BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
TECHNICAL SCIENCES
Vol. 39, No. 4, 1991

Analysis of Steady State of Electric Conduction in Solid with Given Functions of Carrier Mobilities under Conditions of Collision Recombination

by

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Presented by T. ŚLIWIŃSKI on January 10, 1991

Summary. This work presents an analysis of the transport of "hot carriers" of charge in the planar capacitor system. The effect of carrier injection on current-voltage dependence $j = j(U)$ with given carrier mobilities is investigated. It was found that if the boundary functions are quadratic then $j \sim U^4$, whereas if they are of Schottky type then $j(U)$ can be decreasing function $(dU/dj < 0)$. The condition carrier injection, which make the $j \sim U^{3/2}$, was determined.

1. Introduction. Two models of electric conduction in solid may be determined in analysis of a bipolar charge transport, in planar capacitor system with the constant voltage between electrodes [2–4]:

— a fluid model of strong ionization of material, in which a space charge density is characterized by the concentration p of positive charge and by the concentration n of negative charge,

— a structural model of trapped carrier recombination which describe an influence of a structure of material (traps levels) on electric charge transport.

In the both models the Gauss', the continuity, the recombination equations and a given voltage condition are basic equations. In the fluid model the law of bimolecular or trimolecular recombimation (Auger recombination) can be defined by carrier concentrations. In case of Auger recombination (state of "hot carriers") considered in this paper, the mobility μ_p of carriers of positive charge and the mobility μ_n of negative ones must depend on electric field intensity $\boldsymbol{E}$ [1]. Thus, the transport of "hot carriers" in planar capacitor system is described by the equations:

$$(1) \qquad \frac{\varepsilon}{e_0}\frac{\partial E}{\partial x} = p - n$$

$$\frac{\partial}{\partial x}\{\mu_p(E)pE+\mu_n(E)nE\}+\frac{\partial p}{\partial t}-\frac{\partial n}{\partial t}=0 \tag{2}$$

$$\frac{\partial n}{\partial t}=\frac{\partial}{\partial x}\{\mu_n(E)nE\}-(a_R n^2 p+b_R n p^2) \tag{3}$$

with the condition

$$\int_0^L E\,dx=U=\text{const.} \tag{4}$$

where $e_0 = 1.6\cdot 10^{-19}$ C, ε — high frequency permittivity, x — distance from the electrode, t — time, a_R, b_R — coefficients of Auger recombination, L — distance between electrodes, U — voltage applied to electrodes. The function $\mu_p(E)$ and $\mu_n(E)$ may have different form. In this work there will be considered the following functions [1]:

$$\mu_n(E)=\mu_1(E_1/E)^{1/2};\quad \mu_p(E)=\mu_2(E_2/E)^{1/2} \tag{5}$$

where μ_1, μ_2, E_1, E_2 — the constants with dimensions of mobility and electric field intensity, respectively. Two boundary conditions of carrier injections from the electrodes $x=0$ and $x=L$ into the dielectric as well as Eqs (1)–(5) describe the electric conduction.

In this work, we will define the current-voltage dependences of steady state of electric conduction with two mechanism of injection of carriers and with the functions $\mu_p(E)$ and $\mu_n(E)$ of "hot carriers" described by formula (5).

2. Problem solution. Equations (1)–(3) describe the stationary state of electric charge transport with condition $\partial/\partial t=0$:

$$\frac{\varepsilon}{e_0}\frac{dE}{dx}=p-n \tag{6}$$

$$j=e_0\mu_n(E)nE+e_0\mu_p(E)pE;\quad j=\text{const} \tag{7}$$

$$\frac{d}{dx}\{\mu_n(E)nE\}=a_R n^2 p+b_R n p^2 \tag{8}$$

where j — current density. The function $E(x)$ fulfill condition (4).

