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New solution to the problem of bipolar injection in insulator and semiconductor devices

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Content A problem of the space charge transport with two mobilities is presented. It is found that interactions between carriers can be described by the $n(p)$ relationships. Three stable states of the charged capacitor occur. The stability problem is studied by the first approximation method. The laws of Ohm, Fowler-Nordheim, Schottky and Child are obtained.

Neue Lösung des Problems der bipolaren Injektion in Isolierstoffen und Halbleitern

Übersicht Im Beitrag wird das Problem der Raumladung mit zwei Bewegungsrichtungen dargestellt. Es wird festgestellt, daß die gegenseitige Wirkung von Ladungsträgern durch die $n(p)$ Abhängigkeit beschrieben werden kann. Es treten dabei drei stationäre Zustände des geladenen Kondensators auf. Das Problem der Stabilität wird durch die Anwendung der Methode der ersten Approximation untersucht. Es werden die Gesetze von Ohm, Fowler-Nordheim, Scottky und Child hergeleitet.

1 Introduction

The different mathematical methods describing double injection in a solid have been considered. The carrier flow through an insulator has been solved by a regional approximation method [1–5]. In this concept, the transition region between the anode and cathode region has been distinguished. Using quasineutrality assumption, the different relations $n(p)$ between positive and negative charge carrier concentrations have been obtained.

In general, with this assumption set of the solutions is very limited. Moreover, in the case of strong asymmetric double injection the quasineutrality assumption cannot be made [6].

In this work we further present our theoretical analysis describing space charge transport between two electrodes. The purpose of this work is to find new relationships $n(p)$ and to define the current-voltage characteristics for space charge conditions.

2 The basic equations

In our theoretical analysis we make the following assumptions:

(I) The planar capacitor with the anode $x = 0$ and the cathode $x = L$ will be used,

(II) The potential barrier width at the electrodes is small in comparison with the mean free path and there are no surface states at the metal-bulk interfaces,

(III) The carrier diffusion is unimportant [7, 8],

(IV) The carrier mobilities are independent of the electric field intensity,

(V) Interaction between carriers is described by the bimolecular recombination rate [9–11].

With these assumptions the basic equations are the continuity equation, the Gauss equation, the generation-recombination equation and the integral condition. These equations are as follows

$$\frac{\partial}{\partial x}\{[\mu_p p(x,t) + \mu_n n(x,t)]E(x,t)\} + \frac{\partial p(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} = 0 \tag{1}$$

$$\varepsilon_\infty \frac{\partial E(x,t)}{\partial x} = q\{(p(x,t) - p_0) - (n(x,t) - n_0)\} \tag{2}$$

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial}{\partial x}[\mu_n n(x,t)E(x,t)] - \beta(p(x,t)n(x,t) - n_0 p_0) \tag{3}$$

$$\int_0^L E(x,t)\,\mathrm{d}x = V; \quad V = \text{constant}; \ V > 0 \tag{4}$$

where q is the electron charge, ε_∞ the high-frequency permittivity, μ_n and μ_p are the electron and hole mobility, respectively, E the electric field intensity, x the distance from the electrode, t the time, p and n are the hole and electron concentrations, respectively, $n_0 = p_0$ the equilibrium concentrations of carriers, β the recombination coefficient, V the applied voltage.

For the above space charge problem we will find the relationships $n(p)$ and we will define their importance for the current-voltage characteristics.

3 The solution of the problem

In this part we shall show that a function $n(p)$ is valid for the steady and transient state of the current flow. First, let us investigate the steady state.

3.1 The steady state

From (1)–(4) it follows that the equations describing the stationary state have the following form

$$J = q\mu_p p(x)E(x) + q\mu_n nE(x); \quad J = \text{constant} \tag{1a}$$

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$$\varepsilon_\infty \frac{dE(x)}{dx} = q(p(x) - n(x)) \quad (2a)$$

$$\frac{d}{dx}\{\mu_n n(x)E(x)\} - \beta(n(x)p(x) - n_0^2) = 0 \quad (3a)$$

$$\int_0^L E(x)\,dx = V \quad (4a)$$

where J is the electric current density. Next, by combining (1a)–(3a), we obtain the following differential equation

$$-\varepsilon_\infty \mu_n \mu_p \frac{d}{dx}\left(E\frac{dE}{dx}\right) = \frac{\beta J^2}{q(\mu_p + \mu_n)E^2}\left[\left(1 + \frac{\varepsilon_\infty \mu_n E}{J}\frac{dE}{dx}\right) \cdot \left(1 - \frac{\varepsilon_\infty \mu_p E}{J}\frac{dE}{dx}\right) - \frac{q^2(\mu_p + \mu_n)^2 n_0^2 E^2}{J^2}\right] \quad (5)$$

