

Some conditions for strong asymmetric double injection in insulators and semiconductors

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Contents In this paper a space-charge transport model is studied. As a model system the planar capacitor is used. A boundary problem for space charge transport is presented. The effect of the mechanisms of carrier injection on the current-voltage characteristics is shown. It was found that the current-voltage characteristics can be strongly nonlinear.

Einige Bedingungen für stark asymmetrische Doppelinjektion in Isolierstoffen und Halbleitern

Übersicht Ein Modell des Transportes von Raumladungen wird untersucht. Als Modellsystem wurde ein Plattenkondensator verwendet. Randprobleme des Raumladungstransportes werden dargestellt. Der Einfluß des Injektionsmechanismus auf die Strom — Spannungs — Kennlinien wird ebenfalls angegeben. Die Untersuchungen zeigen, daß die Kennlinien starke Nichtlinearitäten aufweisen können.

1

Introduction

The theoretical aspects of double injection in solids have been extensively studied in the past two decades. Fundamental concepts describing double injection in the steady state are a regional approximation method [1–4] and a small signal theory [5–6]. Usually, in those methods the divergence of the electric field has been equal to zero. With this assumption boundary conditions are very limited. In the case of strong asymmetric double injection this assumption is not possible. This assumption ought to be determined by the boundary functions describing the mechanisms of carrier injection from the electrodes into a solid.

The purpose of this work is to present our theoretical analysis of this problem and to find new current-voltage characteristics.

2

The basic equations

In this paper we will consider hole injection from the anode and electron injection from the cathode into a solid. We will assume that the free and valence electrons can be located in the discrete trapping levels and that electron transition between trapping levels occurs. Also, we will assume that the mobilities of carriers are independent of the electric field intensity and that the carrier diffusion is unimportant. For such interactions between positive

and negative charge carriers we shall define the basic equations describing the current flow between the anode and the cathode. As a model system, the planar capacitor will be used. The basic equations of the transport of space charge in solids (insulators or semiconductors) are the Gauss' equation, the continuity equation, the generation-recombination eqs. (7–10) and the field integral, which are written for the planar capacitor system:

$$\varepsilon \frac{\partial E(x, t)}{\partial x} = q \{ (p(x, t) - (n(x, t) + n_{n1}(x, t) + n_{n2}(x, t))) \} \quad (1)$$

$$\frac{\partial}{\partial x} \{ [\mu_n n(x, t) + \mu_p p(x, t)] E(x, t) \} + \frac{\partial p(x, t)}{\partial t} - \frac{\partial n(x, t)}{\partial t} - \frac{\partial n_{n1}(x, t)}{\partial t} - \frac{\partial n_{n2}(x, t)}{\partial t} = 0 \quad (2)$$

$$\frac{\partial n_{n1}(x, t)}{\partial t} = c_n N_{n1} n(x, t) + c_{21} N_{n1} n_{n2}(x, t) \quad (3)$$

$$- v_n n_{n1}(x, t) - c_{12} N_{n2} n_{n1}(x, t); \quad N_{n1} \gg n_{n1}$$

$$\frac{\partial n_{n2}(x, t)}{\partial t} = v_p N_{n2} + c_{12} N_{n2} n_{n1}(x, t) - c_{21} N_{n1} n_{n2}(x, t) \quad (4)$$

$$- C_p p(x, t) n_{n2}(x, t); \quad N_{n2} \gg n_{n2}$$

$$\frac{\partial n(x, t)}{\partial t} = v_n n_{n1}(x, t) - c_n N_n n(x, t) + \frac{\partial}{\partial x} (\mu_n n(x, t) E(x, t)) \quad (5)$$

$$\int_0^L E(x, t) dx = V = \text{constant} \quad V > 0 \quad (6)$$

where q is the electric charge, ε the dielectric constant, t the time, x the distance from the electrode, E the electric field intensity, p and n the free hole and electron concentration, n_{n1} and n_{n2} the trapped electron concentration in the first and second trapping level, $c_n, v_n, C_p, c_{12}, c_{21}$ the generation — recombination parameters of the trapping levels, N_{n1} and N_{n2} the concentrations of traps, μ_p, μ_n the mobilities of free holes and electrons (respectively), L the distance between the electrodes, and V the applied voltage.

