ARCHIVES OF ELECTRICAL ENGINEERING

VOL. LIII No. 2, pp. 121–130 2004

A current flow problem for amorphous solids. A case of weak and mean electric fields

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Manuscript received 2002.09.23, revised version 2004.04.08

A space charge transport problem for a planar capacitor system is presented. For a solid placed between two electrodes it is assumed that the carrier generation and recombination processes are independent of an external electric field. Some singular cases of the current flow trough the system are considered. It is found that the system can act as an almost perfect blocking diode (or a voltage stabiliser), and also, it is found that the negative differential resistance is a singularity of space charge transport through the system.

Keywords: solid, negative resistance, space charge, trapping levels

1. INTRODUCTION

One of the problemes of a space charge transport in a planar capacitor system is to define the interior and the boundary conditions [1–5]. Generally, the interior of a system (the bulk) is characterised by interactions between carriers under conditions of an external electric field. In order to define the electric field corresponding to the given current-voltage characteristic we have to give the boundary conditions describing carrier injection from the electrode into a solid. A current-voltage characteristic can express the electrical properties of the planar capacitor system.

In this paper we will define some new condition for a negative differential resistance as well as we will find some condition for a blocking diode.

2. MATHEMATICAL MODEL

In this work, we will consider the interior of a planar capacitor system in which the trapping levels will be grouped into four energy levels. To this end, the so-called effective parameters such as the frequency parameters c_{21} and c_{t12} as well as the recombination parameters c_{12} and c_{t21} will be used. For the trapped electrons, the concentrations of traps in the first and second trapping level will be represented by N_{t1}

and N_{t2}, respectively. Analogously, for the trapped holes, the concentrations of traps in the first and second trapping level will be equal toP_{t1} and P_{t2}, respectively. The solid will be treated as an unlimited reservoir of traps, that is $P_{t1} \gg p_{t1}$; $P_{t2} \gg p_{t2}$; $N_{t1} \gg n_{t1}$ and $N_{t2} \gg n_{t2}$. The electrodes of a capacitor system will be represented by the anode $x = 0$ and the cathode $x = L$, where L denotes the distance between the electrodes. Moreover, we will assume that the contact voltages at the anode and cathode as well as the diffusion current are negligible [6–7]. For a solid, we will assume that the polarisation effect is characterised by the dielectric constant ε and that the mobilities of free carriers are independent of the electric field intensity E [8–10]. Under these conditions, the basic equations such as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral are written as follows:

$$\frac{\varepsilon}{q}\frac{\partial E(x,t)}{\partial x} = p(x,t) + p_{t1}(x,t) + p_{t2}(x,t) - [n(x,t) + n_{t1}(x,t) + n_{t2}(x,t)]\,, \tag{1}$$

$$\begin{aligned} &\frac{\partial}{\partial x}\left\{\left[\mu_n n(x,t) + \mu_p p(x,t) + \mu_{t1} n_{t1}(x,t) + \mu_{t2} n_{t2}(x,t)\right] E(x,t)\right\} + \frac{\partial p(x,t)}{\partial t} + \\ &\quad + \frac{\partial p_{t1}(x,t)}{\partial t} + \frac{\partial p_{t2}(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0, \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{\partial n(x,t)}{\partial t} &= \nu_{t1} P_{t1} + \nu_{n1} n_{t1}(x,t) - c_n N_{t1} n(x,t) - C_r n(x,t) p_{t1}(x,t) \\ &\quad + \frac{\partial}{\partial x}\left[\mu_n n(x,t) E(x,t)\right], \end{aligned} \tag{3}$$

$$\frac{\partial p_{t1}(x,t)}{\partial t} = \nu_{t1} P_{t1} + c_{t21} P_{t1} p_{t2}(x,t) - c_{t12} P_{t2} p_{t1}(x,t) - C_r n(x,t) p_{t1}(x,t), \tag{4}$$

