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Analysis of double injection in solids. Further results

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In this work we further continue to analyse electric conduction with two given boundary conditions. The space charge is formed by free holes and electrons as well as by trapped electrons. We have determined the conditions in which the current-voltage dependence can be strongly nonlinear and discontinuous.

1. INTRODUCTION

Numerous analytical methods describing double injection in semiconductors and insulators have been made. Fundamental concepts for double injection are a regional approximation method [1–4] and a small signal theory [5–6]. Those concepts contain fundamental physical processes, but analytical methods are not mathematically clear for imperfect crystals and amorphous solids. Usually, in these methods the divergence of the electric field has been equal to zero. With this assumption boundary conditions are very limited. In the case of strong asymmetric injection this assumption is not possible. This assumption ought to be determined by the mechanisms of carrier injection. In the works [7–9] we showed that space charge is determined by the mechanisms of carrier injection from the anode and from the cathode into a solid. In this paper we further continue to present a concept of bipolar space charge which takes into account different boundary functions describing the mechanisms of carrier injection from the electrodes into a solid. The purpose of this work is to present our theoretical analysis for double injection problem and to find new current-voltage characteristics for space-charge conditions.

2. BASIC EQUATIONS

In this work we will consider hole injection from the anode and electron injection from the cathode into a solid. We will assume that the free and valence electrons can be located in the discrete trapping levels. Also, we will assume that electron transition between the trapping levels occurs. As the model system, the planar capacitor will be used. The basic equations of the transport of space charge in solids are the Gauss equation, the continuity equation, the generation-recombination equations [10–12] and the field integral, which are written for the planar capacitor system

$$\varepsilon \frac{\partial E(x,t)}{\partial x} = q\{p(x,t) - [n(x,t) + n_{t1}(x,t) + n_{t2}(x,t)]\} \tag{1}$$

$$\begin{gathered} \frac{\partial}{\partial x}\{[\mu_n n(x,t) + \mu_p p(x,t)]E(x,t)\} + \frac{\partial p(x,t)}{\partial t} + \\ -\frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0, \end{gathered} \tag{2}$$

$$\begin{gathered} \frac{\partial n_{t1}(x,t)}{\partial t} = c_n N_{t1} n(x,t) + c_{21} N_{t1} n_{t2}(x,t) + \\ - v_n n_{t1}(x,t) - c_{12} N_{t2} n_{t1}(x,t); \quad N_{t1} \gg n_{t1}, \end{gathered} \tag{3}$$

$$\begin{gathered} \frac{\partial n_{t2}(x,t)}{\partial t} = v_p N_{t2} + c_{12} N_{t2} n_{t1}(x,t) - c_{21} N_{t1} n_{t2}(x,t) + \\ - C_p p(x,t) n_{t2}(x,t); \quad N_{t2} \gg n_{t2}, \end{gathered} \tag{4}$$

$$\frac{\partial n(x,t)}{\partial t} = v_n n_{t1}(x,t) - c_n N_{t1} n(x,t) + \frac{\partial}{\partial x}(\mu_n n(x,t)\, E(x,t)) \tag{5}$$

$$\int_0^L E(x,t)\mathrm{d}x = V = \mathrm{const} \quad V > 0, \tag{6}$$

where q is the electric charge, ε — dielectric constant, x — distance from the electrode, E — electric field intensity, t — time, p and n — free hole and electron concentrations, n_{t1} and n_{t2} — trapped electron concentrations in the first and second trapping level, c_n; v_n; C_p; c_{12}; c_{21} — generation-recombination parameters of the trapping levels, N_{t1} and N_{t2} — concentrations of traps, μ_p, μ_n — mobilities of free holes and electrons respectively, L — distance between the electrodes, and V — applied voltage. In equations (2)–(5) the diffusion current is neglected [13]. This is acceptable for a planar capacitor system. In (1)–(5) the number of trapping levels is equal to 2. This number can be generalized, but mathematical problem becomes too difficult.

