithms and architectures, based on the methods of artificial neural networks [4-7]. Tank and Hopfield [4] show how optimization problems can be solved by highly interconnected networks of simple analog processors. They state that a consideration of such circuits provides a methodology for assigning function to anatomical structure in real neural circuits. They also illustrate the use of the neural networks for signal processing problems. Kennedy

and Chua [5] extend the model proposed by Tank and Hopfield to the general nonlinear problem.

The purpose of this paper is to present new algorithms and along with them new architectures of analogue neuron-like adaptive processors for online estimation of parameters of sinusoidal signals, which are distorted by higher harmonics and corrupted by noise. For steady-state conditions we have developed neural networks which enable us to estimate the amplitudes and the frequency of the fundamental component of signals. When estimating the basic waveform of currents during short circuits the exponential DC component distorts the results. Assuming the known frequency, we have developed adaptive neural networks which enable us to estimate the amplitudes of the basic components as well as the amplitudes and the time constant of a DC component. The problem of estimation of signal parameters is formulated as an unconstrained optimization problem and solved by using the gradient descent continuous-time method [7]. Basing on this approach we have developed systems of nonlinear differential equations that can be implemented by analog adaptive neural networks. The solution of the optimization problem bases on some principles given by Tank and Hopfield [4] as well as by Kennedy and Chua [5]. The developed networks contain elements which are similar to the adaptive threshold elements of the perceptron presented by Widrow in [6].

2 Statement of the problem

Consider the following sinusoidal signal distorted by a DC exponential component:

$$x(t) = X_a \sin(\omega t) + X_b \cos(\omega t) + X_c \exp(-X_d t) \quad (1)$$

in which

- X_a, X_b are the amplitudes of the sinusoidal signal
- $\omega = 2\pi f$ where f is the frequency (50 or 60 Hz)
- X_c, X_d are the parameters of the DC component.

Let $y(t)$ denote the noise-corrupted measurement of $x(t)$, i.e.

$$y(t) = x(t) + e(t) \quad (2)$$

where $e(t)$ is the error. This error includes higher harmonics, random noise and distortion caused, for example, by measurement instruments.

Consider the practical case where the signal of interest $y(t)$ is measured during a finite duration of time and only N samples of this signal $y(t)|_{t=mT} = y(mT) = y_m$, are available. Hence, the error $e_m = e(mT)$ at the moment $t = mT$ can be expressed as

$$e_m = y_m - x_m \quad (3)$$

where

$$x_m = x(mT), \text{ and } T \text{ is the sampling interval.}$$

We are looking for an on-line algorithm which can provide the desired parameters on the basis of data samples y_m . To formulate the problem we must construct an appropriate energy function $E(\mathbf{X})$, where \mathbf{X} is the vector of the estimated parameters. The lowest energy state will correspond to the desired solution.

In general, the optimization problem can be formulated as follows:

"find a vector \mathbf{X} which minimizes the scalar energy function"

$$E(\mathbf{X}) = \sum_{m=1}^N \sigma_m[e_m(\mathbf{X})] \quad (4)$$

where $\sigma_m[e_m(\mathbf{X})]$ represents a suitably chosen loss function.

In practice, the following cases have special importance [7, 8, 9, 10]:

1. for $\sigma_m[e_m] = |e_m|$ the estimation problem is referred as the least absolute value (L_1 -norm) signal model fitting;
2. for $\sigma_m[e_m] = e_m^2$ we obtain the standard least-squares (L_2 -norm) optimization problem;
3. taking $\sigma_m[e_m] = k_m e_m^2$, with $k_m > 0$, we have a well-known weighted least-squares problem;
4. for the loss function $\sigma_m[e_m] = (1/\gamma) \ln\{\cosh(\gamma e_m)\}$ we obtain iteratively reweighted the least-squares problem, also called the robust least-squares criterion;
5. for $E(\mathbf{X}) = \max_{1 \leq m \leq N} \{|e_m|\}$ the optimization problem is minimax (L_∞ - or Chebyshev norm) model fitting.