In eqs. (1)–(5) the number of trapping levels is equal to 2. This number can be generalized, but the mathematical problem becomes too difficult. In order to avoid this problem we have introduced into our analysis the so-called effective coefficients c_{12} and c_{21} describing allowed electron transitions between the first and second trapping levels. In eqs. (1)–(5) we have assumed that

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$N_t \gg n_p$ that is, the bulk acts as an unlimited reservoir of carriers. In this paper the conduction model (1)–(6) will be used to investigate the steady state.

3 The solution of the problem

From (1)–(6) it follows that the steady state of electric conduction is described by

$$\frac{\varepsilon dE(x)}{q dx} = p(x) - n(x) - n_{n1}(x) - n_{n1}(x) \quad (1a)$$

$$J = qE(x)[\mu_n n(x) + \mu_p p(x)]; \quad J = \text{constant} \quad (2a)$$

$$C_n n(x) - v_n n_{n1}(x) = C_{12} n_{n1}(x) - C_{21} n_{n2}(x) \quad (3a)$$

$$C_{12} n_{n1}(x) - C_{21} n_{n2}(x) = C_p p(x) n_{n2}(x) - v_p N_{t2} \quad (4a)$$

$$\frac{d}{dx} [\mu_n n(x) E(x)] = C_n n(x) - v_n n_{n1}(x) \quad (5a)$$

$$\int_0^L E(x) dx = V \quad (6a)$$

where $C_n = c_n N_{t1}$, $C_{12} = c_{12} N_{t2}$, $C_{21} = c_{21} N_{t1}$.

We shall consider two particular forms of the electric field distributions $E(x)$ determining the space charge density. These forms are as follows:

- (a) the space charge distribution $\varepsilon dE/dx$ is determined by two mechanisms of carrier injection, that is $E(x) = E(x, J, C_1, C_2)$,
- b) the space charge distribution $\varepsilon dE/dx$ is determined by one mechanism of carrier injection, that is $E(x) = E(x, J, C_1)$, where C_1 and C_2 are constants of integration.

It is very well known that electron and hole emission from the metal into the bulk can be described by the following mechanisms [11–14]:

- a) electron emission from the bulk over the top of the barrier into the metal (the thermionic emission current),
- b) quantum–mechanical tunneling through a part of the barrier (the thermionic–field–emission current),
- c) quantum–mechanical tunneling through the barrier from the edge of the depletion region (the field–emission current),
- d) recombination in the depletion region (the recombination current).

In this work we also assume that the potential barrier width is small in comparison with the mean free path. In this case the current density J of injected carriers depends on the barrier height and on the electric field intensity E_0 at the injecting contact. This boundary function can be written as $J = f_0(E_0)$, where f_0 is the function describing the mechanism of carrier injection from the electrode into the bulk. For the function $E = E(x, J, C_1, C_2)$ the voltage condition (6a) is of the form

$$V = \int_0^L E(x, J, C_1, C_2) dx = V(J, C_1, C_2) \quad (6b)$$

and

$$E(0) = E(x=0, J, C_1, C_2) \quad \text{and} \quad E(L) = E(x=L, J, C_1, C_2) \quad (6c)$$

If the current-voltage characteristics are to be evaluated, it is necessary to give two boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes $x = 0$ and $x = L$ into the bulk, respectively. Thus, the current voltage dependence can have the parametric form

$$V = V[E(0), E(L)] \quad \text{and} \quad J = f_0[E(0)] \quad \text{and} \quad J = f_L[E(L)] \quad (6d)$$

for double injection. In the case when $E = E(x, J, C_1)$, then (6d) results in

$$V = V[E(0)] \quad \text{and} \quad J = f_0[E(0)] \quad (6e)$$

for injected holes or

$$V = V[E(L)] \quad \text{and} \quad J = f_L[E(L)] \quad (6f)$$

for injected electrons.