In order to avoid this problem we have introduced into our analysis the so-called effective coefficients c_{12} and c_{21} describing allowed electron transition between the first and second trapping levels. In equations (1)–(5) we have assumed that $N_t \gg n_t$, that is, the bulk acts as an unlimited reservoir of carriers. For such the space-charge transport model we shall find the steady state current-voltage characteristics.

3. PROBLEM SOLUTION

From (1) – (6) it follows that the steady state of electric conduction is described by

$$\frac{\varepsilon}{q}\frac{\mathrm{d}E(x)}{\mathrm{d}x}=p(x)-n(x)-n_{t1}(x)-n_{t2}(x), \tag{1a}$$

$$J=qE(x)[\mu_n n(x)+\mu_p p(x)]; \quad J=\text{const.}, \tag{2a}$$

$$C_n n(x)-v_n n_{t1}(x)=C_{12}n_{t1}(x)-C_{21}n_{t2}(x), \tag{3a}$$

$$C_{12}n_{t1}(x)-C_{21}n_{t2}(x)=C_p p(x)n_{t2}(x)-v_p N_{t2}, \tag{4a}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\mu_n n(x)E(x)]=C_n n(x)-v_n n_{t1}(x), \tag{5a}$$

$$\int_0^L E(x)\mathrm{d}x=V=\text{const.}; \quad V>0, \tag{6a}$$

where $C_n=c_nN_{t1}$, $C_{12}=c_{12}N_{t2}$, $C_{21}=c_{21}N_{t1}$.

We shall consider such the case of electric conduction in which the space charge distribution $\varepsilon\mathrm{d}E/\mathrm{d}x$ is determined by the two mechanisms of carrier injection, that is $E(x)=E(x, J, C_1, C_2)$, where C_1 and C_2 are constants of integration. It is very known that electron and hole emission from the metal into the bulk can be described by the following mechanisms [14 – 17]:

(a) electron emission from the bulk over the top of barrier into the metal (thermionic emission current),

(b) quantum-mechanical tunneling through the part of barrier (thermionic-field-emission current),

(c) quantum-mechanical tunneling through the barrier from the edge of depletion region (field-emission current),

(d) recombination in the depletion region (recombination current).

If the potential barrier width is small in comparison with the mean free path, then the current density J of injected carriers depends on the barrier height and on the electric field intensity E_0 at the injecting contact. This boundary function can be written as $J=\mathrm{f}_0(E_0)$, where f_0 is the function describing the mechanism of carrier injection from the electrode into the bulk. For the function $E=E(x, J, C_1, C_2)$ the voltage condition (6a) is of the form

$$V=\int_0^L E(x, J, C_1, C_2)\mathrm{d}x=V(J, C_1, C_2) \tag{6b}$$

and

$$E(0)=E(x=0, J, C_1, C_2) \text{ and } E(L)=E(x=L, J, C_1, C_2). \tag{6c}$$

If the current-voltage characteristics are to be evaluated, it is necessary to give two boundary functions $J=\mathrm{f}_0[E(0)]$ and $J=\mathrm{f}_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes $x=0$ and $x=L$ into the bulk, respectively. Thus, the current voltage dependence can have the parametric form

$$V=V(E(0),E(L)) \text{ and } J=\mathrm{f}_0[E(0)] \text{ and } J=\mathrm{f}_L[E(L)]. \tag{7}$$

Therefore, we shall find the functions $J=J(V)$ or $V=V(J)$ which can be evaluated by the use of (7).

In this section we shall consider a few of the cases of internal interaction between carriers for which the analytical form of $E(x)$ and of the current-voltage characteristics can be found.