Equation (5) is easily solved for the case of identical values of the carrier mobilities $\mu_n = \mu_p = \mu$. With this assumption, we shall make use of the following substitutions

$$y = E^2; \quad \frac{dy}{dx} = w(y); \quad \frac{d^2y}{dx^2} = w\frac{dw}{dy} \quad (6)$$

By combining (5) and (6), we obtain the Bernoulli type differential equation

$$-w\frac{dw}{dy} = \frac{\beta J^2}{q\varepsilon_\infty \mu^3 y}\left[1 - \left(\frac{\varepsilon_\infty \mu}{2J}w\right)^2 - \frac{(2q\mu n_0)^2}{J^2}y\right] \quad (7)$$

It is well known that the Bernoulli differential equation always leads to the linear differential equation. In our case, using the substitution [12]

$$z = 1 - \left(\frac{\varepsilon_\infty \mu}{2J}w\right)^2 \quad (8)$$

we get the following linear equation

$$\frac{dz}{dy} - \frac{\varepsilon_\infty \beta z}{2q\mu y} = -\frac{2\varepsilon_\infty \beta \mu q n_0^2}{J^2} \quad (9)$$

for which the general integral has the form

$$z = Cy^\chi - \frac{2\varepsilon_\infty \beta \mu q n_0^2}{J^2(1-\chi)}y; \quad \chi = \frac{\varepsilon_\infty \beta}{2q\mu}; \quad \chi \neq 1 \quad (10)$$

where C is a constant of integration. In the case when $\chi = 1$, the above solution must be replaced by

$$z = Cy - \left(\frac{\sigma_\Omega}{J}\right)^2 y \ln y; \quad \sigma_\Omega = 2q\mu n_0; \quad \text{and } \chi = 1 \quad (11)$$

Now, we shall show that a function $n(p)$ exists. From (1a)–(2a), (6) and (8), it follows that the carrier concentration product can be expressed by

$$n(x)p(x) = \frac{J^2 z}{4q^2\mu^2 y} \quad (12)$$

Hence, on the basis (10) and (11) we obtain the following two relationships

$$n(x)p(x) = n(0)p(0) = \text{constant} \quad \text{for } n_0 = 0 \quad \text{and } \chi = 1 \quad (13)$$

as well as

$$n(x)p(x) = \frac{\chi n_0^2}{\chi - 1} = \text{constant}; \quad \text{and } C = 0 \quad (14)$$

The relationship (13) is written for the perfect insulator. In this case the product np depends on the boundary conditions. The relationship (14) is independent of the boundary conditions. In the particular case when $\chi \gg 1$ the solution (14) leads to the law of mass action $n(x)p(x) = n_0^2$. It is worth noting that this law is also determined by the Fermi-Dirac distribution function which is written at the thermodynamic equilibrium conditions. Since the function (14) is independent of the mechanisms of carrier injection from the electrodes into the bulk, the equations (1a), (2a), (4a) and (14) can define the new model of electric conduction. We may suppose that this is possible when these equations determine the stable state. In what follows, we shall investigate the stability of (1a), (2a), (4a) and (14).