In what follows, we shall find the functions $J = J(V)$ or $V = V(J)$ which can be evaluated by the use of (6d)–(6f).

In this section we shall consider a few of the cases of internal interactions between carriers for which the analytical form of $E(x)$ and of the current-voltage characteristics can be found.

3.1 The carrier recombination conditions

In this section we shall take into account a case when electron transition from the conduction band via trapping levels to the valence band is dominant. In this case we assume that $v_n = v_p = c_{21} = 0$ in (1a)–(5a). Next, introducing the new variables A_1, A_2, A_3, A_4 in the form

$$\begin{aligned} A_1 &= q\mu_n p E/J; & A_2 &= q\mu_n n E/J; \\ A_3 &= q\mu_n n_{n1} E/J; & A_4 &= q\mu_n n_{n2} E/J \end{aligned} \quad (7)$$

we obtain the following system of equations

$$rA_1 + A_2 = 1; \quad r = \mu_p/\mu_n \quad (8)$$

$$\frac{\varepsilon\mu_n E dE}{J dx} = A_1 - A_2 - A_3 - A_4 \quad (9)$$

$$\mu_n E \frac{dA_2}{dx} = C_n A_2 \quad (10)$$

$$C_n A_2 = C_{12} A_3 \quad (11)$$

$$C_p A_1 A_4 = q\mu_n E C_{12} A_3/J \quad (12)$$

which results in

$$-\frac{\varepsilon C_n dE}{rJ dA_1} = \frac{A_1}{A_2} - 1 - \frac{C_n}{C_{12}} - \frac{q\mu_n C_n E}{C_p J A_1} \quad (13)$$

The determination of the function $E(x)$ from (13) is a complex problem. The analytical form of $E(x)$ will be found when (i) $C_n \leq C_{12}$ and $c_n = C_p$, and (ii) $N_{t1} \gg n$ and $N_{t1} \gg p$, and (iii)

$A_1 \ll 1/(1+r)$ (a case of strong asymmetric double injection).

With these assumptions, we can obtain

$$\frac{dE}{dA_1} = \frac{q\mu_p E}{\varepsilon C_p A_1} \quad (13a)$$

therefore

$$E = K_1 A_1^m; \quad m = \frac{q\mu_p}{\varepsilon C_p} \quad (13b)$$

where K_1 is a constant of integration. Next, taking into account (13b), (10) and (8), we have

$$\frac{\mu_n K_1 (1 - A_2)^m}{r^m A_2} dA_2 = C_n dx \quad (14)$$

The solution of eq. (14) has the form

$$\ln A_2 + \sum_{i=1}^m \frac{1}{i} \binom{m}{i} (-A_2)^i = \frac{r^m C_n x}{\mu_n K_1} + K'; \quad K' = \text{constant} \quad (14a)$$

for natural numbers m . Using the power series representation

$$\ln A_2 = \ln(1 - rA_1) = - \sum_{i=1}^{\infty} \frac{(rA_1)^i}{i}$$

and limiting the power series to the power $m+1$, the result (14a) takes the form

$$- \sum_{i=1}^m \left[\frac{(1 - A_2)^i}{i} - \frac{1}{i} \binom{m}{i} (-A_2)^i \right] - \frac{(rA_1)^{m+1}}{m+1} = \frac{r^m C_n x}{\mu_n K_1} + K' \quad (14b)$$