3.1. The carrier recombination conditions

In this case we assume that $v_n=v_p=c_{21}=0$ in (1a)–(5a). Next, introducing the new variables A_1, A_2, A_3, A_4 in the form

$$A_1=q\mu_n pE/J; \quad A_2=q\mu_n nE/J; \quad A_3=q\mu_n n_{t1}E/J; \quad A_4=q\mu_n n_{t2}E/J,$$

we obtain the following system of equations

$$rA_1+A_2=1; \quad r=\mu_p/\mu_n, \tag{8}$$

$$\frac{\varepsilon\mu_n E}{J}\frac{\mathrm{d}E}{\mathrm{d}x}=A_1-A_2-A_3-A_4, \tag{9}$$

$$\mu_n E\frac{\mathrm{d}A_2}{\mathrm{d}x}=C_nA_2, \tag{10}$$

$$C_nA_2=C_{12}A_3, \tag{11}$$

$$C_pA_1A_4=q\mu_nEC_{12}A_3/J. \tag{12}$$

Equations (8)–(12) result in

$$-\frac{\varepsilon C_n}{rJ}\frac{\mathrm{d}E}{\mathrm{d}A_1}=\frac{A_1}{A_2}-1-\frac{C_n}{C_{12}}-\frac{q\mu_nC_nE}{C_pJA_1}. \tag{13}$$

Determination of the function $E(x)$ from (13) is a complex problem. Analytical form of $E(x)$ will be found when: (i) $C_n \leqslant C_{12}$ and $c_n=C_p$, and (ii) $N_{t1} \gg n$, and $N_{t1} \gg p$, and (iii) $A_1 \ll 1/(1+r)$ (a case of strong asymmetric injection).

With these assumptions equation (13) takes the simpler form

$$\frac{\mathrm{d}E}{\mathrm{d}A_1}=\frac{q\mu_pE}{\varepsilon C_pA_1}, \tag{13a}$$

therefore

$$E=K_1A_1^{\chi}, \quad \chi=\frac{q\mu_p}{\varepsilon C_p}. \tag{13b}$$

Then, taking into account (13b), (10) and (8), we have

$$\frac{\mu_n K_1 (1-A_2)^{\chi}}{r^{\chi} A_2} \mathrm{d}A_2 = C_n \mathrm{d}x . \tag{14}$$

The solution of equation (14) has the form

$$\ln A_2 + \sum_{i=1}^{\chi} \frac{1}{i}\binom{\chi}{i}(-A_2)^i = \frac{r^{\chi} C_n x}{\mu_n K_1} + K'; \quad K' = \text{const.} \tag{14a}$$

for natural values of χ. Using the power series representation

$$\ln A_2 = \ln(1 - rA_1) = -\sum_{i=1}^{\infty} \frac{(rA_1)^i}{i}$$

and limiting the power series to the power $\chi+1$, the result (14a) takes the form

$$-\sum_{i=1}^{\chi}\left[\frac{(1-A_2)^i}{i} - \frac{1}{i}\binom{\chi}{i}(-A_2)^i\right] - \frac{(rA_1)^{\chi+1}}{\chi+1} = \frac{r^{\chi} C_n x}{\mu_n K_1} + K' . \tag{14b}$$

Let us notice that the sum of terms containing the power $(-A_2)^k$ in the expression $\sum_{i=1}^{\chi}(1-A_2)^i/i$, where $k=1, 2,\ldots,\chi$, takes the value

$$\frac{(-A_2)^k}{k} + \frac{(-A_2)^k\binom{k+1}{k}}{k+1} + \frac{(-A_2)^k\binom{k+2}{k}}{k+2} + \ldots \frac{(-A_2)^k\binom{\chi}{k}}{\chi} =$$

$$= (-A_2)^k\left[\frac{(k-1)!}{k!} + \frac{k!}{k!\,1!} + \frac{(k+1)!}{k!\,2!} + \ldots \frac{(\chi-1)!}{k!\,(\chi-k)!}\right] =$$

$$= (-A_2)^k \frac{1}{k}\left[1 + \binom{k}{1} + \binom{k+1}{2} + \binom{k+2}{3} + \ldots \binom{\chi-1}{\chi-k}\right] = (-A_2)^k \frac{1}{k}\binom{\chi}{k}.$$