3.2
The transient state

In order to define the problem of the stability we must take into account the equations describing the transient state of the current flow. To this end, we shall use the dimensionless variable system

$$Q_1 = \frac{p}{K^{1/2}}; \quad Q_2 = -\frac{n}{K^{1/2}}; \quad K^2 = \frac{\chi n_0^2}{\chi - 1}; \quad x^* = \frac{x}{L} \quad (15)$$

$$E^* = \frac{\varepsilon_\infty E}{qLK^{1/2}}; \quad V^* = \frac{\varepsilon_\infty V}{qL^2K^{1/2}}; \quad t^* = \frac{\mu q K^{1/2} t}{\varepsilon_\infty}$$

Therefore, the transient state is described by the following equations

$$\frac{\partial}{\partial x^*}\{(Q_1 - Q_2)E^*\} + \frac{\partial Q_1}{\partial t^*} + \frac{\partial Q_2}{\partial t^*} = 0 \quad (16)$$

$$\frac{\partial E^*}{\partial x^*} - Q_1 - Q_2 = 0; \quad Q_1 Q_2 + 1 = 0 \quad (17)$$

with condition

$$\int_0^1 E^* dx^* = V^* \quad (18)$$

where E^*, Q_1 and Q_2 depend on the distance x^* and the time t^*. Next, let us define the equations of the steady state for the model

(16)–(18). These equations have the form

$$\frac{d}{dx^*}\{(Q_1^*(x^*)-Q_2^*(x^*))E_s^*(x^*)\}=0 \tag{19}$$

$$\frac{dE_s^*(x^*)}{dx^*}-Q_1^*(x^*)-Q_2^*(x^*)=0;\quad Q_1^*(x^*)Q_2^*(x^*)+1=0 \tag{20}$$

with the integral condition

$$\int_0^1 E_s^*(x^*)\,dx^*=V^* \tag{21}$$

Equations (19)–(21) correspond to (1a), (2a), (4a) and (14). From (19)–(21) we obtain three solutions. These solutions are as follows
a) the first solution is the uniform electric field E_s^*

$$E_s^*(x^*)=V^*;\quad Q_1^*(x^*)=-Q_2^*(x^*)=1\quad\text{and } 0\leqq x^*\leqq 1 \tag{22}$$

b) the second and third solutions are the monotonic functions E_s^*

$$E_s^*\frac{dE_s^*}{dx^*}=\pm[J^{*2}-4E_s^{*2}]^{1/2};\quad Q_2^*=-\frac{J^*\pm[J^{*2}-4E_s^{*2}]^{1/2}}{2E_s^*}\quad\text{and } 0\leqq x^*\leqq 1 \tag{23}$$

where J^* is the constant which depends on the current density J in the form

$$J^*=\varepsilon_\infty J/(q^2\mu LK). \tag{23a}$$

Let us now introduce the small parameter $\varepsilon=\varepsilon(x^*,t^*)$ into the function $Q_2^*(x^*)=-f(x^*)$ in order to obtain the functions $Q_2=Q_2(x^*,t^*)$ and $Q_1=Q_1(x^*,t^*)$ in the form

$$Q_2=-f+\varepsilon;\quad Q_1=-\frac{1}{Q_2}=\frac{1}{f}+\frac{\varepsilon}{f^2}+\frac{\varepsilon^2}{f^3}+\cdots;\quad f=f(x^*) \tag{24}$$

Next, we must introduce the assumptions defining the ε-function space. These assumptions are as follows

- (i) all the functions $\varepsilon=\varepsilon(x^*,t^*)$ are infinitesimal in the region $0\leqq x^*\leqq 1$ and $t^*\geqq 0$,
- (ii) all the derivatives of the functions $\varepsilon=\varepsilon(x^*,t^*)$ are continuous and limited in the region $0\leqq x^*\leqq 1$ and $t^*\geqq 0$,
- (iii) in the ε-function space there exist the uniform convergent sequences ε_ν $(\nu=1,2,3\ldots)$,
- (iv) the boundary values of $\varepsilon(0,t^*)$ are equal to zero for $t^*\geqq t_0^*$, where t_0^* is a finite number.

