

## **A small signal problem and anomalous electric conduction in the metal-solid-metal system. Further analytical and numerical results**

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A new approach to a small signal theory is presented. An electrical transport in a discharging capacitor system is analysed. A solid in which there can exist the deep or shallow trapping levels is considered. It is found that the current flow is stimulated by the trapped electrons and that the system can act as an anomalous reservoir of electrical energy (a case of solar cell).

### 1. INTRODUCTION

One of the fundamental problems of a space charge theory is to define the physical conditions describing the contact and internal processes [1–10]. As a mathematical problem, it is necessary to find the conditions for the existence of the solutions [11–20]. As one of the approximation methods describing the current flow through a solid, a small signal theory has been proposed [21–22]. That concept has been based on the quasineutrality assumption. With this assumption, a boundary problem is neglected. In this paper, we will make a new approach to a small signal method, taking into account a space charge problem.

Practically, the most interesting case occurs when the direction of the current flow in a discharging capacitor system is changed (a case of an anomalous conduction) [23–25]. The purpose of this paper is to explain an anomalous conduction in a discharging capacitor system, using a small signal method.

### 2. THE MATHEMATICAL MODEL

For our space charge problem, we make the following assumptions:

- (I) The system of atoms forming the given crystalline structure is very chaotic (this property corresponds to the different structure defects caused by pollutants and by impurities),
- (II) Consequently, the concentration of atoms is possibly maximum (that is, the coordinate number is equal to 8 or to 12), and the splitting of the energy states (the Zeeman internal effect) occurs,

(III) The energy separations in the band gap are  $\leq 0.1$  eV (under these conditions, allowed electron transitions between the valence level and the conduction level via trapping levels are very quick) [26–28],

(IV) The given crystalline structure is permanently perturbed (that is, the propagation of the perturbation of the kinetic and potential energy distribution of atoms occurs),

(V) The perturbation of the equilibrium state condition is sufficiently small, so that a law of mass action holds [29–30],

(VI) As a model system, we consider a planar capacitor system (Fig. 1).

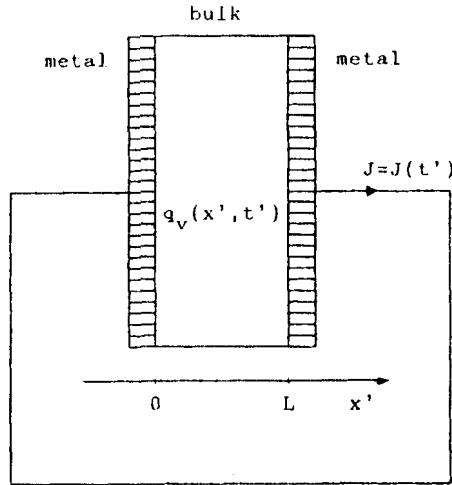


Fig. 1. The planar (discharging) capacitor system. Here,  $q_v(x', t')$  denotes the space charge density determined by the stationary state of solid conduction in a charging capacitor system with the anode  $x' = 0$  and the cathode  $x' = L$ ; the direction of the current  $J(t')$  is anomalous

Taking into consideration the above assumptions, for a discharging capacitor system, the basic equations such as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral are written as

$$\frac{\varepsilon}{q} \frac{\partial E'(x', t')}{\partial x'} = (p'(x', t') - p'_0) - (n'(x', t') - n'_0) - \sum_{i=1}^m (n'_{ii}(x', t') - n'_{ii,0}), \quad (1)$$

$$p'_0 = n'_0 + \sum_{i=1}^m n'_{ii,0},$$

$$\frac{\partial}{\partial x'} \{ [\mu_p p'(x', t') + \mu_n n'(x', t')] E'(x', t') \} + \frac{\partial p'(x', t')}{\partial t'} - \frac{\partial n'(x', t')}{\partial t'} + \quad (2)$$

$$- \sum_{i=1}^m \frac{\partial n'_{ii}(x', t')}{\partial t'} = 0,$$

$$\frac{\partial n'_{ii}(x', t')}{\partial t'} = C_{ni}n'(x', t') - v_{ni}n'_{ii}(x', t'); \quad N_{ii} \gg n'_{ii}; \quad i = 1, 2, \dots, m, \quad (3)$$

$$n'(x', t')p'(x', t') = p'_0n'_0 = K, \quad (4)$$

$$\int_0^L E'(x', t') dx' = 0. \quad (5)$$

Here,  $q = 1.602 \times 10^{-19}$  C,  $\varepsilon$  is the dielectric constant,  $x'$  is a distance from the electrode,  $t'$  is the time,  $E'$  is the electric field intensity,  $p'$  and  $n'$  are the free hole and electron concentrations, respectively,  $n'_{ii}$  denotes the trapped electron concentration in the  $i$ -th trapping level,  $N_{ii}$  denotes the concentration of traps in the  $i$ -th trapping level,  $\mu_p$  and  $\mu_n$  are the hole and electron mobilities, respectively,  $v_{ni}$  and  $C_{ni}$  are the generation-recombination parameters in the  $i$ -th trapping level,  $p'_0$ ,  $n'_0$ ,  $n'_{ii,0}$  are the equilibrium concentrations of carriers,  $m$  is the number of the trapping levels, and  $L$  is the distance between the electrodes.