The proper choice of the optimization criterion used depends on the distribution of the noise error in the sampled data. The standard least-squares criterion is optimal for a normal (Gaussian) distribution of the noise. Often, the signals of voltages and currents encountered in power systems are notoriously contaminated by impulsive noise and large isolated errors (outliers) caused by malfunctioning of some sensors or transient components. To reduce the influence of the outliers we can use the iteratively reweighted least-squares criterion. In the presence of large impulsive noise, an alternative approach is to use the least absolute value criterion. On the other hand, the minimax criterion is an appropriate to be used if the errors are uniformly distributed and the samples are relatively free from outliers.

3.

3 Estimation under steady-state condition

The frequency in electrical power systems can change over a small range due to generation-load mismatches. Some power system protection and control applications require accurate and fast estimates of the frequency. Most digital techniques for on-line measuring frequency have acceptable accuracy if the voltage waveforms are not distorted. On the other hand, under steady-state conditions we don't expect any exponential DC component. Thus, in this section we will develop adaptive neural networks for estimation of the amplitudes X_a, X_b and the angular frequency ω of sinusoidal signals distorted by random noise and harmonics, assuming $X_c = 0$.

Up to now, the L_1 - and L_∞ -norm optimization criteria have seldom been used for parameter estimation, probably because their nondifferentiability causes numerical and analytical

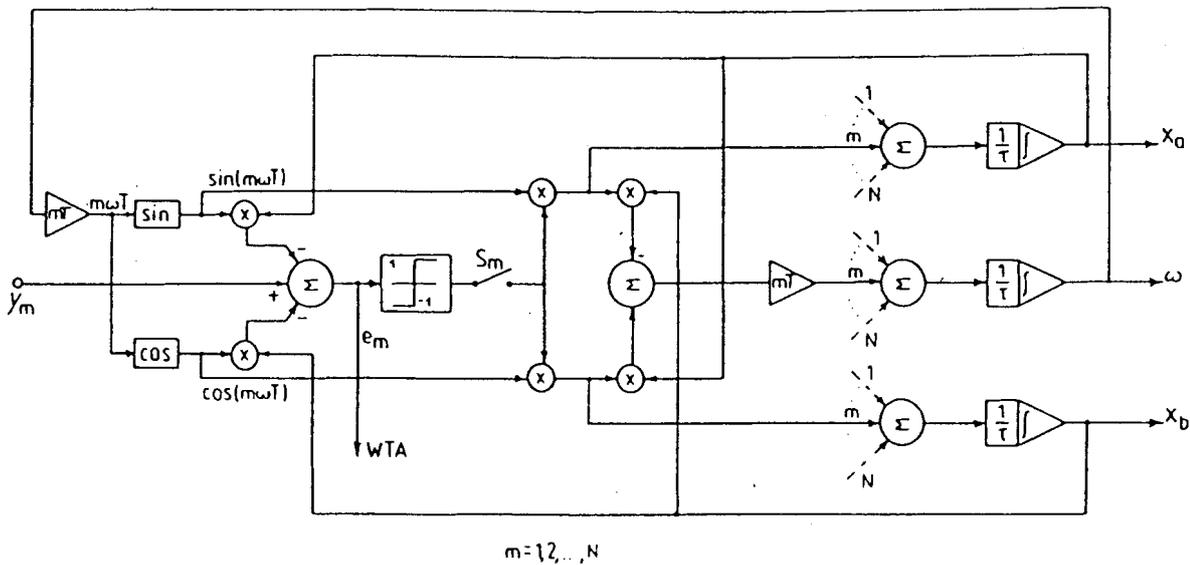


Fig. 1. Artificial neural network for estimation the amplitudes and the frequency of noisy sinusoidal signals

difficulties. Fortunately, the minimax and the least absolute value optimization problems can be easily reformulated as equivalent differentiable optimization problems and implemented by using artificial neural networks.

Minimax criterion

The minimax estimation problem can be reformulated as follows:
 "find a vector of parameters X which minimizes the energy function"

$$E(X_a, X_b, \omega) = \max_{1 \leq m \leq N} \{|e_m(X_a, X_b, \omega)|\} \quad (8)$$

Using the steepest descent continuous-time optimization algorithm we obtain the set of nonlinear equations [7, 10]:

$$\frac{dX_a}{dt} = \frac{1}{\tau} \sum_{m=1}^N S_m \text{sign}(e_m) \sin(m\omega T) \quad (9)$$

$$\frac{dX_b}{dt} = \frac{1}{\tau} \sum_{m=1}^N S_m \text{sign}(e_m) \cos(m\omega T) \quad (10)$$

$$\frac{d\omega}{dt} = \frac{1}{\tau} \sum_{m=1}^N S_m m T \text{sign}(e_m) [X_a \cos(m\omega T) - X_b \sin(m\omega T)] \quad (11)$$

where the coefficient τ represents the time constant of integrators, and

$$S_m = \begin{cases} 1 & \text{if } |e_m| = \max\{|e_i|\} \\ 0 & \text{otherwise.} \end{cases}$$