In order to find the functions $j=j(U)$ or $U=U(j)$ we must assume two boundary functions $j=f_0[E(0)]$ and $j=f_L[E(L)]$ describing the mechanisms of injection of carriers from the electrodes $x=0$ and $x=L$ into the dielectric. Analysis of Eqs (6)–(8) gives the function $E(x)$ which is described by the differential equation:

(9)

$$\frac{-\varepsilon\alpha_2[e_0(r+1)]^2\alpha_1^3 W\,dW}{\left(j-\frac{2}{3}\varepsilon\alpha_2 W\right)\left(j+\frac{2}{3}\varepsilon\alpha_1 W\right)\left\{(a_R+b_R)j+\frac{2}{3}\varepsilon(\alpha_1 b_R-\alpha_2 a_R)W\right\}}=\frac{3dz}{2z}$$

where:

(10) $$z = E^{3/2}; \qquad \frac{dz}{dx} = W(z); \quad \frac{d^2 z}{dx^2} = W\frac{dz}{dx}$$

(11) $$\alpha_1 = \mu_1 E_1^{1/2}; \quad \alpha_2 = \mu_2 E_2^{1/2}; \quad r = \frac{\alpha_2}{\alpha_1}.$$

We will consider the case of internal parameters which fulfil the relation:

(12) $$\alpha_1 b_R - \alpha_2 a_R = 0 \quad \text{or} \quad \frac{b_R}{a_R} = \frac{\alpha_2}{\alpha_1} = r$$

Thus Eq. (9) has the form:

(13) $$\frac{C_1 dW}{W - \frac{3j}{2\varepsilon\alpha_2}} + \frac{C_2 dW}{W + \frac{3j}{2\varepsilon\alpha_1}} = \frac{2(a_R + b_R)j\varepsilon}{3e_0^2\alpha_1^2(r+1)^2} \cdot \frac{dz}{z};$$

$$C_1 = \frac{1}{r+1}; \quad C_2 = \frac{r}{r+1}$$

or, after substituting (10), the equivalent form:

(14) $$\left(j + \varepsilon\alpha_1 E^{1/2}\frac{dE}{dx}\right)^r \left(j - \varepsilon\alpha_2 E^{1/2}\frac{dE}{dx}\right) = AE^{\chi};$$

$$\chi = \frac{(a_R + b_R)\varepsilon j}{e_0^2\alpha_1^2(r+1)} = \frac{\varepsilon a_R j}{e_0^2\alpha_1^2}$$

where A — an integral constant. Equation (14) defines the dependence between concentrations of carriers and an electric field intensity in the form:

(15) $$p^r n E^{r_1} = \text{const.}; \quad r_1 = \frac{r+1}{2} - \chi.$$

In the sequel we introduce the auxiliary parameter $\lambda(x)$ which is described by the formula:

(16) $$\lambda(x) = \frac{e_0\alpha_1 E^{1/2}(x)n(x)}{j}; \quad \lambda_0 = \lambda(0); \quad \lambda_1 = \lambda(L).$$

Taking into account (7), (11), (12), (5), Eq. (8) yields:

(17) $$\frac{d\lambda}{dx} = \frac{b_R j^2 \lambda(1-\lambda)}{(e_0\alpha_2)^2\alpha_1 E^{3/2}}$$

while relation (15) defines the function $E(\lambda)$:

(18) $$E^{\chi} = A_1\lambda(1-\lambda)^r; \quad A_1 = \text{const}$$

where A_1 — a new integral-constant defined by the boundary values $E(0)$ and λ_0 for $x = 0$:

(19) $$A_1 = \frac{E^{\chi}(0)}{\lambda_0(1-\lambda_0)^r}.$$

Thus, condition (4) has the form:

$$U = \int_{\lambda_0}^{\lambda_1} E\Big(\frac{dx}{d\lambda}\Big) d\lambda \tag{20}$$

or the equivalent formula which results from Eqs (17)–(19):

$$U = \left(\frac{e_0\alpha_2}{j}\right)^2 \frac{\alpha_1 E^{5/2}(0)}{b_R\{\lambda_0(1-\lambda_0)^r\}^{5/2\chi}} \int_{\lambda_0}^{\lambda_1} \lambda^{\frac{5}{2\chi}-1}(1-\lambda)^{\frac{5r}{2\chi}-1} d\lambda \tag{21}$$

