

Application of Spectral Analysis to Detection of Space Charge Distribution in Solid Dielectrics

Czesław Stec and Edmund Motyl

Abstract — The paper deals with the signal processing of ultrasonic pulses to obtain information on space charge distribution in solid dielectrics. The linear theory based on the Fourier transform is applied to the study of ultrasonic wave propagation along the measuring path. Application of spectral method simplifies a signal analysis and enables description of the pulse and step responses accurately. The compiled computer procedures, which include FFT computer programs permit direct processing of measured signals that are stored in memory of oscilloscope.

I. INTRODUCTION

The pulsed electroacoustic (PEA) is one of the most effective methods for measuring the space charge distribution in solid dielectrics. The method was originated by Maeno and others [1]. Now the PEA method is applied for investigation of electroinsulating materials, cables, electrets and active dielectrics [2], [3], [4], [5].

The measuring path in PEA method consists of dielectric sample, acoustic delay line and acoustic sensor. The idea of the method is based on the generation of local forces after application of short pulsed voltage to a dielectric with the accumulated space charge inside. The forces cause the local deformation of a material and generate local strains. The strain wave is related to convolutional integral of electrical field and space charge distribution. The strain wave propagates towards piezoelectric sensor placed on its way. Piezoelectric sensor converts the transient acoustic signal to electrical signal, which is recorded by digital storage oscilloscope.

The electrical signal - induced in sensor - depends on strain wave function, which is related to space charge distribution. The analysis of the measured signal allows get information on space charge distribution along the thickness of a dielectric. In this work the spectral analysis is applied to detect the distribution function.

II. THEORY OF THE METHOD

Consider parallel-plate sample of dielectric slab with non-uniform space charge distribution q_v [C/m³] along the sample thickness d [μm] as shown in Fig. 1.

Authors are with the Wrocław University of Technology, Institute of Electrical Engineering Fundamentals (I-7), Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland. e-mail: motyl@elektryk.ie.pwr.wroc.pl

This work was supported by the Grant under contract No 4 T10C 008 22 from the State Committee of Scientific Research (KBN) Poland.

The one-dimensional space charge distribution model is used. The pulsed electrical field $E_p(t)$ [V/m], constant in space is applied to charged dielectric. The attraction of external field with space charge generates forces, which cause local displacement of a material $u(x,t)$ [m] as a result of elastic reaction of matter. This phenomenon is approximately described by known Newton equation for elastic solid in a form:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = v^2 \frac{\partial^2 u(x,t)}{\partial x^2} + \frac{q_v(x)E_p(t)}{\rho_0} \quad (1)$$

where v [m/s] = $(Y/\rho_0)^{0.5}$ is velocity of acoustic wave, ρ_0 [kg/m³] is a mass density and Y [N/m²] is Young modulus.

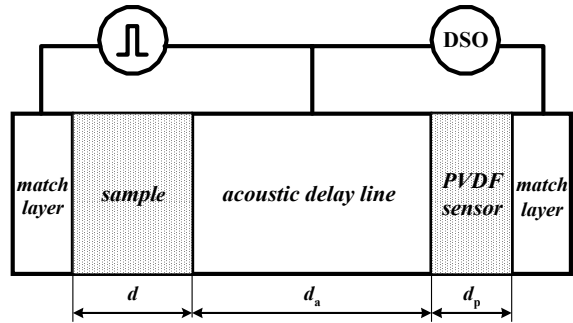


Fig. 1. Principle of measurement system.

The solution of the above wave equation at initial conditions: $u(x,0) = 0$, $u_t(x,0) = 0$ gives [6]

$$u(x,t) = \frac{1}{2v\rho_0} \int_0^t d\tau E_p(\tau) \int_{x-v(t-\tau)}^{x+v(t-\tau)} q_v(\xi) d\xi \quad (2)$$

The local deformation or strain is given by

$$S(x,t) = \frac{\partial u(x,t)}{\partial x} = \quad (3)$$

$$\frac{1}{2v\rho_0} \int_0^t E_p(\tau) [q_v(x+v(t-\tau)) - q_v(x-v(t-\tau))] d\tau$$

The strain wave generated within the sample is the sum of two superposed waves, which propagate through the thickness of the sample in opposite directions. One strain wave component is absorbed by matching layer, which is placed at left side of the sample.