In the last sum we used the well-known properties of Newton's symbol

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}; \quad \binom{n}{k} = \binom{n}{n-k}$$

Thus, equation (14b) leads to

$$-\frac{(rA_1)^{\chi+1}}{\chi+1} - \sum_{i=1}^{\chi}\frac{1}{i} = \frac{r^{\chi} C_n x}{\mu_n K_1} + K' , \tag{14c}$$

therefore

$$A_1 = \frac{1}{r}\left(K_2 - \frac{(\chi+1) r^{\chi} C_n x}{\mu_n K_1}\right)^{\frac{1}{\chi+1}}; \quad K_2 = -(\chi+1)\left(K' + \sum_{i=1}^{\chi}\frac{1}{i}\right). \tag{14d}$$

Finally, using (13b), we obtain the electric field distribution in the following form

$$E(x)=K_3\left(K_2-\frac{(\chi+1)C_n x}{\mu_n K_3}\right)^{\frac{\chi}{\chi+1}};\qquad K_3=\frac{K_1}{r^{\chi}}. \tag{15}$$

Constant K_3 can be defined by the boundary values

$$K_3^{1/\chi}=\frac{\mu_n}{(\chi+1)C_n L}(E^i_{(0)}-E^i_{(L)});\quad i=(\chi+1)/\chi \tag{15a}$$

By calculating the integral (6a) we get the current voltage dependence in the parametric form

$$V=\frac{(\chi+1)L}{(2\chi+1)}\frac{(E^{i+1}_{(0)}-E^{i+1}_{(L)})}{(E^i_{(0)}-E^i_{(L)})};\qquad i=(\chi+1)/\chi \tag{16}$$

and $J=f_0[E(0)]$, $J=f_L[E(L)]$.

For example, when the boundary functions f_0 and f_L describe the tunnel effect in the form $J=a_0E^2(0)$ and $J=a_LE^2(L)$; $(a_L>a_0)$ or f_0 and f_L are linear, then we have $J\sim V^2$ or $J\sim V$, respectively.
Another example, with the Poole injection at the boundaries $x=0$ and $x=L$ so that $J=J_0\exp(b_LE(L))$ and $J=J_0\exp(b_0E(0))$, then the function $J(V)$ is Poole type: $J\sim\exp(\mathrm{const}\cdot V)$. Similarly, proceeding for the Schottky boundary functions f_0 and f_L, we obtain $J\sim\exp(\mathrm{const}\cdot V^{1/2})$. In the case when χ is a rational number $\chi=m/n$, we can make use of the following substitution

$$z^n=1-A_2=rA_1;\quad \mathrm{d}A_2=(-n)z^{n-1}\mathrm{d}z, \tag{17}$$

for which equation (14) takes the form

$$\frac{(-n)z^m z^{n-1}}{1-z^n}\mathrm{d}z=\frac{r^{m/n}C_n}{\mu_n K_1}\mathrm{d}x. \tag{17a}$$

Hence, we get

$$z^m\ln(1-z^n)-m\int_0^z z^{m-1}\ln(1-z^n)\,\mathrm{d}z=\frac{r^{m/n}C_n x}{\mu_n K_1}+K', \tag{17b}$$

where $K'=\mathrm{const}$. Since $z\ll 1$ the function $\ln(1-z^n)$ may be expressed by a power series. By combining (17b), this equation can be written as

$$-z^m\left(z^n+\frac{z^{2n}}{2}+\frac{z^{3n}}{3}+\dots\right)+m\left(\frac{z^{m+n}}{m+n}+\frac{z^{m+2n}}{2(m+2n)}+\right.$$
$$\left.+\frac{z^{m+3n}}{3(m+3n)}+\dots\right)=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'. \tag{17c}$$

Limiting the power series to the first approximation, we obtain

$$-\frac{nz^{m+n}}{m+n}=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'. \tag{17d}$$

Next, taking into consideration the substitution (17) and $\chi+1=(m+n)/n$, we notice that question (17d) results in (14d) with the constant $K_2=-K'(\chi+1)$. On this basis we ascertain that the current voltage dependence (16) is satisfied for rational values of χ.