$$\frac{\partial p_{t2}(x,t)}{\partial t} = c_{t12} P_{t2} p_{t1}(x,t) - c_{t21} P_{t1} p_{t2}(x,t) - \nu_{t2} p_{t2}(x,t) + c_{t2} P_{t2} p(x,t), \tag{5}$$

$$\begin{aligned} \frac{\partial n_{t1}(x,t)}{\partial t} &= c_{21} N_{t1} n_{t2}(x,t) - \nu_{n1} n_{t1}(x,t) - c_{12} N_{t2} n_{t1}(x,t) + c_n N_{t1} n(x,t) \\ &\quad + \frac{\partial}{\partial x}\left[\mu_{t1} n_{t1}(x,t) E(x,t)\right], \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\partial n_{t2}(x,t)}{\partial t} &= \nu_{p2} N_{t2} + c_{12} N_{t2} n_{t1}(x,t) - c_{21} N_{t1} n_{t2}(x,t) - C_p p(x,t) n_{t2}(x,t) \\ &\quad + \frac{\partial}{\partial x}\left[\mu_{t2} n_{t2}(x,t) E(x,t)\right], \end{aligned} \tag{7}$$

$$\int_0^L E(x,t)\,\mathrm{d}x = V; \qquad V = \text{const} > 0, \tag{8}$$

where $q = 1.602 \times 10^{-19}$ C, x is the distance from the electrode, t is the time, n and p are the free hole and electron concentrations, respectively, n_{t1}; n_{t2}; p_{t1}; p_{t2} are the concentrations of the trapped holes and electrons, respectively, μ_{t1} and μ_{t2} are the mobilities of trapped electrons, ν_{p2}; ν_{n1}; ν_{t1}; ν_{t2} denote the frequency parameters, c_n; C_p; C_t; c_{t2} denote the recombination parameters and V is the applied voltage between the electrodes. For such internal processes we shall define the stationary state and we shall find different current-voltage characteristics.

3. STATIONARY STATE

Some electrical properties of the metal-solid-metal system are characterised by a current-voltage function. From (1)–(8) it follows that the stationary state is described by the following equations:

$$\frac{\varepsilon}{q}\frac{\mathrm{d}E(x)}{\mathrm{d}x} = p(x) + p_{t1}(x) + p_{t2}(x) - [n(x) + n_{t1}(x) + n_{t2}(x)], \tag{1a}$$

$$J = qE(x)\left[\mu_n n(x) + \mu_p p(x) + \mu_{t1} n_{t1}(x) + \mu_{t2} n_{t2}(x)\right]; \qquad J = \text{const.}, \tag{2a}$$

$$\nu_{t1}P_{t1} + \nu_{n1}n_{t1}(x) - c_n N_{t1} n(x) - C_t n(x) p_{t1}(x) + \frac{\mathrm{d}}{\mathrm{d}x}[\mu_n n(x)E(x)] = 0, \tag{3a}$$

$$\nu_{t1}P_{t1} + c_{t21}P_{t1}p_{t2}(x) - c_{t12}P_{t2}p_{t1}(x) - C_t n(x) p_{t1}(x) = 0, \tag{4a}$$

$$c_{t12}P_{t2}p_{t1}(x) - c_{t21}P_{t1}p_{t2}(x) - \nu_{t2}p_{t2}(x) + c_{t2}P_{t2}p(x) = 0, \tag{5a}$$

$$c_{21}N_{t1}n_{t2}(x) - \nu_{n1}n_{t1}(x) - c_{12}N_{t2}n_{t1}(x) + c_n N_{t1} n(x) + \frac{\mathrm{d}}{\mathrm{d}x}[\mu_{t1}n_{t1}(x)E(x)] = 0, \tag{6a}$$

$$\nu_{p2}N_{t2} + c_{12}N_{t2}n_{t1}(x) - c_{21}N_{t1}n_{t2}(x) - C_p p(x) n_{t2}(x) + \frac{\mathrm{d}}{\mathrm{d}x}[\mu_{t2}n_{t2}(x)E(x)] = 0, \tag{7a}$$

$$\int_0^L E(x)\,\mathrm{d}x = V, \tag{8a}$$

where J is the current density. The interior and the contacts of the system will be characterised by a current-voltage function in the form $J = J(V)$ or $V = V(J)$. In order to find these functions, we have to give the two boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$ describing the mechanisms of carrier injection from the anode $x = 0$ and the cathode $x = L$ into the bulk, respectively. In what follows, we will consider some possible cases of current flow between the electrodes in a solid.