Let us notice that the sum of terms containing the power $(-A_2)^k$ in the expression $\sum_{i=1}^m (1 - A_2)^i/i$, where $k = 1, 2, \dots, m$, takes the value

$$\begin{aligned} & \frac{(-A_2)^k}{k} + \frac{(-A_2)^k \binom{k+1}{k}}{k+1} + \frac{(-A_2)^k \binom{k+2}{k}}{k+2} + \dots + \frac{(-A_2)^k \binom{m}{k}}{m} \\ &= (-A_2)^k \left[\frac{(k-1)!}{k!} + \frac{k!}{k!1!} + \frac{(k+1)!}{k!2!} + \dots + \frac{(m-1)!}{k!(m-k)!} \right] \\ &= (-A_2)^k \frac{1}{k} \left[1 + \binom{k}{1} + \binom{k+1}{2} + \binom{k+2}{3} + \dots + \binom{m-1}{m-k} \right] \\ &= (-A_2)^k \frac{1}{k} \binom{m}{k} \end{aligned}$$

In the last sum we used the well-known properties of Newton's symbol

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}; \quad \binom{n}{k} = \binom{n}{n-k}$$

Thus, eq. (14b) leads to

$$-\frac{(rA_1)^{m+1}}{m+1} - \sum_{i=1}^m \frac{1}{i} = \frac{r^m C_n x}{\mu_n K_1} + K' \quad (14)$$

therefore

$$\begin{aligned} A_1 &= \frac{1}{r} \left(K_2 - \frac{(m+1)r^m C_n x}{\mu_n K_1} \right)^{1/(m+1)}; \\ K_2 &= -(m+1) \left(K' + \sum_{i=1}^m \frac{1}{i} \right) \end{aligned} \quad (14d)$$

Finally, using (13b), we obtain the electric field distribution in the following form

$$E(x) = K_3 \left(K_2 - \frac{(m+1)C_n x}{\mu_n K_3} \right)^{m/(m+1)}; \quad K_3 = \frac{K_1}{r^m} \quad (15)$$

The constant K_3 can be defined by the boundary values

$$K_3^{1/m} = \frac{\mu_n}{(m+1)C_n L} \left(E_{(0)}^i - E_{(L)}^i \right); \quad i = (m+1)/m \quad (15a)$$

By calculating the integral (6a), we get the current-voltage dependence in the parametric form

$$V = \frac{(m+1)L}{(2m+1)} \frac{\left(E_{(0)}^{i+1} - E_{(L)}^{i+1} \right)}{\left(E_{(0)}^i - E_{(L)}^i \right)}; \quad i = (m+1)/m \quad (16)$$

$$\text{and } J = f_0[E(0)], \quad J = f_L[E(L)]$$

For example, when the boundary functions f_0 and f_L and describe the tunnel effect in the form $J = a_0 E^2(0)$ and $J = a_L E^2(L)$; ($a_L > a_0$) or the boundary functions f_0 and f_L are linear, then we have $J \sim V^2$ or $J \sim V$, respectively. Another example, with the Poole injection at the boundaries $x=0$ and $x=L$ so that $J = J_0 \exp(b_L E(L))$ and $J = J_0 \exp(b_0 E(0))$; ($b_L > b_0$), the function $J(V)$ is Poole type:

$$J \sim \exp(\text{constant} \cdot V) \quad (16a)$$

Similarly, proceeding for the Schottky boundary functions f_0 and f_L , we obtain

$$J \sim \exp(\text{constant} \cdot V^{1/2}) \quad (16b)$$

We show that the current-voltage characteristic (16) is valid for the rational values of m . Another case of strong asymmetric carrier injection is $A_2 \rightarrow 0$ and $N_i \ll (n+p)$. In this case eq. (13) can be replaced by

$$\frac{r\varepsilon C_n}{J} \frac{dE}{dA_2} = \frac{1}{A_2} - \alpha; \quad 1 + r(1 + C_n/C_{12}) = \alpha \quad (17)$$