The second wave component travels through the right electrode to the piezoelectric sensor. This strain wave component ($dx/dt > 0$) at the wall adjacent to the sensor ($x = 0$) is equal,

$$S_0 = -\frac{1}{2\nu\rho_0} \int_0^t E_p(\tau) q_v(-\nu(t-\tau)) d\tau \quad (4)$$

After introducing the function $g(t) = -q_v(-\nu t)/2\nu\rho_0$ for time limit $0 \leq t < d/\nu$, the relative strain at the left electrode of sensor can be described as follows,

$$S_0(t) = \int_0^t E_p(\tau) g(t-\tau) d\tau = E_p(t) * g(t) \quad (5)$$

The above equation is applied to continuous functions. In case of Dirac-like pulsed function, $E_p(t) = E_0\Delta t_0\delta(t)$, where $E_0\Delta t_0$ is the area of the electric field pulse. The pulsed response is given by equation,

$$k_0(t) = \frac{-E_0}{2\nu\rho_0} \Delta t_0 q_v(-\nu t) = g(t) \quad (6)$$

that is valid for, $0 \leq t < d/\nu$.

The above relations allow to define the spectral transfer functions of the particular elements of system in frequency domain.

III. REACTION OF THE PIEZOELECTRIC SENSOR

The short – circuit current generated in detector is

$$i_p(t) = \frac{A}{a} \int_0^a e_{33}(x) \frac{\partial S}{\partial t} dx \quad (7)$$

The strain $S_0(t)$ at left wall of detector is the source of pressure wave and determines the boundary condition. The strain wave in detector is $S(x, t) = S_0(\nu_0 t - x)$. Considering this equation and homogeneity of material, one obtains the short-current response of sensor

$$i_p(t) = \frac{Ae_{33}}{a} \int_0^a \frac{\partial S_0(\nu_p t - x)}{\partial t} dx \quad (8)$$

After successively transformations we have

$$i_p(t) = \frac{Ae_{33}}{a} (-\nu_p) \int_{\nu t}^{\nu t - a} \frac{\partial S_0(x)}{\partial x} dx = \frac{Ae_{33}\nu_p}{a} [S_0(\nu_p t) - S_0(\nu_p t - a)] \quad (9)$$

Indicating the transit time of strain wave in sensor as $\Delta\tau_p = a/\nu_p$, one obtains the resultant convolutional relation

$$i_p(t) = \frac{Ae_{33}\nu_p}{a} [S_0(\nu_p t) - S_0(\nu_p(t - \Delta\tau_p))] = \frac{Ae_{33}\nu_p}{a} S_0(\nu_p t) * [\delta(t) - \delta(t - \Delta\tau_p)] \quad (10)$$

Expressing formally the Dirac impulse function as derivative of unit step function $\delta(t) = [1(t)]'$, so the relation in square brackets is of pulse function $k(t) = 1(t) - 1(t - \Delta\tau_p)$ with pulse width $\Delta\tau_p$. Considering the above relations, one obtains the sensor response in

form of convolutional integrals,

$$i_p(t) = \frac{Ae_{33}\nu_p}{a} S_0(\nu_p t) * [\delta(t) - \delta(t - \Delta\tau_p)] = \frac{Ae_{33}\nu_p}{a} S_0(\nu_p t) * k'(t) = \frac{Ae_{33}\nu_p}{a} S_0'(\nu_p t) * k(t) \quad (11)$$

The pulsed response of piezoelectric sensor evaluated from Eq. (11) is

$$h(t) = \frac{Ae_{33}\nu_p}{a} (k(t))' \quad (12)$$

and depends on thickness a and velocity of strain wave ν_p in sensor. The above form of equations concerns the thin as well the thick sensor, on condition of its acoustic matching (lack of oblique waves). The obtained relations are convenient for transformation in frequency domain.

IV. SPECTRAL ANALYSIS

From the above considerations the block-diagram of system shown in Fig. 1 can be built. It is shown in Fig. 2 and the transfer functions of particular segments are given as follows:

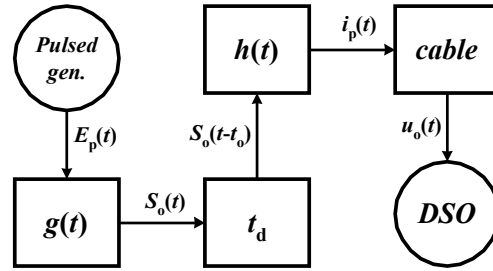


Fig. 2. Block-diagram of the measurement system.

a) The transfer function of electrical field – strain for sample,

$$T_{ES}(j\omega) = \frac{S_0(j\omega)}{E_p(j\omega)} = G(j\omega) = F[g(t)] \quad (13)$$

b) The transfer function of delay time for electrode,

$$T_{SS}(j\omega) = \frac{F[S_0(t-t_d)]}{F[S_0(t)]} = \exp(-j\omega t_d) \quad (14)$$

c) The transfer function of strain – current for sensor,

$$T_{SI}(j\omega) = \frac{I_p(j\omega)}{S_0(j\omega)} = H(j\omega) = F[h(t)] \quad (15)$$

d) The transfer function of current – voltage for cable,

$$T_{IU}(j\omega) = \frac{U_0(j\omega)}{I_p(j\omega)} = Z(j\omega) \quad (16)$$

where $Z(j\omega)$ is wave impedance of cable loaded with input impedance of oscilloscope.