The set of differential equations can be implemented by a neuron-like network shown in Fig. 1. The network consists of basic computing units: integrators, summers, multipliers, signum activation functions and trigonometric functions generators. The switches S are controlled by a special subnetwork called Winner-Take-All (WTA) circuit. The function of the WTA is to select the largest in absolute value instantaneous error. The sign of the selected error is transmitted for further processing, while the other error signals are completely inhibited by opening corresponding switches.

Least absolute value criterion

The network shown in Fig. 1 can easily be modified to perform the parameters estimation according to L_1 - and L_2 -norm criteria. By closing all the switches S or by removing them and the associated WTA circuit, the network will act according to the least absolute value criterion, realizing the set of differential equations:

$$\frac{dX_a}{dt} = \frac{1}{\tau} \sum_{m=1}^N \text{sign}(e_m) \sin(m\omega T) \quad (12)$$

$$\frac{dX_b}{dt} = \frac{1}{\tau} \sum_{m=1}^N \text{sign}(e_m) \cos(m\omega T) \quad (10)$$

$$\frac{d\omega}{dt} = \frac{1}{\tau} \sum_{m=1}^N mT \text{sign}(e_m) [X_a \cos(m\omega T) - X_b \sin(m\omega T)] \quad (11)$$

Least-squares criterion

In order to estimate parameters according to the least-squares criterion all signum activation functions must be replaced by linear functions, and all switches S must be closed or removed. In this case the neural network can be described by a system of differential equations:

$$\frac{dX_a}{dt} = \frac{1}{\tau} \sum_{m=1}^N e_m \sin(m\omega T) \quad (12)$$

$$\frac{dX_b}{dt} = \frac{1}{\tau} \sum_{m=1}^N e_m \cos(m\omega T) \quad (13)$$

$$\frac{d\omega}{dt} = \frac{1}{\tau} \sum_{m=1}^N mT e_m [X_a \cos(m\omega T) - X_b \sin(m\omega T)] \quad (14)$$

Computer simulation

Extensive computer simulation experiments have confirmed that the neural network shown in Fig. 1 allows us to estimate in real-time desired parameters of noisy sinusoidal signals. Owing to limited space, we shall present only some illustrative results. For all examples presented in this paper, we have chosen the following parameters: the number of samples $N = 30$, the integration time constant for all three integrators was $\tau = 20 \cdot 10^{-8} \text{ s}$ for the L_1 - and L_2 -norm, and $\tau = 2 \cdot 10^{-8} \text{ s}$ for the L_∞ -norm. Let us consider a sinusoidal signal $x(t) = 140 \sin(\omega t) + 60 \cos(\omega t)$, $\omega = 100\pi$, contaminated by uniformly distributed noise. Fig. 2 shows the trajectories of estimated parameters for the sampling window $NT = 30 \text{ ms}$ (sampling frequency $f_s = 1000 \text{ Hz}$) and the noise level of 2%. The figure shows that the trajectories of the estimated parameters X_a and X_b converge to almost the same values, independent of the criterion used. The best results have been obtained using the minimax criterion (Figs. 3 and 4a). In the presence of higher harmonics the L_2 -norm shows the best accuracy (Fig. 3b).