Thus, in this space the distance $\rho(\varepsilon_1,\varepsilon_2)$ between two arbitrary points ε_1 and ε_2 can be determined by

$$\rho(\varepsilon_1,\varepsilon_2)=\max_{x^*,t^*}\left|\frac{\partial\Delta\varepsilon}{\partial t^*}\right|+\max_{x^*,t^*}\left|\frac{\partial\Delta\varepsilon}{\partial x^*}\right|+\max_{x^*,t^*}|\Delta\varepsilon|;\quad \Delta\varepsilon=\varepsilon_1-\varepsilon_2$$

In this ε-function space we can define the so-called orbital stability. Generally, the stability problem can be solved by the variational method (the first approximation method). The main idea of the first approximation method is to find the difference between the left-hand side of (16)–(17) and (19)–(20) in the form

$$\Delta F=F(\varepsilon)-F(\varepsilon=0)=\delta F(\varepsilon)+\delta^2F(\varepsilon)=0;$$

$$\lim_{\rho\to 0}\delta^2F(\varepsilon)/\rho(\varepsilon,0)=0;$$

$$\int_0^1(E^*(\varepsilon)-E_s^*)\,dx^*=\int_0^1(\delta E^*(\varepsilon)+\delta^2E^*(\varepsilon))\,dx^*=0;$$

$$\lim_{\rho\to 0}\delta^2E^*(\varepsilon)/\rho(\varepsilon,0)=0$$

where $F(\varepsilon)$ is the left-hand side of (16)–(17), $F(\varepsilon=0)$ the left-hand side of (19)–(20), $\delta F(\varepsilon)$ the main linear part of ΔF, $\delta^2F(\varepsilon)$ the nonlinear part of ΔF. Now, let us define the variation δE^*. This functional is of the form

$$\delta E^*=\int_0^{x^*}\left(1+\frac{1}{f^2}\right)\varepsilon\,dx^*-\int_0^1 dx^*\int_0^{x^*}\left(1+\frac{1}{f^2}\right)\varepsilon\,dx^*$$

$$=\int_{x_1^*}^{x^*}\left(1+\frac{1}{f^2}\right)\varepsilon\,dx^*;\quad x_1^*\in\langle 0;1\rangle$$

We shall assume that the variation δE^* is arbitrary small, that is $\delta E^*=0$. This condition is possible in the ε-function space. Practically, this condition denotes that the perturbation of the electric field intensity is infinitesimal. The stability of the system can be determined by the following implication

$$\text{if } \delta F(\varepsilon)=0\quad\text{and } \delta E^*=0$$

$$\text{then } \lim_{t^*\to\infty}\varepsilon(x^*,t^*)=0\quad\text{and } \lim_{\varepsilon\to 0}\rho(\varepsilon)=0$$

for $0\leqq x^*\leqq 1$, where $\rho(\varepsilon)=\rho(\varepsilon,0)$.

Let us illustrate this problem for (22) and (23). First, we shall investigate the stability of (22). In this case we must assume that $f\equiv 1$. Hence, on the basis (24) and (16)–(17) we ascertain that the condition $\delta F(\varepsilon)=0$ has the form

$$\frac{\partial\varepsilon}{\partial t^*}+\frac{\partial E^*}{\partial x^*}=0\quad\text{and}\quad\frac{\partial E^*}{\partial x^*}=2\varepsilon$$

therefore

$$\frac{\partial\varepsilon}{\partial t^*}+2\varepsilon=0 \tag{25}$$

Hence, the small parameter ε is of the form

$$\varepsilon(x^*,t^*)=\varepsilon(x^*,0)\exp(-2t^*);\quad 0\leqq x^*\leqq 1 \tag{25a}$$

Now, we can define the functions $\varepsilon(x^*,0)$ for which the condition $\delta E^*=0$ is satisfied. As an example, let us take into account the following function

$$\varepsilon(x^*,0)=\begin{cases}\varepsilon_1(x^*) & \text{for } x^*\in\langle 0;x_0^*\rangle\\ 0 & \text{for } x^*\in\langle x_0^*;1\rangle\end{cases}\quad\text{and}\quad x_0^*\to 0+$$

where ε_1 is the arbitrary differentiable function. In the particular case, when ε_1 is of the form $\varepsilon_1 = \phi(x^*, v)\sin(vx^*)$ or $\varepsilon_1 = \phi(x^*, v)\cos(vx^*)$, where ϕ is the arbitrary differentiable and infinitesimal function as $v \to \infty$, then we have $\delta E^* = 0$ for all values $x_0^* \in \langle 0; 1\rangle$. This property follows from one of the Dirichlet theorems. We can easily verify that the variations $\delta^2 E^*(\varepsilon)/\rho(\varepsilon)$ and $\rho(\varepsilon)$ are arbitrarily small as $\varepsilon \to 0$. Thus, the solution (22) is stable. Now, let us investigate the stability of the system for (23). We must notice that the solution (23) defines the positive or negative space charge density. As an example, let us consider the function $f(x^*)$ describing the negative space charge density, that is $f(x^*) > 1$ and $\mathrm{d}f/\mathrm{d}x^* > 0$ for $x^* \in \langle 0; 1\rangle$. To this end, we shall introduce the new symbols