3.2. Absence of electron transition between trapping levels

In this case we must assume that $C_{12}=C_{21}=0$ in (1a)–(5a). With this assumption we have

$$\begin{gathered}A_2(x)=A_2(L)=\text{const.};\quad A_1=(1-A_2)\mu_n/\mu_p;\quad A_3=A_2/\theta_0;\\ \theta_0=v_n/C_n;\quad A_4=\frac{\theta_p(q\mu_n E)^2}{A_1 J^2};\quad \theta_p=v_p N_{t2}/C_p.\end{gathered} \tag{18}$$

Hence, on the basis (1a)–(5a), we obtain the following differential equation

$$\begin{gathered}\frac{\varepsilon\mu_n E}{J}\frac{\mathrm{d}E}{\mathrm{d}x}=\alpha-\alpha_1 E^2;\quad \alpha_1=\frac{\theta_p}{A_1}\left(\frac{q\mu_n}{J}\right)^2;\\ \alpha=A_1-A_2-A_3=\text{const.},\end{gathered} \tag{19}$$

for which the general integral has the form

$$E=\{\alpha_0 J^2+C\exp(-2\beta_0 x/J)\}^{1/2} \tag{20}$$

$$\alpha_0=\frac{\alpha A_1}{\theta_p(q\mu_n)^2};\quad \beta_0=\frac{\theta_p q^2\mu_n}{\varepsilon A_1},$$

where C is a constant of integration. Next, expressing the constants of integration by the boundary values $E(0)$ and $E(L)$, we get

$$E^2(0)=a_0 J^2+C;\quad E^2(L)=\alpha_0 J^2+C\exp(-2\beta_0 L/J). \tag{20a}$$

Let us notice that equations (20a) result in the transcendental equation for the constant α or A_1. For (19) the voltage condition (6a) can be written as

$$V=\int_0^L E(x)\mathrm{d}x=\int_{E(0)}^{E(L)} E\frac{\mathrm{d}x}{\mathrm{d}E}\mathrm{d}E=\frac{\varepsilon\mu_n}{J}\int_{E(0)}^{E(L)}\frac{E^2}{\alpha-\alpha_1 E^2}\mathrm{d}E. \tag{21}$$

In what follows, we shall discuss the importance of parameter α for (21). In the case when $\alpha>0$ relation (21) leads to

$$V=\frac{\varepsilon\mu_n a}{2Ja_1^3}\ln\left|\frac{(a_1E(L)+a)(a_1E(0)-a)}{(a_1E(L)-a)(a_1E(0)+a)}\right|+\frac{\varepsilon\mu_n}{Ja_1^2}\{E(0)-E(L)\}, \tag{21a}$$

where: $a_1=\alpha_1^{1/2}$; $a=|\alpha|^{1/2}$.
If the electrode $x=0$ injects an infinite quantity of holes, that is $E(0)\rightarrow 0$, then (21a) and (20a) become

$$V=\frac{\varepsilon\mu_n a}{2Ja_1^3}\ln\left|\frac{a_1E(L)+a}{a_1E(L)-a}\right|-\frac{\varepsilon\mu_n}{Ja_1^2}E(L) \tag{21aa}$$

and

$$E(L)=C(\exp(-2\beta_0L/J)-1);\quad C=-\alpha_0J^2. \tag{21ab}$$

For values $a_1E(L)\ll a$, we can expand the logarithm to obtain

$$V=\frac{\varepsilon\mu_n a}{Ja_1^3}\left(\frac{a_1}{a}E(L)+\frac{1}{3}\frac{a_1^3}{a^3}E^3(L)+\ldots\right)-\frac{\varepsilon\mu_n}{a_1^2J}E(L).$$