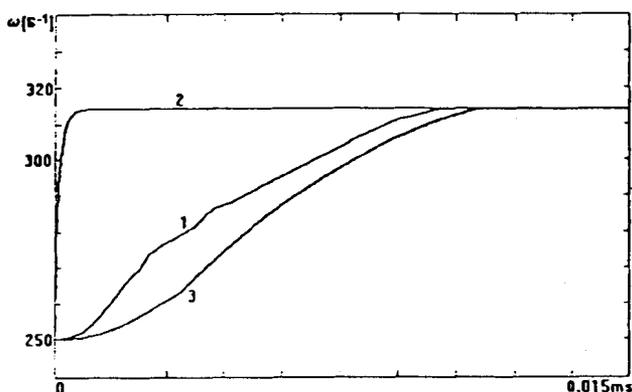
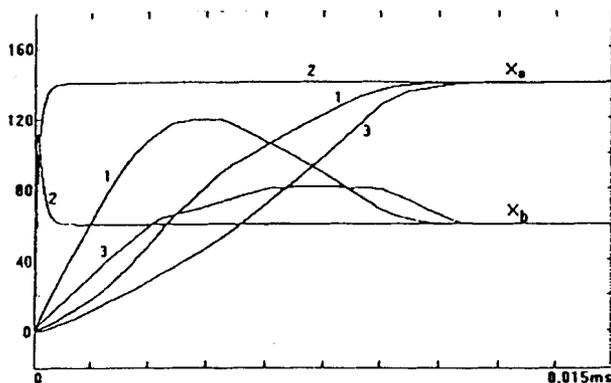


Fig. 2. Computer simulated state trajectories of estimated parameters of a signal $x(t) = 140 \sin(\omega t) + 60 \cos(\omega t)$ corrupted by noise (2%), using: 1) L_1 -norm, 2) L_2 -norm, 3) L_∞ -norm; $N = 30$, $NT = 30 \text{ ms}$.

4 Estimation of the parameters under short-circuit condition of short-circuit currents

During short circuit the waveform of currents can be additionally distorted by an exponential DC component. For the application the sinusoidal signal model has to be extended with an exponential term. We have assumed that at the beginning of a short circuit the frequency remains constant. In this section we shall present adaptive neural networks which enable us to estimate the amplitudes X_a and X_b of the basic waveform as well as the amplitude X_c and the time constant $T_t = 1/X_d$ of the exponential component.

Robust least-squares criterion

The criterion is preferable if additive impulsive noise is expected.

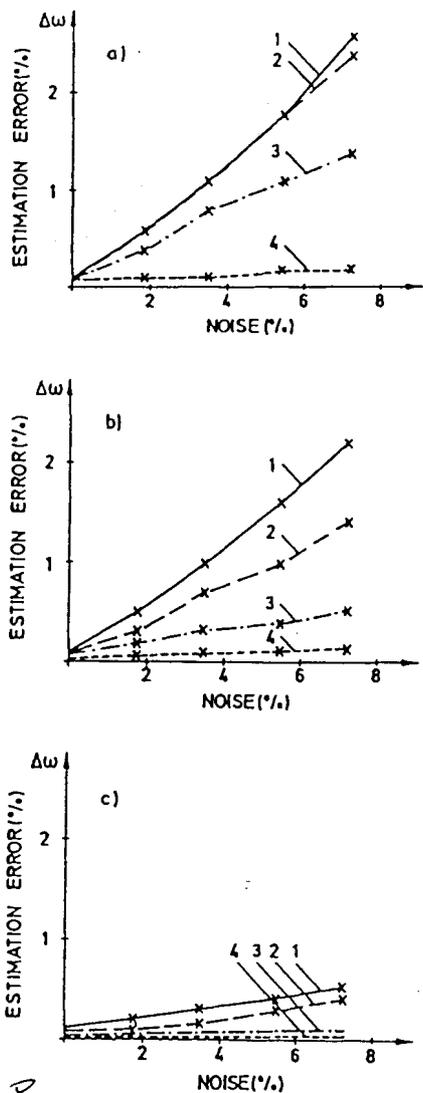


Fig. 8. Errors of the frequency estimation using: a) L_1 - norm, b) L_2 - norm, c) L_∞ - norm criterion; $N = 30$; sampling window: 1) 10 ms ($f_s = 3000$ Hz), 2) 15 ms ($f_s = 2000$ Hz), 3) 20 ms ($f_s = 1500$ Hz), 4) 30 ms ($f_s = 1000$ Hz)

Using the loss function shown in the Section 2 we determine the first derivative of this function with respect to the unknown parameters X_a, X_b, X_c, X_d . The function can be minimized with respect to each parameter by implementing a dynamic gradient system [7]; *which*

$$\frac{dX}{dt} = -\frac{1}{\tau} \nabla E(X), \tau > 0 \quad (15)$$

where τ is the integration time constant and

$$\nabla E(X) = \begin{bmatrix} \frac{\partial E(X)}{\partial X_a} & \frac{\partial E(X)}{\partial X_b} & \frac{\partial E(X)}{\partial X_c} & \frac{\partial E(X)}{\partial X_d} \end{bmatrix}^T$$