A current flow with $\mu_n = \mu_{t1} = \mu_{t2} = 0$ and $C_t = 0$

In this section we will assume that the mean free path of the electron is very small and that the additional portion of the kinetic energy, which is given to the trapped electron by en external electric field, is too small. For the trapped holes and electrons, we additionally introduce into our analysis the following assumption

$$\tau_{t1}^{-1} = c_{t12}P_{t2} = c_{t21}P_{t1}; \qquad \tau_1^{-1} = c_{21}N_{t1} = c_{12}N_{t2}, \tag{9}$$

where, τ_{t1} and τ_1 denote the time constants. Next, our problem will be considered by the use of the new variables

$$\begin{aligned} \lambda_1 &= \frac{q\mu_p pE}{J}; \quad \lambda_2 = \frac{q\mu_p p_{t1}E}{J}; \quad \lambda_3 = \frac{q\mu_p p_{t2}E}{J}; \\ \lambda_4 &= \frac{q\mu_p nE}{J}; \quad \lambda_5 = \frac{q\mu_p n_{t1}E}{J}; \quad \lambda_6 = \frac{q\mu_p n_{t2}E}{J}. \end{aligned} \tag{10}$$

Under these conditions, equations (1a)–(7a) are written as follows:

$$\frac{\varepsilon\mu_p E}{J}\frac{dE}{dx} = \lambda_1 + \lambda_2 + \lambda_3 - (\lambda_4 + \lambda_5 + \lambda_6), \tag{11}$$

$$\lambda_1 = 1, \tag{12}$$

$$v_{t1}P_{t1}\frac{q\mu_p E}{J} + \frac{1}{\tau_{t1}}(\lambda_3 - \lambda_2) = 0, \tag{13}$$

$$\frac{1}{\tau_{t1}}(\lambda_2 - \lambda_3) - v_{t2}\lambda_3 + c_{t2}P_{t2}\lambda_1 = 0, \tag{14}$$

$$v_{p2}N_{t2}\frac{q\mu_p E}{J} + v_{t2}\lambda_3 - c_{t2}P_{t2}\lambda_1 - \frac{C_p J}{q\mu_p E}\lambda_1\lambda_6 = 0, \tag{15}$$

$$c_n N_{t1}\lambda_4 - v_{n1}\lambda_5 + \frac{1}{\tau_1}(\lambda_6 - \lambda_5) = 0, \tag{16}$$

$$\frac{1}{\tau_1}(\lambda_5 - \lambda_6) + v_{p2}N_{t2}\frac{q\mu_p E}{J} - \frac{C_p J}{q\mu_p E}\lambda_1\lambda_6 = 0. \tag{17}$$

From (12)–(16) we get (respectively)

$$\lambda_3 = \frac{c_{t2}P_{t2}}{v_{t2}} + \frac{v_{t1}P_{t1}}{v_{t2}}\frac{q\mu_p E}{J}, \tag{18}$$

$$\lambda_2 = \frac{c_{t2}P_{t2}}{v_{t2}} + \left(\tau_{t1} + \frac{1}{v_{t2}}\right)v_{t1}P_{t1}\frac{q\mu_p E}{J}, \tag{19}$$

$$\lambda_6 = \frac{v_{p2}N_{t2} + v_{t1}P_{t1}}{C_p}\left(\frac{q\mu_p E}{J}\right)^2, \tag{20}$$