where the boundary parameters λ_0 and λ_1 satisfy the equation:

$$\int_{\lambda_0}^{\lambda_1} \lambda^{m_1}(1-\lambda)^{m_2} d\lambda = \frac{j^2 b_R L A_1^{-\frac{3}{2\chi}}}{(e_0\alpha_2)^2\alpha_1}; \quad m_1 = \frac{3}{2\chi} - 1; \quad m_2 = \frac{3r}{2\chi} - 1. \tag{22}$$

Formulas (21), (22) and boundary functions $j = f_0[E(0)]$ and $j = f_L[E(L)]$ define the current-voltage dependence in the parametric form under conditions of the trimolecular recombination. If the values n and p meet the condition $\alpha_1 n \ll \alpha_2 p \Rightarrow \lambda \ll 1$ or $1 - \lambda \approx 1$, then formula (21) is reduced to the form:

$$U \approx \frac{2\varepsilon\alpha_2 E^{5/2}(0)}{5j}\left\{\left(\frac{\lambda_1}{\lambda_0}\right)^{\frac{5}{2\chi}} - 1\right\} \tag{23}$$

and there is satisfied the proportion:

$$\frac{E^\chi(0)}{\lambda_0} \approx \frac{E^\chi(L)}{\lambda_1}. \tag{24}$$

In this case, function $j(U)$ is described in the parametric form:

$$U = \frac{2\varepsilon\alpha_2(E^{5/2}(L) - E^{5/2}(0))}{5j} \quad \text{and} \quad j = f_0[E(0)] \quad \text{and} \quad j = f_L[E(L)]. \tag{25}$$

In case of $\alpha_2 p \ll \alpha_1 n$, the parameter α_2 is replaced by the parameter $-\alpha_1$ in formula (25). We will consider two cases of boundary functions f_0 and f_L.

Example 1. The particular case of the tunnel effect: $j = a_0 E^2(0)$ and $j = a_L E^2(L)$, where a_0, a_L — boundary constants. Then, the dependence $j = j(U)$ has the form:

$$j \sim U^4; \quad a_0 > a_L. \tag{26}$$

Example 2. The Schottky function is defined on two boundaries: $j = j_0 \exp(b_0 E^{1/2}(0))$ and $j = j_L \exp(b_L E^{1/2}(L))$ where j_0, j_L, b_0, b_L — boundary constants. We assume additionally that $j_0 = j_L$. Then, the function

$U = U(j)$ has the form:

$$(27) \qquad U = \frac{2\varepsilon\alpha_2}{5}\left(\frac{1}{b_L^5} - \frac{1}{b_0^5}\right)\frac{1}{j}\left(\ln\frac{j}{j_0}\right)^5; \quad b_0 > b_L; \quad j > j_0$$

The derivative dU/dj

$$(28) \qquad \frac{dU}{dj} = \frac{2\varepsilon\alpha_2}{5}\left(\frac{1}{b_L^5} - \frac{1}{b_0^5}\right)\frac{1}{j^2}\left(\ln\frac{j}{j_0}\right)^4\left(5 - \ln\frac{j}{j_0}\right)$$

is negative, $dU/dj < 0$ for $j > j_0 e^5 = 148.4 j_0$. In these two cases the condition $\alpha_1 n \ll \alpha_2 p$ is satisfied when $\alpha_2/\alpha_1 < 1$. According to [3, 4], where there was considered the electrical charge transport under condition of constant carrier mobilities, formula (25) has the form:

$$(29) \qquad U \sim \frac{E^3(L) - E^3(0)}{j} \quad \text{and} \quad j = f_0[E(0)] \quad \text{and} \quad j = f_L[E(L)]$$
$$\mu_n = \text{const.}; \qquad \mu_p = \text{const.}.$$

Thus, we infer that formula (26) characterizes trimolecular recombination. Formula (27) must be replaced by the form which follow from formula (29):

$$(30) \qquad U \sim \frac{1}{j}\left(\ln\frac{j}{j_0}\right)^6; \quad \mu_n = \text{const.}; \quad \mu_p = \text{const.}.$$

In this case $dU/dj < 0$ for $j > e^6 j_0 = 403,4 j_0$. We infer that the characteristic $j(U)$ can be the decreasing function independently of whether carrier mobilities are constant or they are functions of E as in (5).