From (1), (2) and (5) it follows that the relation for the current density  $J(t')$  has the following form

$$J(t') = \frac{q}{L} \int_0^L [\mu_p p'(x', t') + \mu_n n'(x', t')] E'(x', t') dx'. \quad (6)$$

In order to reduce the number of the functions in (1)–(5), we introduce into our problem the so-called effective generation-recombination parameters defined by

$$C_n = \sum_{i=1}^m C_{ni}; \quad v_n = \frac{\sum_{i=1}^m v_{ni} n'_{ii,0}}{n'_{i0}}; \quad n'_{i0} = \sum_{i=1}^m n'_{ii,0}. \quad (7)$$

Under these conditions, equations (3) can be replaced by the following expression

$$\frac{\partial n'_i(x', t')}{\partial t'} = C_n n'(x', t') - v_n n'_i(x', t'); \quad n'_i = \sum_{i=1}^m n'_{ii} \quad (8)$$

where  $n'_i$  denotes the total concentration of trapped electrons. In what follows, for the current flow between the two electrodes, (7) and (8) will be used. Our problem will be solved by the use of the following normalised variable system

$$D = \frac{\varepsilon E'}{qLK^{1/2}}; \quad x = \frac{x'}{L}; \quad Q_1 = \frac{p'}{K^{1/2}}; \quad Q_2 = -\frac{n'}{K^{1/2}}; \quad Q_{2t} = -\frac{n'_t}{K^{1/2}} \quad (9)$$

$$Q_1^0 = \frac{p'_0}{K^{1/2}}; \quad Q_2^0 = -\frac{n'_0}{K^{1/2}}; \quad Q_{2t}^0 = -\frac{n'_{i0}}{K^{1/2}}; \quad t = \frac{q\mu_p K^{1/2} t'}{\varepsilon}; \quad \tau_n = \frac{q\mu_p K^{1/2}}{\varepsilon C_n}$$

$$\tau_g = \frac{q\mu_p K^{1/2}}{\varepsilon v_n}; \quad j = \frac{\varepsilon J}{q^2 \mu_p LK}; \quad r = \frac{\mu_n}{\mu_p}.$$

Thus, equations describing the space charge transport between the two electrodes take the form

$$\frac{\partial D(x, t)}{\partial x} = Q_1(x, t) + Q_2(x, t) + Q_{2t}(x, t) - (Q_1^0 + Q_2^0 + Q_{2t}^0), \quad (10)$$

$$\frac{\partial}{\partial x} \{ [Q_1(x, t) - rQ_2(x, t)] D(x, t) \} + \frac{\partial Q_1(x, t)}{\partial t} + \frac{\partial Q_2(x, t)}{\partial t} + \frac{\partial Q_{2t}(x, t)}{\partial t} = 0, \quad (11)$$

$$\frac{\partial Q_{2t}(x, t)}{\partial t} = \frac{Q_2(x, t)}{\tau_n} - \frac{Q_{2t}(x, t)}{\tau_g}, \quad (12)$$

$$Q_1(x, t) Q_2(x, t) = -1, \quad (13)$$

$$\int_0^1 D(x, t) dx = 0, \quad (14)$$

and the current density  $j = j(t)$  is

$$j = \int_0^1 (Q_1 - rQ_2) D dx = - \int_0^1 F \frac{(1 - rQ_2^2)}{Q_2^2} \frac{\partial Q_2}{\partial x} dx; \quad F = F(x, t) = \int_0^x D(s, t) ds. \quad (15)$$

Using the Taylor theorem to the function  $F(x, t)$ , we have

$$\begin{cases} F(x, t) = D(0, t)x + \frac{x^2}{2} \frac{\partial D(x=a, t)}{\partial x}; & a \in (0, x) \\ F(x, t) = D(1, t)(x-1) + \frac{(x-1)^2}{2} \frac{\partial D(x=b, t)}{\partial x}; & b \in (x, 1). \end{cases} \quad (15a)$$