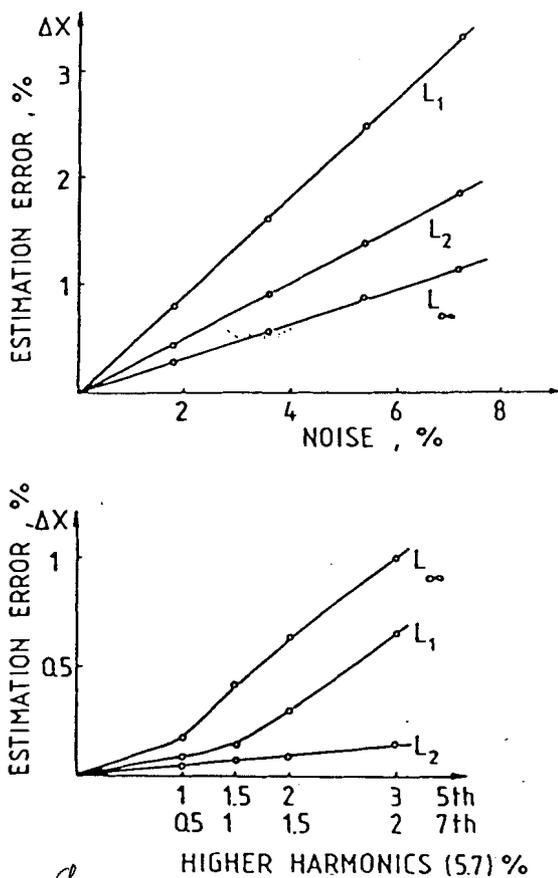


Fig. 9. Errors of the amplitude estimation; signal contaminated by a) uniformly distributed noise, and b) 5 th and 7 th harmonics; $N = 30$; $NT = 20$ ms

The system of eqns. (15) represents the known continuous-time steepest descent algorithm. The gradient system can be rewritten in a scalar form, as a system of differential equations [9]:

$$\frac{dX_a}{dt} = \frac{1}{\tau} \sum_{m=1}^N \epsilon_m \sin(m\omega T) \quad (16)$$

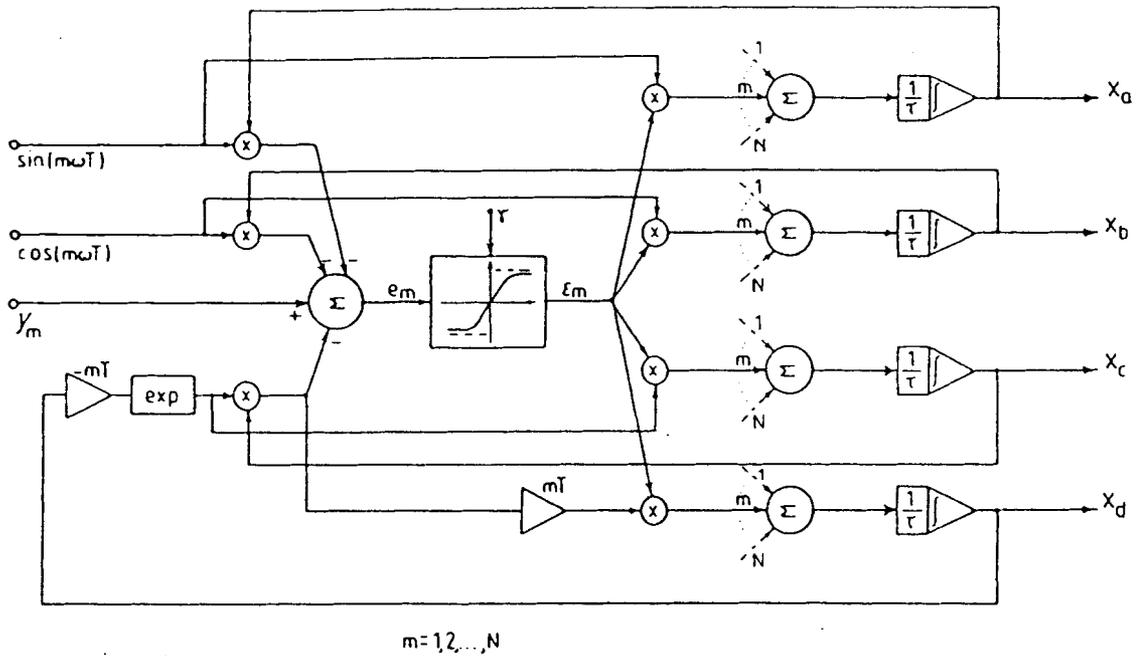
$$\frac{dX_b}{dt} = \frac{1}{\tau} \sum_{m=1}^N \epsilon_m \cos(m\omega T) \quad (17)$$