$$a = a(x^*) = 1 + \frac{1}{f^2_{(x^*)}}; \quad b = b(x^*) = \frac{1}{f^2_{(x^*)}} - 1;$$

$$g = g(x^*) = f(x^*) + \frac{1}{f(x^*)}$$

With these symbols, the operator $F(\varepsilon)$ is defined by

$$\frac{\partial (Q_1 + Q_2)}{\partial t^*} = a\frac{\partial \varepsilon}{\partial t^*} + \cdots; \quad Q_1 - Q_2 = g + \varepsilon b + \cdots;$$

$$\frac{\partial}{\partial x^*}\{(Q_1 - Q_2)E^*\} = \left(\frac{\mathrm{d}g}{\mathrm{d}x^*} + \frac{\partial(\varepsilon b)}{\partial x^*}\right)(E_s^* + \delta E^*) + $$
$$+ (g + \varepsilon b)\left(\frac{\mathrm{d}E_s^*}{\mathrm{d}x^*} + a\varepsilon\right) + \cdots$$

Thus, taking into account the condition $\delta E^* = 0$, we can write the condition $\delta F(\varepsilon) = 0$ in the following form

$$a\frac{\partial \varepsilon}{\partial t^*} + \frac{\partial (bE_s^*\varepsilon)}{\partial x^*} + ag\varepsilon = 0 \tag{26}$$

Next, using the substitution $\vartheta = \ln|\varepsilon b E_s^*|$ and the theory of characteristics, we can write the following ordinary equations

$$\frac{\mathrm{d}x^*}{\mathrm{d}t^*} = \frac{bE_s^*}{a} \quad \text{and} \quad \frac{\mathrm{d}\vartheta}{\mathrm{d}t^*} = -g$$

Hence

$$t^* = \varphi(x^*) - \varphi(x^*(t^* = 0))$$

and

$$\vartheta = -\int_0^{t^*} g[x^*(s)]\,\mathrm{d}s - G_1(x^*(t^* = 0))$$

therefore

$$\varepsilon(x^*, t^*) = \pm\frac{1}{b(x^*)E_s^*(x^*)}\exp\left\{G[\varphi(x^*) - t^*] - \int_0^{t^*} g[x^*(s)]\,\mathrm{d}s\right\}$$

or

$$\varepsilon(x^*, t^*) = \varepsilon(0, t^* - \varphi(x^*))\frac{b(0)E_s^*(0)}{b(x^*)E_s^*(x^*)}\exp\left\{\int_{t^*}^{t^* - \varphi(x^*)} g[x^*(s)\,\mathrm{d}s\right\} \tag{26a}$$

where

$$\varphi(x^*) = \int_0^{x^*} \frac{a(s)\,\mathrm{d}s}{b(s)E_s^*(s)}; \quad G = -G_1[\varphi^{-1}]$$

and G_1 is an arbitrary differentiable function defining the initial values of the function ϑ, and φ^{-1} is the inverse function. If the condition $\delta E^* = 0$ is to be satisfied, we must assume that the values of G_1 are infinitely large. Let us notice that the functions $b(x^*)$, $E_s^*(x^*)$, $\varphi(x^*)$ and $g(x^*)$ are limited. Since

$$\int_{t^*}^{t^* - \varphi(x^*)} g[x^*(s)]\,\mathrm{d}s = -g[x^*(c)]\varphi(x^*); \quad t^* < c < t^* - \varphi(x^*)$$

and $\varepsilon(0, t^*) = 0$ for $t^* \geqq t_0^*$, therefore the function $\varepsilon(x^*, t^*)$ is equal to zero for $t^* \geqq t_0^* + \max_{x^*}|\varphi(x^*)|$ and for $x^* \in \langle 0; 1\rangle$. Analogously, proceeding for the function $f(x^*) < 1$, we ascertain that the steady state of (16)–(18) is stable.