Substituting  $x = 0$  or  $x = 1$  into (15a), we get

$$\begin{cases} D(0, t) = -\frac{1}{2} \frac{\partial D(x=a, t)}{\partial x} \\ D(1, t) = \frac{1}{2} \frac{\partial D(x=b, t)}{\partial x} \end{cases}; \quad a, b \in (0, 1), \quad (15aa)$$

and

$$\text{if } x \in \langle 0, 1 \rangle \quad \text{and} \quad \frac{\partial D(x, t)}{\partial x} < 0 \Rightarrow F(x, t) > 0 \quad \text{for } x \in (0, 1), \quad (15ab)$$

$$\text{if } x \in \langle 0, 1 \rangle \quad \text{and} \quad \frac{\partial D(x, t)}{\partial x} > 0 \Rightarrow F(x, t) < 0 \quad \text{for } x \in (0, 1). \quad (15ac)$$

On the basis of (15a)–(15ac), we can write the following implications

$$\text{if } x \in \langle 0, 1 \rangle \text{ and } \left( \frac{\partial Q_2}{\partial x} > 0 \wedge \frac{-1}{\sqrt{r}} < Q_2 < 0 \wedge \frac{\partial D}{\partial x} < 0 \right) \Rightarrow j(t) < 0, \quad (15b)$$

$$\text{if } x \in \langle 0, 1 \rangle \text{ and } \left( \frac{\partial Q_2}{\partial x} < 0 \wedge Q_2 < \frac{-1}{\sqrt{r}} \wedge \frac{\partial D}{\partial x} < 0 \right) \Rightarrow j(t) < 0, \quad (15c)$$

$$\text{if } x \in \langle 0, 1 \rangle \text{ and } \left( \frac{\partial Q_2}{\partial x} < 0 \wedge \frac{-1}{\sqrt{r}} < Q_2 < 0 \wedge \frac{\partial D}{\partial x} < 0 \right) \Rightarrow j(t) > 0, \quad (15d)$$

$$\text{if } x \in \langle 0, 1 \rangle \text{ and } \left( \frac{\partial Q_2}{\partial x} < 0 \wedge \frac{-1}{\sqrt{r}} < Q_2 < 0 \wedge \frac{\partial D}{\partial x} > 0 \right) \Rightarrow j(t) < 0, \quad (15e)$$

$$\text{if } x \in \langle 0, 1 \rangle \text{ and } \frac{\partial Q_2}{\partial x} \equiv 0 \Rightarrow j(t) \equiv 0. \quad (15f)$$

The next step is to find the physical conditions defining the initial distributions of the space charge density  $Q(x, 0) = Q_1(x, 0) + Q_2(x, 0) + Q_{2i}(x, 0)$ . In what follows, we will assume that these initial distributions are determined by the steady state of solid conduction in the form

$$\frac{dD_s(x)}{dx} = Q_1(x, 0) + \theta(Q_2(x, 0) - Q_2^0) - Q_1^0; \quad \theta = 1 + \frac{\tau_g}{\tau_n}, \quad (16)$$

$$Q_1(x, 0) \cdot Q_2(x, 0) = -1; \quad j_s = [Q_1(x, 0) - rQ_2(x, 0)] D_s(x); \quad j_s = \text{const} > 0, \quad (17)$$

$$\int_0^1 D_s(x) dx = \text{const} > 0. \quad (18)$$

For (16)–(18), according to Fig. 1,  $x = 0$  is the anode and  $x = 1$  denotes the cathode. In other words, for our space charge problem, we assume that a time  $t = 0-$  exists for which the transition from the steady current — charging capacitor conditions to the transient current — discharging capacitor conditions occurs. Taking into account this technical possibility, for our problem, we assume that the free negative charge density is continuous, that is  $Q_2(x, 0-) = Q_2(x, 0+)$ . With this assumption, the current flow in the discharging capacitor system will be investigated.

### 3. THE SOLUTION OF THE PROBLEM

Now, we will consider the two particular cases of the carrier flow through the bulk. Additionally, we will assume that  $r = 1$  (that is,  $\mu_p = \mu_n$ ). First, let us consider the case when the electrons are not localised in the bulk.

#### 3.1. The current flow through a solid without trapped charge $Q_{2i}(x, t) = 0$ .

In this case of electric conduction, from (10)–(14), we obtain the following quasi-linear partial differential equation

$$(1 + Q_2^2) \frac{\partial Q_2}{\partial t} + D(1 - Q_2^2) \frac{\partial Q_2}{\partial x} - (Q_2^4 - 1) = 0. \quad (19)$$