$$\lambda_5 = \frac{v_{p2}N_{t2} + v_{t1}P_{t1}}{C_p}\left(\frac{q\mu_p E}{J}\right)^2 + v_{t1}P_{t1}\tau_1\frac{q\mu_p E}{J}, \tag{21}$$

$$\lambda_4 = \frac{\nu_{n1}(\nu_{p2}N_{t2} + \nu_{t1}P_{t1})}{c_n N_{t1} C_p}\left(\frac{q\mu_p E}{J}\right)^2 + \nu_{t1}P_{t1}\frac{q\mu_p E}{J}\left(\frac{1}{c_n N_{t1}} + \frac{\tau_1 \nu_{n1}}{c_n N_{t1}}\right). \tag{22}$$

Next, (18)–(22) and (11) result in

$$\frac{\varepsilon\mu_p E}{J}\frac{\mathrm{d}E}{\mathrm{d}x} = a_0 - a_1\left(\frac{E}{J}\right) - a_2\left(\frac{E}{J}\right)^2, \tag{23}$$

where:

$$a_0 = 1 + \frac{2c_{t2}P_{t2}}{\nu_{t2}}; \qquad a_1 = \left(\frac{1}{c_n N_{t1}} + \frac{\tau_1\nu_{n1}}{c_n N_{t1}} + \tau_1 - \tau_{t1} - \frac{2}{\nu_{t2}}\right) q\mu_p \nu_{t1} P_{t1};$$
$$a_2 = \left(2 + \frac{\nu_{n1}}{c_n N_{t1}}\right)\frac{q^2\mu_p^2}{C_p}(\nu_{p2}N_{t2} + \nu_{t1}P_{t1}). \tag{24}$$

Using a new variable $z = E/J$, (23) takes the following form

$$\varepsilon\mu_p J z\frac{\mathrm{d}z}{\mathrm{d}x} = a_0 - a_1 z - a_2 z^2. \tag{25}$$

In the case when $\frac{4a_0a_2}{a_1^2} \gg 1$ or $a_1 = 0$, the general integral of (25) is of the form

$$E(x) = J\sqrt{\frac{a_0}{a_2} + C\exp\left(\frac{-2a_2x}{\varepsilon\mu_p J}\right)}; \qquad C = (E(0)/J)^2 - \frac{a_0}{a_2}. \tag{26}$$

When the anode injects an infinite quantity of holes, that is $E(0) = 0$, (26) results in

$$E(x) = J\sqrt{\frac{a_0}{a_2}\left\{1 - \exp\left(\frac{-2a_2x}{\varepsilon\mu_p J}\right)\right\}}. \tag{27}$$

Additionally, when $2a_2L \ll \varepsilon\mu_p J$ (a case in which the positive charge carrier concentration is much greater than the negative charge carrier concentration), a function $E(x)$ takes the simpler form

$$E(x) = \sqrt{\frac{2a_0 J x}{\varepsilon\mu_p}}. \tag{27a}$$

Taking into account the voltage condition (8a), we ascertain that the current-voltage characteristic is quadratic, that is

$$J = \frac{9}{8}a_0^{-1}\varepsilon\mu_p\frac{V^2}{L^3}. \tag{28}$$

In the case when $4a_0a_2 \ll a_1^2$, the general integral (25) leads to

$$E(x) = C\exp\left(\frac{-a_2 x}{\varepsilon\mu_p J}\right) - \frac{a_1 J}{a_2}; \qquad C = E(0) + \frac{a_1 J}{a_2}. \tag{29}$$

Next, substituting (29) into (8a), we obtain a function $V = V(J)$ in the following parametric form

$$V = \frac{\varepsilon\mu_p J}{a_2}\left(E(0) + \frac{a_1 J}{a_2}\right)\left\{1 - \exp\left(\frac{-a_2 L}{\varepsilon\mu_p J}\right)\right\} - \frac{a_1 JL}{a_2}; \qquad J = f_0[E(0)], \tag{30}$$

where $J = f_0[E(0)]$ is the boundary function describing the hole injection from the anode into the bulk. Now, we will investigate a function (30). To this end, we introduce a new variable

$$w = \frac{a_2 L}{\varepsilon\mu_p J} \qquad \text{and} \qquad 1 - e^{-w} = w - \frac{w^2}{2!} + \frac{w^3}{3!} - \frac{w^4}{4!} + \ldots \tag{31}$$