Hence, we obtain the function $E(A_2)$ in the form

$$E = \frac{J}{r\varepsilon C_n} \left(\ln(A_2) - \alpha A_2 + C \right); \quad C > 0 \quad (17a)$$

where C is a constant of integration. From (17a) and (10), we get the following dependence

$$\left(\ln(A_2)\right)^2 - 2\alpha A_2 \approx \left(\ln(A_2)\right)^2 = -2C \ln(A_2) + \frac{2r\epsilon C_n^2 x}{\mu_n J} + C_1 \tag{17b}$$

or the equivalent form

$$\left(\ln(A_2) + C\right)^2 = \frac{2r\epsilon C_n^2 x}{\mu_n J} + C_2 \tag{17b}$$

where C_1 and C_2 are constants of integration. Therefore, the function $E(x)$ is of the form

$$E(x) = \frac{J}{r\epsilon C_n} \left(\frac{2r\epsilon C_n^2 x}{\mu_n J} + C_2\right)^{1/2} + C_3 \exp\left\{\left(\frac{2r\epsilon C_n^2 x}{\mu_n J} + C_2\right)^{1/2}\right\} \tag{18}$$

where C_3 is a new constant of integration. When $C_3 \rightarrow 0$ (that is $|\ln(A_2)| \gg \alpha A_2$), then the functions (18) takes the simpler form

$$E^2(x) = \frac{2Jx}{\epsilon\mu_p} + E^2(0) \tag{18a}$$

for which the current-voltage characteristic can be written as

$$V = \frac{\epsilon\mu_p}{3J} \left\{ \left(E^2(0) + \frac{2JL}{\epsilon\mu_p} \right)^{3/2} - E^3(0) \right\} \text{ and } J = f_0[E(0)] \tag{18b}$$

In the particular case when $f_0^{-1}(J) \ll (2JL/\epsilon\mu_p)^{1/2}$, where f_0^{-1} is the inverse function, the current-voltage characteristic (18b) becomes

$$J = \frac{9}{8} \epsilon\mu_p \frac{V^2}{L^3} \tag{19}$$

which is Child's law. The condition $f_0^{-1}(J) \gg (2JL/\epsilon\mu_p)^{1/2}$ determines the uniform electric field $E(x) \equiv V/L$ for $x \in \langle 0; L \rangle$. In the case when $C_2 \rightarrow 0$, then the function (18) results in

$$E(x) = \left(\frac{2Jx}{\epsilon\mu_p}\right)^{1/2} + E(0) \exp\left\{\left(\frac{2r\epsilon C_n^2 x}{\mu_n J}\right)^{1/2}\right\} \tag{20}$$

Hence, on the basis (6a) we obtain the $J(V)$ curve in the other parametric form

$$V = \frac{2}{3} \left(\frac{2JL^3}{\epsilon\mu_p}\right)^{1/2} + 2E(0)L \text{ and } J = f_0[E(0)] \tag{20a}$$

for $J \gg 2r\epsilon C_n^2 L/\mu_n$.

3.2

The carrier generation-recombination conditions with immobile positive charge $\mu_p = 0$

Now, let us assume that electron transition from a normal atom to an adjacent vacancy does not occur. Under these conditions, the vacancies become immobile. With additional assumption of

coefficient equality $C_{12} = C_{21}$, we have

$$A_2 = 1; \quad A_3 = A_4 = \frac{C_n}{v_n} = \frac{1}{\theta_0}; \quad A_1 = \frac{(q\mu_n)^2 v_p N_{t2} \theta_0 E^2}{C_p J^2} \tag{21}$$

Hence, on the basis (9), we obtain the following differential equation

$$\frac{\epsilon\mu_n E dE}{J dx} = \frac{(q\mu_n)^2 v_p N_{t2} \theta_0 E^2}{C_p J^2} - (1 + 2/\theta_0) \tag{22}$$

for which the general integral has the form

$$E = \frac{J}{\beta^{1/2}} \left\{ \theta + C \exp\left(\frac{2\beta x}{\epsilon\mu_n J}\right) \right\}^{1/2}; \tag{23}$$

$$\theta = 1 + 2/\theta_0; \quad \beta = (q\mu_n)^2 v_p N_{t2} \theta_0 / C_p$$

where C is a constant of integration which can be expressed by the boundary value for $x = L$

$$C = \left(\frac{\beta E^2(L)}{J^2} - \theta\right) \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \tag{23a}$$