Taking into consideration only terms up to the power 3, we get

$$V=\frac{\varepsilon\mu_n}{3J\alpha}E^3(L). \tag{21ac}$$

For the values of current density $J\gg 2\beta_0L$, we have

$$E^2(L)\approx -2C\beta_0L/J=2\alpha_0\beta_0LJ=\frac{2\alpha L}{\varepsilon\mu_n}J.$$

Therefore, the current-voltage characteristic has the form

$$J=f_L\left(\frac{3V}{2L}\right), \tag{21ad}$$

where f_L is the boundary function describing the mechanism of electron injection from the electrode $x=L$ into the bulk.
When $\alpha<0$, then (21) leads to

$$V=\frac{\varepsilon\mu_n}{a_1^2J}\left[E(0)-E(L)+\frac{1}{b}\{\mathrm{arctg}[bE(L)]-\mathrm{arctg}[bE(0)]\}\right], \tag{21b}$$

where $b=\frac{a_1}{a}$.
If $E(L)\rightarrow 0$, that is, the electrode $x=L$ injects an infinite large of electrons, then (21b) takes the simpler form

$$V=\frac{\varepsilon\mu_n}{a_1^2J}\left[E(0)-\frac{1}{b}\mathrm{arctg}(bE(0))\right] \tag{21ba}$$

and

$$E^2(0)=\alpha_0 J^2(1-\exp(2\beta_0 L/J); \quad \alpha_0<0. \tag{21bb}$$

With additional assumption $2\beta_0 L \ll J$ and $bE(0)\ll 1$, we can write

$$V=\frac{\varepsilon\mu_n}{a_1^2 J}\left[E(0)-\frac{1}{b}\left(bE(0)-\frac{1}{3}b^3E^3(0)+...\right)\right. \tag{21bc}$$

and

$$E^2(0)=\frac{-2\alpha LJ}{\varepsilon\mu_n}. \tag{21bd}$$

Next, limiting the power series to the power b^3E^3 and using the boundary condition $J=\mathrm{f}_0[E(0)]$, we get the $J(V)$ curve in the form

$$J=\mathrm{f}_0\left(\frac{3V}{2L}\right) \tag{21be}$$

where f_0 is the boundary function describing the mechanism of hole injection from the electrode $x=0$ into the bulk. When $\alpha=0$, that is $A_1=\theta/(1+r\theta)$ and $A_2=1/(1+r\theta)$, where $\theta=1+1/\theta_0$, then the function (21) becomes

$$V=\frac{J}{\beta_0}(E(0)-E(L)) \text{ and } E(0)=E(L)\exp(\beta_0 L/J). \tag{22}$$

Therefore, taking into account the boundary function $J=\mathrm{f}_L[E(L)]$, we get the current-voltage characteristic in the following parametric form

$$V=\frac{JE(L)}{\beta_0}(\exp(\beta_0 L/J)-1) \text{ and } J=\mathrm{f}_L[E(L)]. \tag{23}$$

For the values of $J\gg\beta_0 L$ the function (23) becomes $J=\mathrm{f}_L[E(L)]$. An interesting case is when $\theta_p=0$ in (19) (a case of insulator). In this case the electric field distribution is

$$E^2(x)=C_1x+C_2, \tag{24}$$

where C_1 and C_2 are constants of integration. Hence, on the basis (21) we obtain the $J(V)$ curve in the parametric form

$$V=\frac{2L}{3}\frac{E^3(L)-E^3(0)}{E^2(L)-E^2(0)} \tag{25}$$

and $J=\mathrm{f}_0[E(0)]$ and $J=\mathrm{f}_L[E(L)]$.
Thus, this function is similar to (16). In the case of strong asymmetric double injection the function (26) leads to (21ad) or to (21be).