Continuing the analysis of stationary state with "hot-carriers", we will consider the case of the parameter $r = 1$ or $\alpha_1 = \alpha_2 = \alpha$ and $a_R = b_R$. Then formulas (14), (15) have the form:

$$(31) \qquad \frac{dE^{3/2}}{dx} = \pm\left\{\left(\frac{3j}{2\varepsilon\alpha}\right)^2 - AE^{\chi}\right\}^{1/2}; \quad A = \frac{9\epsilon_0^2}{\varepsilon^2} pn\, E^{1-\chi} = \text{const.}$$

which under condition (4) yields:

$$(32) \qquad U = \int_{E(0)}^{E(L)} E\left(\frac{dx}{dE}\right) dE = \frac{3}{2}\int_{E(0)}^{E(L)} \frac{E^{3/2} dE}{\left\{\left(\frac{3j}{2\varepsilon\alpha}\right)^2 - AE^{\chi}\right\}^{1/2}}$$

$$(33) \qquad \frac{3}{2}\int_{E(0)}^{E(L)} \frac{E^{1/2} dE}{\left\{\left(\frac{3j}{2\varepsilon\alpha}\right)^2 - AE^{\chi}\right\}^{1/2}} = L$$

Thus, formulas (32), (33) and the boundary functions $j = f_0[E(0)]$ and $j = f_L[E(L)]$ define the current-voltage dependences in parametric form, for the internal parameters $\alpha_1 = \alpha_2$ and $a_R = b_R$. It is worth to notice that

$$j^2 - \left(\frac{2\varepsilon\alpha}{3}\right)^2 AE^{\chi} = (e_0\alpha)^2 E(p-n)^2; \quad p \neq n \tag{34}$$

If the concentration of carriers meet the firm inequalities $n \gg p$ or $p \gg n$ then the current-voltage dependence (31)–(33) is:

$$U = \frac{2\varepsilon\alpha}{5j}\left\{\left(E^{3/2}(0) + \frac{3jL}{2\varepsilon\alpha}\right)^{5/3} - E^{5/2}(0)\right\} \quad \text{and} \quad j = f_0[E(0)] \tag{35}$$

Dependence (35) is the current-voltage characteristic under conditions of the monopolar conduction. In the particular case, when $E^{3/2}(0) \ll \frac{3jL}{2\varepsilon\alpha}$ or $f_0^{-1}(j) \ll (\frac{3jL}{2\varepsilon\alpha})^{2/3}$, where f_0^{-1} — the inverse of $j = f_0[E(0)]$, then formula (35) makes the power function $j \sim U^{3/2}$ in this form:

$$j = \frac{10}{9}\left(\frac{5}{3}\right)^{1/2} \varepsilon\alpha \frac{U^{3/2}}{L^{5/2}} \tag{36}$$

If $f_0^{-1}(j) \gg (\frac{3jL}{2\varepsilon\alpha})^{2/3}$ then the function $j = j(U)$ has the form $j = f_0(U/L)$. When the firm inequalities $p \gg n$ or $n \gg p$ are not a case, then the function $j = j(U)$ can be approximated by power-series expansions of the integrands in formulas (32) and (33) with assumption that $p \neq n$:

$$\begin{gathered} U = \frac{\varepsilon\alpha}{j}\int_{E(0)}^{E(L)} E^{3/2}\sum_{i=0}^{\infty}\binom{-1/2}{i}[B(E)]^i dE \\ \int_{E(0)}^{E(L)} E^{1/2}\sum_{i=0}^{\infty}\binom{-1/2}{i}[B(E)]^i dE = \frac{jL}{\varepsilon\alpha}; \quad B(E) = \left(\frac{2\varepsilon\alpha}{3j}\right)^2(-A)E^{\chi} \end{gathered} \tag{37}$$