From (22), (23) and (6a) it follows that the voltage function $V = V(E(L), J)$ has the form

$$V = \frac{\epsilon\mu_n J^2}{\beta^{3/2}} \left\{ \frac{\beta^{1/2}}{J} (E(L) - E(0)) + \frac{\theta^{1/2}}{2} \ln \left| \frac{(\beta^{1/2} E(L) - J\theta^{1/2})(\beta^{1/2} E(0) + J\theta^{1/2})}{(\beta^{1/2} E(L) + J\theta^{1/2})(\beta^{1/2} E(0) - J\theta^{1/2})} \right| \right\} \tag{24}$$

$$E(0) = \frac{J}{\beta^{1/2}} \left\{ \theta \left[1 - \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \right] + \frac{\beta E^2(L)}{J^2} \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \right\}^{1/2} \tag{24a}$$

Therefore, the current voltage characteristic is described parametrically by (24) and (24a) and the boundary function $J = f_L[E(L)]$. In the most simple case when the boundary function is linear $J = (\beta/\theta)^{1/2} E(L)$, we obtain Ohm's law $J = \sigma V/L$ with the conductivity parameter

$$\sigma = \frac{q\mu_n (v_p N_{t2} \theta_0)^{1/2}}{C_p^{1/2} (1 + 2/\theta_0)^{1/2}} \tag{25}$$

An interesting case occurs when the electrode $x = L$ injects an infinite quantity of electrons, that is $E(L) \rightarrow 0$. Then, (24) and (24a) yield

$$V = \frac{\epsilon\mu_n J^2}{\beta^{3/2}} \left\{ \frac{\theta^{1/2}}{2} \ln \left| \frac{1 + \left[1 - \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \right]^{1/2}}{1 - \left[1 - \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \right]^{1/2}} \right| - \theta^{1/2} \left[1 - \exp\left(\frac{-2\beta L}{\epsilon\mu_n J}\right) \right]^{1/2} \right\} \tag{26}$$

and additionally, for the values of the current density $J \gg 2\beta L / (\varepsilon\mu_n)$, we have

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}} \left\{ \frac{\theta^{1/2}}{2} \ln \frac{1+W}{1-W} - \theta^{1/2} W \right\}; \quad W = \left(\frac{2\beta L}{\varepsilon\mu_n J} \right)^{1/2} \quad (26a)$$

Expanding the logarithm as far as terms in W^3 , we obtain finally

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}} \left\{ \theta^{1/2} \left(W + \frac{W^3}{3} \right) - \theta^{1/2} W \right\} = \frac{\varepsilon\mu_n \theta^{1/2} J^2}{3\beta^{3/2}} W^3 \quad (26b)$$

or the equivalent form

$$J = \frac{9}{8} \varepsilon\mu_n \theta^{-1} \frac{V^2}{L^3} \quad (26c)$$

which is also Child's law.

3.3

The hole-electron pair generation

Now we shall consider a case when electron transition from the valence band to the conduction band is dominant. In this case we assume that $C_n = C_{12} = C_p = 0$ in (1a)-(5a). Under these conditions, we obtain the following differential equation

$$\frac{dE}{dx} = \alpha_1 \frac{x - x_0}{E} - \alpha_2 \quad (27)$$

where

$$\alpha_1 = \frac{(\mu_p + \mu_n) q v_p N_{12}}{\varepsilon \mu_n \mu_p}; \quad \alpha_2 = \frac{q v_p N_{12}}{\varepsilon} \left(\frac{1}{C_{21}} + \frac{1}{v_n} \right);$$

$$x_0 = L + \frac{[(r+1)A_2(L) - 1]J}{(r+1)q v_p N_{12}} \quad (27a)$$

where x_0 is the constant of integration. From (27) we get two singular solutions