V. RESULTS

A. Simulation of charge distribution

The one dimensional space charge distribution $q_v(x)$ along the sample thickness as shown in Fig. 3 was considered. The thickness dependence was converted to time according to the relation $x = vt$. It was supposed that every component of the acoustic wave propagates through the dielectric with the same velocity without decrease in amplitude during the propagation. By the Fourier transform the space charge distribution in frequency domain can be converted to frequency domain. The Fourier transform of space charge distribution $q_v(t)$ is shown as $Q_v(\omega)$ in Fig. 4.

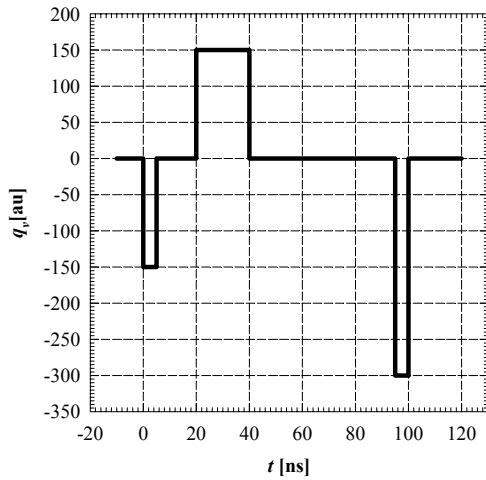


Fig. 3. Simulation of space charge distribution.

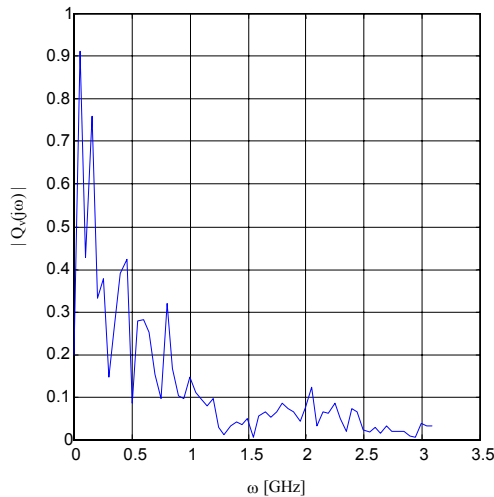


Fig. 4. Transfer function of space charge distribution.

All computations were performed with Matlab®. The algebra of complex variables and FFT procedures allow to formulate transfer functions in frequency domain for the above block-diagram. The transfer function computed for piezoelectric sensor is shown in Fig. 5.

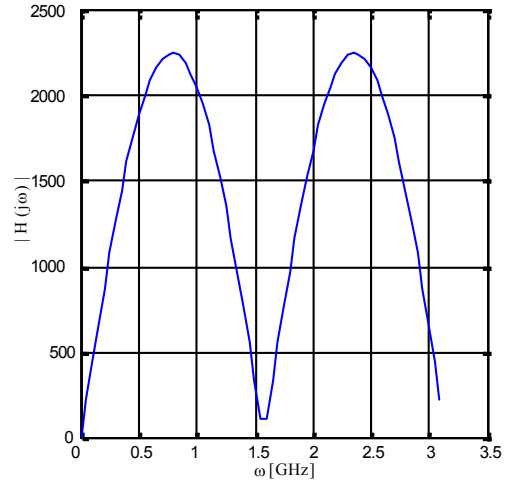


Fig. 5. Transfer function $H(\omega)$ of piezoelectric sensor.

Polivinyldene fluoride (PVDF) polymeric film was used as a piezoelectric sensor. The acoustic transit time is about 4.1 ns for 9 μm PVDF film used in our laboratory. The acoustic velocity of a strain wave in a sensor was assumed $v_p = 2.2 \mu\text{m/ns}$.

The transfer function of cable with characteristic impedance 50Ω is shown in Fig. 6. The coaxial cable was 1 m long and was loaded by input resistance 50Ω and input capacitance 20 pF of oscilloscope.