$$\frac{dX_c}{dt} = \frac{1}{\tau} \sum_{m=1}^N \epsilon_m \exp(-X_d m T) \quad (18)$$

$$\frac{dX_d}{dt} = -\frac{1}{\tau} \sum_{m=1}^N \epsilon_m X_c m T \exp(-X_d m T) \quad (19)$$

where $\epsilon_m = \tanh(\gamma \epsilon_m)$.

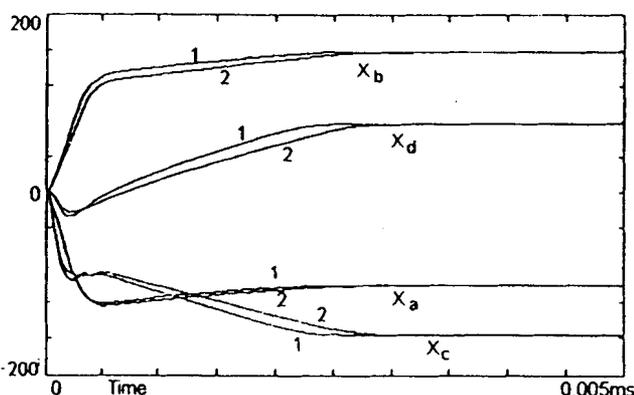
The system of differential equations can be implemented by a neuron-like, adaptive analogue processor shown in Fig. 10. Each channel consists of a sigmoidal function (hyperbolic tangent) generator and exponential function generator. The slope of the sigmoid function depends on the



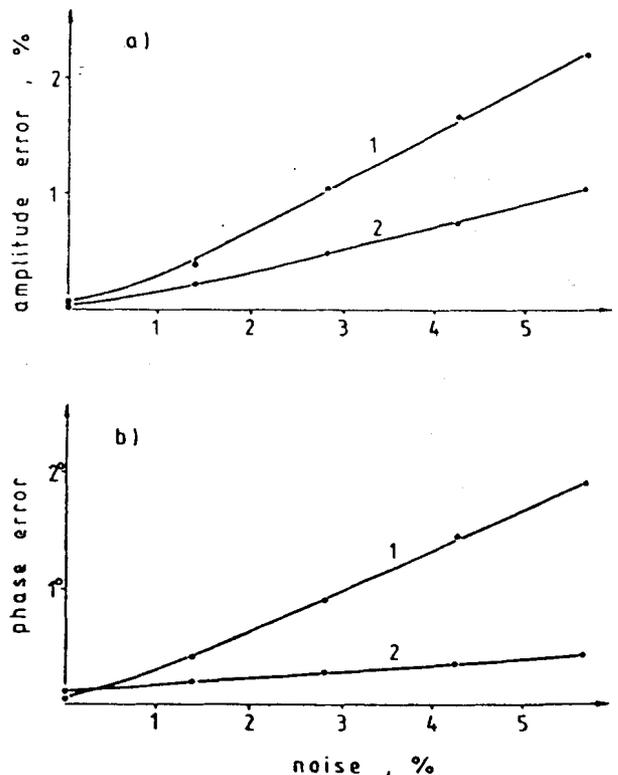
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Fig. 5. Artificial neural network for estimation of parameters of sinusoidal signals with exponential DC components, using the robust least-squares criterion

parameter γ . If γ is small, say less than 0.1, the hyperbolic tangent can be approximated by its argument. The sigmoid function is almost linear in a wide range, and the network acts according to the standard least-squares criterion. If the parameter γ is large, say greater than 1000, then the sigmoid function approximates the hard limiter (signum function) and the network is able to solve the problem according to the least-absolute value criterion.