It results from (34) that the condition $p \neq n$ implies $B(E) < 1$. Formula, defining a dependence between the integral constant and the boundary values $E(0)$ and $E(L)$, can be determined from (37), for values of index $i = 1, 2, 3$ under sum symbol of the integrand in (37). Thus, the analysis of (37) for $i = 1$ gives the current-voltage dependence in the parametric form:

$$\begin{aligned} U = \frac{\varepsilon\alpha}{j}\Bigg\{&\frac{2}{5}[E^{5/2}(L) - E^{5/2}(0)] \\ &+ \frac{\chi + \frac{3}{2}}{\chi + \frac{5}{2}}\left[\frac{jL}{\varepsilon\alpha} - \frac{2}{3}(E^{3/2}(L) - E^{3/2}(0))\right] \cdot \frac{E^{\chi+\frac{5}{2}}(L) - E^{\chi+\frac{5}{2}}(0)}{E^{\chi+\frac{3}{2}}(L) - E^{\chi+\frac{3}{2}}(0)}\Bigg\} \\ &\text{and} \quad j = f_L[E(L)]; \quad j = f_0[E(0)]. \end{aligned} \tag{38}$$

Analogical analysis for $i = 2$ or $i = 3$ results in the more precise dependence $j = j(U)$, but the formulas are too complex. According to formulas

(14) and (25), (38) we note that $\chi \sim j$, and formula (25) results from (38) for $\alpha_2 = \alpha_1 = \alpha$.

3. Conclusion. In this work there is considered the model of electric conduction under condition of existence of "hot carriers". There are neglected contact voltages and surface charge on interface planes between the dielectric and metal. In this conditions, we have made the assumption that the mechanism of the injection of carriers from metal into the dielectric can be described by the function of current density and electric field intensity at the electrodes $x = 0$ and $x = L$ in form: $j = f_0[E(0)]$ and $j = f_L[E(L)]$. The "hot carriers" current-voltage dependence can be the power function $j \sim U^4$, $j \sim U^{3/2}$ or can be the decreasing function $dU/dj < 0$. Respecting the works [3, 4] which present two models: fluid and structural, we ascertain that property $dU/dj < 0$ is no specific quality of Auger recombination. We can prove that the fluid model of the bimolecular recombination is the particular case of the structural model with constant carrier mobilities [3, 4], and the characteristic $dU/dj < 0$ can exist when the boundary function f_0 and f_L are linear, that is $j \sim E(0)$ and $j \sim E(L)$. In this work we have proved that the dependence $j = j(U)$ can be decreasing function when the mechanisms of injection of carriers from the electrodes into the dielectric are described by the Schottky functions.

Acknowledgment. This work was supported by Grant RPBP 02.7 and guided by the Department of Fundamental Research at the Institute of Electrotechnology of Polish Academy of Science. The author wishes to thank Professor T. Śliwiński for his valuable remarks during the preparation of the paper.

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REFERENCES

[1] M. A. Lampert, P. Mark, *Current injection in solid*, Academic Press, London, 1970.

[2] B. Świstacz, *Transient states of electric conduction in solid dielectric including carrier recombination. I. Ideal dielectric without traps*, Bull. Pol. Ac.: Tech., **33** (1985) 587–596.

[3] B. Świstacz, *Electron-Ion Conductance in the Metal-Dielectric-Metal System with Two Given Boundary Conductions. I. Carrier generation with three mobilities. II. Generation-recombination with two mobilities*, ibid., **38** (1990) 145–161.

[4] B. Świstacz, *Analiza stanu stacjonarnego transportu nośników ładunków różnoimiennych, w warunkach bimolekularnej rekombinacji w ciele stałym, z dwoma zadanymi warunkami brzegowymi*, Rap. Ser. PRE. **384**, OINT/I-7, 1989, Politech. Wroclawska.