$$E_1(x) = z_1(x - x_0); \quad E_2(x) = z_2(x - x_0) \quad (27b)$$

where

$$z_{1,2} = -\frac{\alpha_2}{2} \pm \frac{1}{2} (\alpha_2^2 + 4\alpha_1)^{1/2}; \quad z_1 > 0; \quad z_2 < 0 \quad (27c)$$

for $x_0 \leq 0$ or $x_0 \geq L$, respectively. Using the integral condition (6a), we can find the relation between the voltage V and the boundary parameter x_0 in the form

$$V = \frac{1}{2} (z_2(L - x_0)^2 - z_2 x_0^2) = \frac{L}{2} (2E(L) - z_2 L) \text{ for } x_0 \geq L \quad (28)$$

or

$$V = \frac{1}{2} (z_1(L - x_0)^2 - z_1 x_0^2) = \frac{L}{2} (2E(0) + z_1 L) \text{ for } x_0 \leq 0 \quad (29)$$

Hence, it follows that the current-voltage dependence $J(V)$ is determined by the boundary function $J = f_L[E(L)]$ or $J = f_0[E(0)]$ describing the mechanism of carrier injection from the electrode $x = L$ or $x = 0$ into the bulk. Since $E(x) \geq 0$ for $x \in \langle 0, L \rangle$, therefore this dependence is as follows

$$J = f_L[(V - V_{02})/L] \quad \text{for } V \geq V_{02} = |z_2|L^2/2 \quad (30)$$

or

$$J = f_0[(V - V_{01})/L] \quad \text{for } V \geq V_{01} = |z_1|L^2/2 \quad (31)$$

From (27a) it follows that the constant of integration $A_2(L)$ depends on the boundary values $E(L)$ in the form

$$A_2(L) = \frac{q v_p N_{12} E(L)}{|z_2|J} + \frac{1}{r+1}; \quad E(L) = z_2(L - x_0)$$

Since $x_0 \geq L$ and $A_2(L) < 1$, therefore the boundary function f_L and the boundary parameter $E(L)$ must satisfy the following condition

$$E(L) < \frac{r|z_2|f_L[E(L)]}{q(r+1)v_p N_{12}} \quad (32)$$

Analogously, proceeding for the conduction $x_0 \leq 0$ and $A_2(0) < 1$, we can find the conduction for the boundary function f_0 and the boundary parameter $E(0)$. It is worth noting that the Schottky function and the Fowler-Nordheim function as well as the power boundary function fulfill the condition (32).

4

Discussion

In this section we wish to refer to two analytical methods which have solved the problem of double injection in the steady state. On this basis we can compare our analysis with those methods and as well indicate the difficulties of the bipolar space charge theory.

A regional approximation method for double injection in insulators and semiconductors has been used by Lampert and Schwob. In this method, the positive and negative as well as quasineutral charge regions have been distinguished in the space $x \in \langle 0, L \rangle$. These regions are as follows

(i) the first region is $x \in \langle 0, x_1 \rangle$ (the anode region) in which $\varepsilon dE/dx = f_1(n, p) > 0$,

(ii) the second region is $x \in \langle x_1, x_2 \rangle$ in which $\varepsilon dE/dE = f_2(n, p) \approx 0$,

(iii) the third region is $x \in \langle x_2, L \rangle$ (the cathode region) in which $\varepsilon dE/dx = f_3(n, p) < 0$

where f_1, f_2 , and f_3 are the given functions.

With the boundary conductions such as (a) $E(0) = E(L) = 0$ and (aa) continuity of the electric field at the junction planes $x = x_1$ and $x = x_2$, the current voltage characteristics have been obtained. These functions can be $J \sim V$, $J \sim V^2/L^3$ (Child's law) $J \sim V^3/L^5$, $J \sim V^{l+1}/(V_0 - V)^l$ where V_0 and l are the constant parameters characterizing the material.