Since

$$\frac{\varepsilon\mu_p a_1 J^2}{a_2^2} w = \frac{a_1 JL}{a_2} \qquad \text{and} \qquad \frac{\varepsilon\mu_p J}{a_2} w = L,$$

therefore, when w is infinitesimal, we can write

$$V = LE(0) + (LE(0) + (a_1 JL)/a_2)\left(-\frac{w}{2} + \frac{w^2}{6}\right); \qquad w \ll 1. \tag{32}$$

The condition $w \ll 1$ is equivalent to $J \gg J_g$, where

$$J_g = \frac{a_2 L}{\varepsilon\mu_p} = \left(2 + \frac{v_{n1}}{c_n N_{t1}}\right)\frac{q^2\mu_p L}{\varepsilon C_p}\left(v_{p2}N_{t2} + v_{t1}P_{t1}\right). \tag{33}$$

In a particular case, when $\frac{a_1}{a_2}J \gg E(0) = f_0^{-1}(J)$ (a case of a current source), where $f_0^{-1}(J)$ is the inverse function, (32) is replaced by

$$V = LE(0) + \frac{a_1 JL}{a_2}\left(-\frac{w}{2} + \frac{w^2}{6}\right). \tag{34}$$

For quadratic boundary function describing the tunnel effect $J = f_0[E(0)]$ in the form $J = \alpha_0^2 E^2(0)$, where α_0^2 is a boundary parameter, the current-voltage characteristic (34) becomes

$$V = \frac{L}{\alpha_0}\sqrt{J} - \frac{a_1 L^2}{2\varepsilon\mu_p} + \frac{a_1 L J_g^2}{6a_2}\frac{1}{J}; \qquad J \gg J_g \qquad \text{and} \qquad J \gg \left(\frac{a_2}{\alpha_0 a_1}\right)^2. \tag{35}$$

Thus the derivative $\mathrm{d}V\mathrm{d}J$ of (35) can be negative, that is

$$V(J) > 0 \quad \text{and} \quad \frac{\mathrm{d}V}{\mathrm{d}J} < 0 \quad \text{for} \quad J \gg J_g \quad \text{and} \quad \alpha_0^2 >> \frac{9a_2\varepsilon\mu_p}{a_1^2 L}. \tag{35a}$$

For physical interpretation, we ascertain that, in the case of a blocking contact, the differential resistance of the system can be negative. The similar property occurs when the boundary function is of the Poole form, namely $J = f_0[E(0)] = J_0 \exp(b_0 E(0))$, where J_0 and b_0 are the boundary parameters. In this case, a function $V(J)$ takes the form

$$V = \frac{L}{b_0} \ln \frac{J}{J_0} - \frac{a_1 L^2}{2\varepsilon\mu_p} + \frac{a_1 L J_g^2}{6a_2} \frac{1}{J}; \qquad J \gg J_g \quad \text{and} \quad \frac{a_1 J}{a_2} \gg \frac{1}{b_0} \ln \frac{J}{J_0}, \tag{36}$$

for which there is

$$\text{if} \quad J \gg J_g \wedge b_0 \gg \frac{6\varepsilon\mu_p}{a_1 L} \wedge \ln \frac{J}{J_0} \gg 3 \quad \Rightarrow \quad V(J) > 0 \wedge \frac{\mathrm{d}V}{\mathrm{d}J} < 0. \tag{36a}$$