We have simulated the network on computer and extensively tested for a variety of sinusoidal signals corrupted by noise and distorted by exponential DC components. The simulations fully confirmed correctness of the presented approach. For the results presented in Figs. 6 and 7 we have chosen



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Fig. 6. Trajectories of estimated parameters of the signal: $x(t) = -100 \sin \omega t + 160 \cos \omega t - 160 \exp(-80t)$ contaminated by white noise (2%) and additive impulsive noise, without (1) and with wild noise (2), $\gamma = 1$



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Fig. 7. Errors of the amplitude (a) and phase (b) estimation by using the network in Fig. 5; $N = 30$, $\gamma = 1$, sampling window: 1) 15 ms, 2) 30 ms

the number of samples $N = 30$ and the integration time constant for all integrators $\tau = 2 \times 10^{-8}$ s. Fig. 11 shows trajectories of the estimated parameters for the signal without and with outliers. The figure illustrates that the trajectories converge to almost the same values, independent of impulsive noise.

Minimax criterion

The optimization problem can be transformed into an equivalent differentiable minimization problem:

"minimize X_0 subject to $-X_0 \leq e_m \leq X_0$, where $X_0 \geq 0$ "

The optimal value of X_0 is simultaneously the minimum of the energy function. By applying the standard penalty function approach [7] the constrained minimization problem can be mapped into an unconstrained problem:

"minimize $E_\infty(X)$, where"

$$E_\infty(X) = vX_0 + \frac{k}{2} \sum_{m=1}^N [\min\{0, e_m + X_0\}]^2 + \frac{k}{2} \sum_{m=1}^N [\max\{0, e_m - X_0\}]^2 \quad (20)$$

$v > 0; k > 0$ are penalty terms (typically $v = 1, k = 10$).

By applying the gradient strategy i.e. the steepest descent continuous time algorithm [7], we obtain a gradient system:

$$\frac{dX_0}{dt} = -\frac{1}{\tau_0} \left(v - k \sum_{m=1}^N |g_m(e_m, X_0)| \right) \quad (21)$$

$$\frac{dX_i}{dt} = -\frac{1}{\tau} \sum_{m=1}^N g_m(e_m, X_0) \frac{\partial e_m}{\partial X_i} \quad (22)$$

for $i = a, b, c, d$; where $\tau_0 > 0, \tau_i > 0$

$$g_m(e_m, X_0) = \begin{cases} e_m - X_0 & \text{if } e_m > X_0 \\ 0 & \text{if } -X_0 \leq e_m \leq X_0 \\ e_m + X_0 & \text{if } e_m < -X_0 \end{cases}$$

Taking into account (1) the gradient system (21, 22) can be rewritten as a system of differential equations:

$$\frac{dX_0}{dt} = -\frac{1}{\tau_0} \left\{ \frac{v}{k} - \sum_{m=1}^N [(e_m + X_0)S_{m1} - (e_m - X_0)S_{m2}] \right\} \quad (23)$$

$$\frac{dX_a}{dt} = \frac{1}{\tau} \sum_{m=1}^N [(e_m + X_0)S_{m1} + (e_m - X_0)S_{m2}] \sin(m\omega T) \quad (24)$$

$$\frac{dX_b}{dt} = \frac{1}{\tau} \sum_{m=1}^N [(e_m + X_0)S_{m1} + (e_m - X_0)S_{m2}] \cos(m\omega T) \quad (25)$$

$$\frac{dX_c}{dt} = \frac{1}{\tau} \sum_{m=1}^N [(e_m + X_0)S_{m1} + (e_m - X_0)S_{m2}] \exp(-X_0 m T) \quad (26)$$

$$\frac{dX_d}{dt} = \frac{1}{\tau} \sum_{m=1}^N [(e_m + X_0)S_{m1} + (e_m - X_0)S_{m2}] \cdot X_0 (-mT) \exp(-X_0 m T) \quad (27)$$

where $S_{m1} = \begin{cases} 1 & \text{if } e_m < -X_0 \\ 0 & \text{otherwise} \end{cases}$

$S_{m2} = \begin{cases} 1 & \text{if } e_m > X_0 \\ 0 & \text{otherwise} \end{cases}$