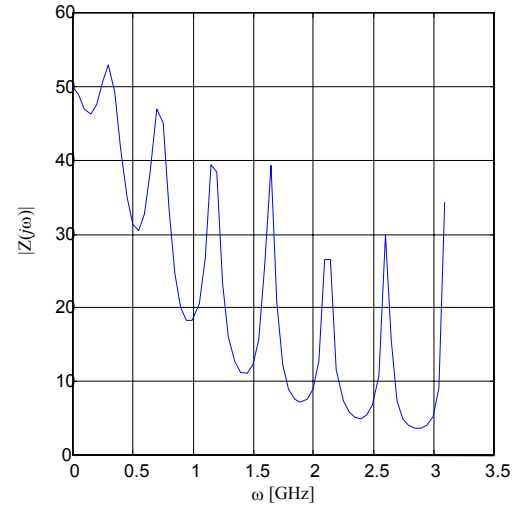


Fig. 6. Current – voltage transmittance of cable connected to oscilloscope.

Equations 13 – 16 were used for computed the spectral characteristic of output signal that is measured by oscilloscope. The amplitude of the above signal is shown in Fig. 7. The output signal recorded by oscilloscope is computed using inverse FFT. It is shown in Fig. 8. The measured signal is distorted on its propagation path from sample to oscilloscope as it can be seen by comparison of Fig. 8 and Fig. 3.

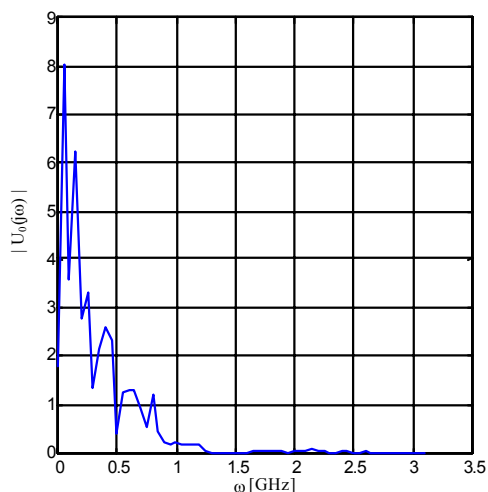


Fig. 7. Spectral amplitude of measured signal.

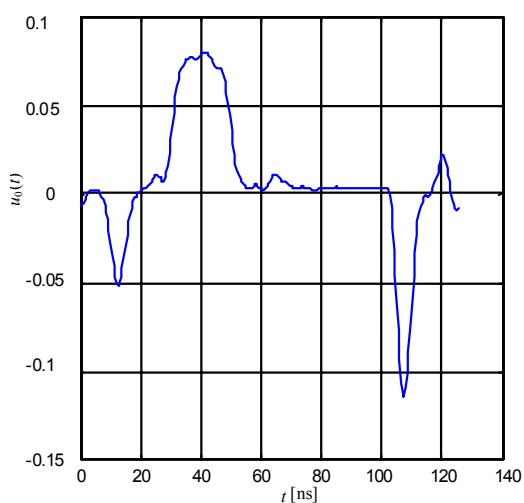


Fig. 8. Signal measured by oscilloscope.

B. Identification of space charge distribution

The inverse problem was examined in this part of paper. The acoustic signal is measured using electroacoustic method and information on space charge distribution along the sample thickness is searched. For solving this problem the Matlab® program files were copying using data recorded by digital oscilloscope. From the above data the FFT transformations were created. Procedures of function shifting, interpolation and change of sampling rate were realised. The final result was spectral function of space charge distribution. The inverse FFT of the above result gave the investigated run of the space charge distribution in time domain. To obtain the spatial dependence of charge distribution, the horizontal time axis have to be converted into thickness according to the relation $x = vt$. The another calibration procedure has to be applied in order to calibrate vertical axis in space charge density $[C/m^3]$.

Well-defined requirements should be fulfilled for application of electroacoustic (EA) methods for investigation of thin samples: 30-150 μm in thickness.

The transit time can be estimated as 20 ns to 100 ns if acoustic wave velocity is approximately equal 2.2 $\mu m/ns$. The oscilloscope with at least 500 MHz bandwidth scope is needed to obtain resolution 1/50 and the parameters of measuring line should be precisely chosen.

VI. CONCLUSIONS

Application of spectral method simplifies a signal analysis and enables description of the pulse and step responses accurately. The compiled computer procedures, which include FFT computer programs permit direct processing of measured signals that are stored in memory of oscilloscope. Effectiveness of applied computer method was verified. It concerns the accuracy matching of the elements along the measuring path and the relation between measuring signal in time and space charge distribution along the sample thickness by spectral transmittance. The effects of sensor capacitance and input oscilloscope capacitance on time constant of measuring line were estimated by iterative simulation. The noises generated in the system can be reduced by application of well-chosen low band-pass frequency filter.

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