In the case of the Schottky boundary function $J = f_0[E(0)] = J_1 \exp(\sqrt{b_1 E(0)})$, where J_1 and b_1 are the boundary parameters, we now have

$$V = \frac{L}{b_1}\left(\ln \frac{J}{J_1}\right)^2 - \frac{a_1 L^2}{2\varepsilon\mu_p} + \frac{a_1 L J_g^2}{6a_2} \frac{1}{J}; \qquad J \gg J_g; \qquad \frac{a_1 J}{a_2} \gg \frac{1}{b_1}\left(\ln \frac{J}{J_1}\right) \tag{37}$$

and

$$\text{if} \quad J \gg J_g \wedge b_1 \gg \frac{12\varepsilon\mu_p}{La_1} \wedge \left(\ln \frac{J}{J_1}\right)^2 \gg 6 \quad \Rightarrow \quad V(J) > 0 \wedge \frac{\mathrm{d}V}{\mathrm{d}J} < 0. \tag{37a}$$

Now, let us esteem a function (30) when the variable w is very great $w \gg 1$ (this is acceptable when $J < J_g$ and $J \to 0$). Under these conditions, the current-voltage dependence (30) is written as

$$V = \frac{a_1 \varepsilon\mu_p J^2}{a_2^2} + \frac{\varepsilon\mu_p J}{a_2}\left(E(0) - \frac{a_1 L}{\varepsilon\mu_p}\right) \tag{38}$$

and $J = f_0[E(0)]$. For example, if a boundary function $J = f_0[E(0)] = \sigma_0 E(0)$ is linear, then the function (38) is strongly non-linear in the form

$$V = \frac{a_1 \varepsilon\mu_p J^2}{a_2^2} + \frac{\varepsilon\mu_p J}{a_2}\left(\frac{J}{\sigma_0} - \frac{a_1 L}{\varepsilon\mu_p}\right). \tag{38a}$$

According to (29)–(30), let us notice that a boundary condition $E(0) = 0$ is not possible (since, there is $E(x) < 0$ and $V(J) < 0$). Taking into account this fact, (29)–(30) must be replaced by (respectively)

$$E(x) = C \exp\left(\frac{-a_2 x}{\varepsilon\mu_p J}\right) + \frac{|a_1| J}{a_2}; \qquad C = E(0) - \frac{|a_1| J}{a_2} \tag{39}$$

and

$$V = \frac{\varepsilon\mu_p J}{a_2}\left(E(0) - \frac{|a_1|J}{a_2}\right)\left\{1 - \exp\left(\frac{-J_g}{J}\right)\right\} + \frac{|a_1|LJ}{a_2}; \qquad J_g = \frac{a_2 L}{\varepsilon\mu_p}. \tag{40}$$

Therefore, when the anode injects an infinite amount of holes into a solid, that is $E(0) = 0$, the current-voltage dependence (40) is of the form

$$V = \frac{|a_1|LJ}{a_2} - \frac{\varepsilon\mu_p|a_1|J^2}{a_2^2}\left\{1 - \exp\left(\frac{-J_g}{J}\right)\right\} \tag{41}$$

and

$$\frac{dV}{dJ} = \frac{|a_1|L}{a_2}F(y); \qquad F(y) = \left[1 + \exp(-y) - \frac{2}{y}\left(1 - \exp(-y)\right)\right]; \qquad y = J_g/J, \tag{41a}$$

as well as

$$\lim_{y\to\infty} F(y) = 1; \qquad \lim_{y\to 0} F(y) = 0+; \qquad F(y) > 0 \quad \text{for} \quad y > 0. \tag{41b}$$

Hence, on the basis of (41a) we ascertain that the system acts as an almost perfect blocking diode (or a voltage stabiliser).