A small signal theory for diffusion problem has been presented by Manificier and Henisch. In this method, the (i)-(iii) space

charge regions have also been distinguished. With the condition $x_1 \rightarrow 0$ and $x_2 \rightarrow L$ for $E(0) = E(L) = 0$ or $E(0) = E(L) = V/L$, the diffusion problem equations have been written as linearized equations. With such assumptions, the functions $p(x)$ and $n(x)$ as well as $V = V(\mu_n, \mu_p)$ have been found.

These concepts contain fundamental physical processes, but analytical methods are not mathematically clear. In general, from the Gauss' equation and the continuity equation it follows that the electric conduction problem is to find the space charge distribution in the bulk. On this basis we ascertain that the quasineutrality assumption used by these theories are not understood. Moreover, with this assumption the set of solutions is very limited.

From our considerations it follows that the space charge density $\varepsilon dE/dx$ is determined by the transport equations (1a)–(6a) and the boundary functions $f_0[E(0)]$ and $f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes into the bulk. This is the fundamental difference between our methodology and those theories. Also, from our considerations it follows that the bipolar space charge transport can be described by the singular solutions (27b) which are determined by one boundary condition.

In order to explain this mathematical detail, let us now return to (1)–(6). Using the theory of characteristics, we ascertain that the initial conditions $p(x, 0)$, $n(x, 0)$, $n_1(x, 0)$, $n_2(x, 0)$ and the boundary conditions $p(0, t)$ and $n(L, t)$ determine the transient state. The fundamental problem of our considerations is to find the physical aspect for the boundary values of $p(0, t)$ and $n(L, t)$. In this paper we assumed that the convection current $J(x, t)$ and the electric field intensity are continuous at the plane $x = 0$ and $x = L$. From the field theory it follows that this assumption is equivalent to $q_s(t)|_{x=0}^{x=L} = 0$ where q_s is the surface charge density at the plane $x = 0$ and $x = L$. With this assumption we can write the following boundary conditions

$$J(0, t) = q[\mu_n n(0, t) + \mu_p p(0, t)]E(0, t) = f_0[E(0, t)] \quad (33)$$

$$J(L, t) = q[\mu_n n(L, t) + \mu_p p(L, t)]E(L, t) = f_L[E(L, t)] \quad (34)$$

where

$$\varepsilon E(L, t) = \int_0^L q_v(x, t) dx + \varepsilon E(0, t);$$

$$\varepsilon L E(0, t) = \varepsilon V - \int_0^L (L - x) q_v(x, t) dx; \quad (35)$$

$$q_v = q(p - n - n_1 - n_2)$$

With these boundary conditions the current voltage characteristics (6d)–(6f) are valid. In the case of (30) there must be

$$p(0, t) = \frac{\mu_n}{q(\mu_n + \mu_p)} \left\{ \frac{f_L[E(L, t)]}{\mu_n E(0, t)} + \varepsilon(\alpha_2 + z_2) \right\}$$

and

$$n(L, t) = \frac{\mu_n}{q(\mu_n + \mu_p)} \left\{ \frac{f_L[E(L, t)]}{\mu_n E(L, t)} - \frac{\varepsilon \mu_p}{\mu_n} (\alpha_2 + z_2) \right\} \quad (36)$$

Analogously, using the boundary function f_0 and the parameter z_1 , we can find the boundary values $p(0, t)$ and $n(L, t)$ for (31). Thus, we see that the singular solutions (27b) can exist.

5

Conclusions

In this paper we have presented a problem of space charge transport with one or two mobilities. In this problem we showed that there exist electric field intensity distributions $E(x)$ which can be decreasing or increasing for $x \in \langle 0; L \rangle$. For this set of the functions $E(x)$ the $J(V)$ curves can be determined by two boundary conditions or one condition. Usually, the current-voltage characteristics are of the parametric form and are nonlinear. From (24) it follows that the $J(V)$ curve can be discontinuous. The conduction model (1)–(6) can explain the experimental current-voltage characteristics for the insulators such as ZnS, CdS, SiO₂, TiO₂, Al₂O₃, anthracene, polyethylene and for the typical semiconductors such as Ge and Si.

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