Now, we can define the importance of (13) and (14) for the current-voltage characteristics

3.3 The current-voltage characteristics

It is worth noting that the solution (22) defines Ohm's law $J^* = 2V^*$. The similar result can follow from (23). In this case, the function $E_s^*(x^*)$ has the form

$$E_s^*(x^*) = \tfrac{1}{2}\{J^{*2} - 16(x^* + C)^2\}^{1/2} \tag{27}$$

where C is a constant of integration. Next, using the boundary condition $J^* = 2E_s^*(x^* = 0)$ and (21), we obtain the following current-voltage dependence

$$V^* = \frac{1}{16}\left\{J^{*2}\arcsin\left(\frac{4}{J^*}\right) + 4[J^{*2} - 16]^{1/2}\right\} \tag{27a}$$

In the particular case when $J^* \gg 4$, then $J^* = 2V^*$, that is $J = \sigma V/L$, where the conductivity parameter σ is

$$\sigma = \left(\frac{\chi}{\chi - 1}\right)^{1/2}\sigma_\Omega \tag{28}$$

In the special case when $\chi \gg 1$, that is $\varepsilon_\infty\beta \gg 2q\mu$, the conductivity parameter σ becomes the ohmic conductivity $\sigma_\Omega = 2q\mu n_0$. The other interesting result follows from (1a), (2a), (4a) and (13). In this case, the electric field intensity distribution satisfies the equation

$$E\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{1}{\varepsilon_\infty\mu}(J^2 - A_1E^2)^{1/2} \tag{29}$$

therefore

$$E = A\left\{J^2 - \left(\frac{A_1}{\varepsilon_\infty\mu}\right)^2(x + B)^2\right\}^{1/2} \quad A = A_1^{-1/2} \tag{29a}$$

where A and B are constants of integration. Making use of the boundary conduction $(\mathrm{d}E/\mathrm{d}x)|_{x=0} = 0$, we obtain the current-voltage dependence $J = J(V)$ in the following parametric form

$$V = \frac{\varepsilon_\infty \mu E_{(0)}^3}{2J}\left\{\arcsin\left(\frac{JL}{\varepsilon_\infty \mu E_{(0)}^2}\right) + \frac{L}{\varepsilon_\infty \mu E_{(0)}^2}\left[J^2 - \left(\frac{J^2 L}{\varepsilon_\infty \mu E_{(0)}^2}\right)^2\right]^{1/2}\right\} \quad (30)$$

and

$$J = f_0[E(0)]$$

where $f_0[E(0)]$ is the boundary function describing the mechanism of carrier injection from the electrode $x = 0$ into the bulk. Moreover, with the boundary condition $(dE/dx)|_{x=0} = 0$, we have

$$E(L) = E(0)\left\{1 - \left(\frac{JL}{\varepsilon_\infty \mu E_{(0)}^2}\right)^2\right\}^{1/2} \quad (30a)$$

Let us consider some special cases of f_0. The boundary function is quadratic $J = a_0 E^2(0)$, where a_0 is the boundary parameter. In this case we have $J \sim V^2$. In the particular case when $a = \varepsilon_\infty \mu / L$ we get Child's law in the form

$$J = \frac{16}{\pi^2}\varepsilon_\infty \mu \frac{V^2}{L^3} \quad (31)$$

and $E(L) = 0$. From (30), we can obtain the other functions $J(V)$, namely

$$J = f_0[V/L] \quad \text{for} \ \frac{\varepsilon_\infty \mu V^2}{L^3} \gg f_0[V/L] \quad \text{or} \ f_0^{-1}(J) \gg \left(\frac{LJ}{\varepsilon_\infty \mu}\right)^{1/2} \quad (32)$$

where f_0^{-1} is the inverse function. For instance, if f_0 is the Fowler-Nordheim function in the form $f_0 = a_0 E_0^2 \exp(-b_0/E_0)$, where a_0 and b_0 are the boundary parameters and $E_0 = E(0)$, then we have

$$J = \frac{a_0 V^2}{L^2}\exp(-b_0 L/V) \quad \text{for} \quad a_0 \ll \frac{\varepsilon_\infty \mu}{L} \quad (32a)$$