4. CONCLUSIONS

We have presented a model (1)–(8) describing the space charge transport in a planar capacitor system. Here, we have assumed that impurities or pollutants can cause the additional trapping levels for positive and negative charge carriers. As a singular case of electric conduction in a solid, we have analysed an electric field distribution corresponding to the low level of hole injection from the anode into the bulk. From (29) and (39) it follows that a negative space charge with the density $q_v = \varepsilon\frac{dE}{dx} < 0$ can be distributed in the bulk, while we have assumed that the holes are mobile with the mobility μ_p. From (1)–(8) it follows that this particular integral of (25) is physically acceptable when an initial space charge is negative (which must be given in order to determine the transient state) [11], that is

$$q_v(x,0) = q\left\{p(x,0) + p_{t1}(x,0) + p_{t2}(x,0) - [n(x,0) + n_{t1}(x,0) + n_{t2}(x,0)]\right\} < 0. \tag{42}$$

Now let us return to (29). Here, the electric field distribution defines the negative space charge in a solid when the internal parameters satisfy the condition $4a_0a_2 \ll a_1^2$. A function (29) is defined when the recombination parameter C_p in the second trapping level (for the trapped electrons) satisfies the following condition

$$\tau_1^{-2} \ll \frac{1}{18}\nu_{p2}N_{t2}C_p \qquad \text{or} \qquad \tau_1^{-2} \ll \frac{1}{18}\nu_{t1}P_{t1}C_p. \tag{43}$$

This formula describes the internal condition for a negative space charge determined by (29) in the bulk with one mobility $\mu_p \neq 0$. From (43) it follows that the free positive space charge is neutralised by the trapped electrons localised in the second trapping level when the concentrations of traps N_{t2} and the recombination parameter C_p are sufficiently great. Under these conditions, we ascertain that the system can act as an almost perfect blocking diode or a negative resistance can occur.

According to [4, 5, 8, 9], we can compare our methodology with a regional approximation method and with a small signal theory. A Dumke's theory for the switching effect has been presented by Lampert [4]. In this theory, a solid with deep impurities has been presented. It was assumed that the total current density could be defined by the photonelectron current and by the equilibrium hole current density. Under these conditions the current–voltage characteristic has been written as

$$VJ = \text{const.} \quad \text{or} \quad J = aV^2 + bJV \quad \text{or} \quad J = aV^2 + b_1 J^2 V,$$

where a, b, b_1 are the material parameters. This theory is not mathematically clear since the boundary problem has not been considered. Also, Lampert considered a model of the minority carrier flow and a model of bimolecular recombination. Using a regional approximation method, Lampert has shown that the "S" type function $J(V)$ with NDR (Negative Differential Resistance) can occur. Moreover, here, a Child's law $J \propto V^2/L^3$ is possible (a case of blocking diode or voltage stabiliser). A regional approximation method for the minority carrier flow in perfect and imperfect crystal structure with non-ohmic contacts has been presented by Kao [8–9]. Here, it was found that NDR regime occurs only for the imperfect crystals.

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ZAGADNIENIE PRZEWODNICTWA ELEKTRYCZNEGO W STRUKTURACH AMORFICZNYCH. PRZYPADEK SŁABYCH I ŚREDNICH PÓL ELEKTRYCZNYCH

Rozważa się transport ładunku elektrycznego w układzie płaskiego kondensatora, zakładając że procesy generacyjno–rekombinacyjne nie zależą od zewnętrznego pola elektrycznego. Stwierdza się, że układ zachowuje się jak prawie idealna dioda blokująca (stabilizator napięcia), a ujemna rezystancja jest osobliwością transportu ładunku przestrzennego.

ПРОБЛЕМА ЭЛЕКТРИЧЕСКОГО ТОКА В АМОРФНЫХ ТВЕРДЫХ ТЕЛАХ. СЛУЧАЙ НЕ ОЧЕНЬ СИЛЬНЫХ ЭЛЕКТРИЧЕСКИХ ПОЛЕЙ

В статье представляется транспорт объемного заряда в плоском конденсаторе. Процесс захвата электрона не зависит от электрического поля. Рассматривается дальнейший подход к задачи электрического тока в котором вольтамперная характеристика может уменьшаться. Удостоверяется, что соотношение металл-твердое тело-металл сохранятся как запорный диод.