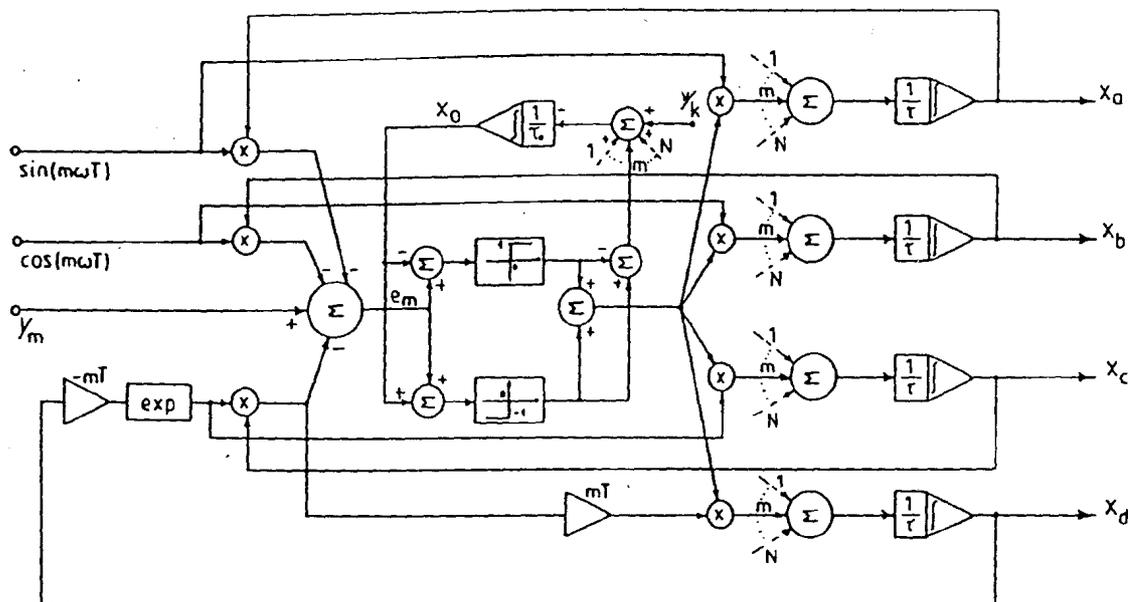
The system of differential equations can be implemented by an analogue neural network whose functional block diagram is shown in Fig. 8. The network has also been simulated on computer and extensively tested. Good agreement with theoretical considerations has been obtained. As an example Fig. 14 shows the trajectories of estimated parameters of a sinusoidal signal distorted by an exponential DC component. *and Fig. 15 errors of the amplitude estimation for*

5 Conclusions

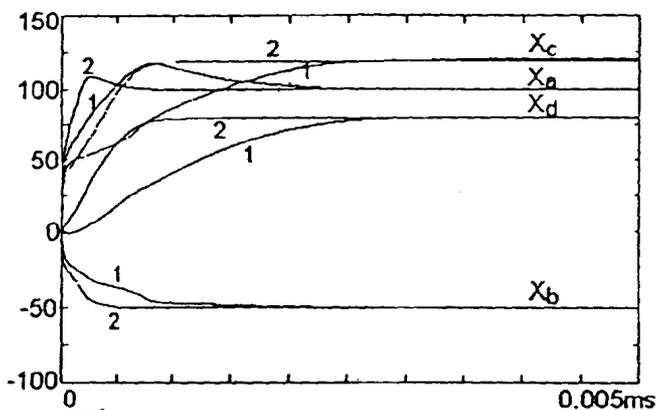
Adaptive analogue neural networks represent a very promising approach for high-speed estimation of parameters of signals. In this paper new algorithms and architectures of neuron-like adaptive circuits have been proposed. The algorithms for steady-state conditions enable us to estimate the amplitudes and the frequency of the fundamental component of voltages and currents. The algorithms for short-circuit conditions allow us to estimate the amplitudes of the basic component and the parameters of a DC exponential component of currents. The choice of a proper network also depends on the expected distribution of noise in the measured signal. Extensive computer simulation experiments confirmed the validity and performance of the proposed algorithms.

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 Fig. 8. Artificial neural network for estimation of parameters of sinusoidal signals with exponential DC components, by using the minimax criterion



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 Fig. 9. Computer simulated trajectories of estimated parameters of the signal:

$x(t) = 100 \sin \omega t - 50 \cos \omega t + 120 \exp(-80t)$ corrupted by noise (2%). $N = 20$, sampling window: 1) 20 ms, 2) 40 ms

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