If f_0 is linear, that is $f_0 = \sigma_0 E(0)$, where σ_0 is the boundary parameter, then we obtain Ohm's law $J = \sigma_0 V/L$ for $V \gg (\sigma_0 L^2 / \varepsilon_\infty \mu)$ or $J \gg (\sigma_0^2 L / \varepsilon_\infty \mu)$. Analogously, proceeding for the Schottky function $f_0 = J_0 \exp(b_1 E_0^{1/2})$, where J_0 and b_1 are the boundary parameters, we ascertain that there exist the finite numbers V_1 and V_2, such that

$$J = J_0 \exp[b_1 (V/L)^{1/2}] \quad \text{for} \quad V \in \langle V_1, V_2 \rangle \quad (32b)$$

and $\varepsilon_\infty \mu L^{-3} \gg J_0$.

4
Discussion

A problem of the $n(p)$ relationships has been presented by Schwob [4–5]. The fundamental assumption of his method is the condition $\varepsilon_\infty dE/dx \approx 0$ for $x \in (0, L)$. Using the boundary conditions $E(0) = E(L) = 0$ and $\mu_p = \mu_n = \mu$, Schwob determined the different functions $n(p)$ and interpreted them in the n–p plane. In this plane, he distinguished four regions which correspond to the configurations *p–i–n*, *n–i–p*, *n–i–n*, *p–i–p*. In this concept, the current-voltage characteristics $V(J)$ has been described by

$$V = \int_0^L \frac{J}{\mu_p p + \mu_n n}\frac{dx}{ds}ds; \quad (ds)^2 = (dn)^2 + (dp)^2; \quad s = s(x) \quad (33)$$

On this basis, it was found that the quadratic function has the form

$$J = 32\varepsilon_\infty \mu V^2/(\pi^2/L^3) \quad (33a)$$

It is worth noting that this function has the similar form to (31). In general, let us notice that the fundamental assumptions of this method with the boundary conditions $E(0) = E(L) = 0$ are not mathematically clear when the potential barrier width at the electrodes is much smaller than the distance L. In this case, the voltage condition (4) and quasineutrality assumption must lead to the function $E(x) \equiv V/L$. Also, from (1)–(4) it follows that the fundamental problem of the current flow between the anode and the cathode is to determine the space charge density distribution. According to our methodology, we have shown that there exist two fundamental problems for the space charge transport problem. These problems are as follows

(1) The problem of the boundary conditions describing the mechanisms of carrier injection from the metal into the bulk,
(2) The problem of the stability of the stationary state.

It is worth noting that the space charge transport equations are nonlinear. Therefore, there can exist some solutions describing the steady state. On this basis, we ascertain that the problem (2) ought to be taken into account.

This problem has not been discussed by a regional approximation method [1–5]. Thus, the space charge problem requires the further mathematical considerations.

5
Conclusions

From our considerations, it follows that equations (1a)–(4a) can result in the relationships between n, p and E in the form $f_{np}(n, p, E, J, n_0, C) = 0$, where C is a constant of integration. In a particular case, the function f_{np} defines the relationship $n(p)$ in the form $n(x)p(x) = \chi n_0^2/(\chi - 1)$. This relationship corresponds to three stable states of the current flow through the bulk. These states are described by the uniform electric field and the increasing or decreasing electric field intensity distribution $E(x)$. Generally, for the monotonic function $E(x)$ the current-voltage characteristic $J = J(V)$ is nonlinear. In particular, this characteristic becomes linear $J = \sigma V/L$, where σ depends on the ohmic conductivity σ_Ω in the form (28). For the perfect insulator, that is $n_0 = 0$, the function f_{np} becomes $n(x)p(x) = n(x=0)p(x=0)$. With the boudary condition $p(0) = n(0)$, the current-voltage dependence becomes Child's, Fowler-Nordheim's and Schottky's law in the form (31)–(32b).

The present results can explain the experimental characteristics $J(V)$ for insulator and semiconductor materials such as anthracene, polyethylene, Al_2O_3, ZnS, TiO_2, GaAs, Ge, and Si.

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