4. DISCUSSION

In this section we compare our analysis with others, which have been used for solids. A regional approximation method for double injection (bipolar space charge problem) in insulators and semiconductors has been used by Lampert and Schwob [1–4]. In this method, the positive and negative as well as quasineutral charge regions have been distinguished in the region $x \in \langle 0,L \rangle$. These regions are as follows:

(i) the first region is $x \in \langle 0,x_1 \rangle$ (the anode region) in which $\varepsilon \frac{dE}{dx} = f_1(n,p) > 0$,

(ii) the second region is $x \in \langle x_1,x_2 \rangle$ in which $\varepsilon \frac{dE}{dx} = f_2(n,p) \approx 0$,

(iii) the third region is $x \in \langle x_2,L \rangle$ (the cathode region) in which $\varepsilon \frac{dE}{dx} = f_3(n,p) < 0$

where, f_1, f_2 and f_3 are the given functions. With the boundary conditions such as: (a) $E(0)=E(L)=0$; (aa) continuity of the electric field at the junction planes $x=x_1$ and $x=x_2$, the current voltage characteristics have been obtained. These functions can be: $J \sim V$; $J \sim V^2/L^3$ (Child's law); $J \sim V^3/L^5$; $J \sim V^{l+1}/(V_0 - V)^l$ where V_0 and l are the constant parameters characterizing the material.

A small signal theory for diffusion problem has been presented by Manifacier and Henisch [5, 6]. In this method, the (i)–(iii) space charge regions have also been distinguished and $x_1 \to 0$ and $x_2 \to L$ when $E(0)=E(L)$. With condition $\varepsilon \frac{dE}{dx} = f_2(n,p) \approx 0$, the diffusion problem equations have been written as the linearized equations. The fundamental problem of this method is to find the function $p(x)$ and $n(x)$ for the various boundary parameters $\frac{dn}{dx}$ and $\frac{dp}{dx}$ at the planes $x=0$ and $x=L$. Usually, in this method the current voltage characteristics is linear $J \sim V$.

According to our considerations, we notice that the space charge density $q_v(x) = \varepsilon \frac{dE}{dx}$ is determined by the transport equations (1a)–(6a) and the boundary functions $f_0[E(0)]$ and $f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes into the bulk. This is the fundamental difference between our methodology and the regional approximation method as well as the small signal theory. From the Gauss equation and the continuity equation it follows that the fundamental problem of electric conduction in solids is to find the space charge density distribution [18–22]. On this basis we ascertain that the quasineutrality assumed by the small signal theory and by the regional approximation method is not quite understood.

5. CONCLUSIONS

In our work we have presented some results of the analysis of bipolar conduction in a metal-solid-metal system in the steady state. We have characterized the generation-recombination processes. The generation-recombination processes can determine the function $E(x)$ in the form (15) and (20). This function $E(x)$ can be decreasing or increasing for $x \in \langle 0;L \rangle$. For such the electric field distributions the $J(V)$ curves are of the parametric form. From (16), (21ad), (21be), (23), (25) it follows that the function $J=J(V)$ can have different forms such as: $J \sim V$; $J \sim V^2$; $J \sim \exp(\mathrm{const} \cdot V)$, $J \sim \exp(\mathrm{const} \cdot V^{1/2})$ or the others.

The conduction model (1)–(6) can explain the experimental current-voltage characteristics for the insulators such as ZnS, CdS, SiO_2, TiO_2, Al_2O_3, anthracene, polyethylene and for the typical semiconductors such as Ge and Si [23–25].

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ANALIZA PODWÓJNEGO WSTRZYKIWANIA W CIAŁACH STAŁYCH

W pracy kontynuowana jest analiza przewodnictwa elektrycznego przy dwóch zadanych warunkach brzegowych. Ładunek przestrzenny jest formowany przez swobodne dziury i elektrony oraz przez elektrony spułapkowane. Wyznaczono warunki, w których zależność prądowo-napięciowa jest silnie nieliniowa i nieciągła.

АНАЛИЗ ДВОЙНОГО ВПРЫСКИВАНИЯ В ТВЁРДОМ ТЕЛЕ

В работе продолжается анализ электрической проводимости для двух заданных крайных условий. Пространственный заряд формируется через свободные дурки и электроны а также через уловленные электроны. Обозначено условия при которых напряжённо — токовая зависимость сильно нелинейна и непрогресивна.