Hence, on the basis of the theory of characteristics, we can write the ordinary differential equations

$$\frac{dx}{dt} = D \frac{1 - Q_2^2}{1 + Q_2^2}, \quad (19a)$$

$$\frac{dQ_2}{dt} = -1 + Q_2^2. \quad (19b)$$

Therefore, along the characteristics  $x(t)$  the free negative charge density has the form

$$Q_2(x(t), t) = \frac{A \exp(-2t) + 1}{A \exp(-2t) - 1} \quad \text{for } t \geq 0, \quad (20)$$

where

$$A = \frac{Q_2(x(0), 0) + 1}{Q_2(x(0), 0) - 1} \quad \text{for } 0 \leq x(0) \leq 1, \quad (20a)$$

and

$$x(t) = \int_0^t D(x(s), s) \frac{1 - Q_2^2(x(s), s)}{1 + Q_2^2(x(s), s)} ds + x(0). \quad (20b)$$

Taking into account the physical conditions, we have to define a set corresponding to the given discharging capacitor system. This set is as follows

$$\Omega = \{(x, t) : 0 \leq x \leq 1 \wedge t \geq 0\}. \quad (20c)$$

The next step is to define the uniqueness condition in the set  $\Omega$ . According to the Picard theorem used to (19a), for the two characteristics  $x_1(t)$  and  $x_2(t)$  the uniqueness condition is written as

$$\text{if } x_1(0) > x_2(0) \Rightarrow x_1(t) > x_2(t) \quad \text{for } t \geq 0, \quad (21)$$

or in the equivalent form

$$\left. \frac{dx(t)}{dx(0)} \right|_{t=\text{const}} > 0 \quad \text{for } t \geq 0. \quad (21a)$$

The property (21a) is fundamental for this paper. Now, in order to find the space charge distribution, we must return to (16)–(18). From these equations it follows that the initial space charge can be negative or positive or the electric field distribution  $D(x, 0)$  is uniform. First, let us take into consideration a case when the initial negative space charge is distributed in the bulk, that is

$$D_s(x) \frac{dD_s(x)}{dx} = -\sqrt{j_s^2 - 4D_s^2(x)}; \quad Q_2(x, 0) = -\frac{j_s + \sqrt{j_s^2 - 4D_s^2(x)}}{2D_s(x)}; \quad D_s(x) > 0. \quad (22)$$

Therefore

$$Q_2(x, 0) < -1; \quad \frac{\partial Q_2(x, 0)}{\partial x} < 0; \quad \frac{\partial D(x, 0)}{\partial x} < 0 \quad \text{for } 0 \leq x \leq 1. \quad (22a)$$

Hence, on the basis of (20a) and (20), we have

$$0 < A < 1; \quad \text{for } 0 \leq x \leq 1 \quad (22b)$$

and

$$Q_2(x, t) = Q_2(x(t), t) < -1 \quad \text{for } (x, t) \in \Omega. \quad (22c)$$

Thus, from (10), (13) and (22c) it follows that the negative space charge

$$\frac{\partial D(x, t)}{\partial x} = \frac{Q_2^2(x(t), t) - 1}{Q_2(x(t), t)} < 0 \quad \text{for } (x, t) \in \Omega \quad (22d)$$

is distributed in the bulk. Now, referring to (15aa), we can write

$$D(0, t) > 0 \wedge D(1, t) < 0 \quad \text{for } t \geq 0. \quad (22e)$$

this property denotes that the total charge between the two electrodes is negative. Using (22a)–(22d), we can find the other properties of  $D(x, t)$  and of  $Q_2(x, t)$ . To this end, we introduce into (20) and (20a) the following symbols  $V = A \exp(-2t)$  or  $V = Q_2(x, 0)$  as well as  $Y = A$  or  $Y = Q_2(x(t), t)$ . With these symbols, (20) and (20a) are written as

$$Y = \frac{V + 1}{V - 1}. \quad (23)$$

Hence, on the basis of (20a), (20) and (22a), we get

$$\frac{dA}{dx(0)} = \frac{dY}{dV} \cdot \frac{dV}{dx(0)} = \frac{-2}{(V - 1)^2} \cdot \frac{\partial Q_2(x(0), 0)}{\partial x(0)} > 0 \quad \text{for } 0 \leq x(0) \leq 1 \quad (23a)$$

and

$$\frac{\partial Q_2(x(t), t)}{\partial x(0)} = \frac{dY}{dV} \cdot \frac{dV}{dx(0)} = \frac{-2 \exp(-2t)}{(V - 1)^2} \cdot \frac{dA}{dx(0)} < 0 \quad \text{for } 0 \leq x(t) \leq 1. \quad (23b)$$

Therefore, (23b) and (21a) yield

$$\frac{\partial Q_2(x(t), t)}{\partial x(t)} = \frac{\partial Q_2(x(t), t)}{\partial x(0)} \cdot \frac{1}{\frac{dx(t)}{dx(0)}} < 0; \quad \text{for } (x, t) \in \Omega. \quad (23c)$$

Thus, we can write the following property

$$Q_2(x, t) < -1 \wedge \frac{\partial Q_2(x, t)}{\partial x} < 0 \wedge \frac{\partial D(x, t)}{\partial x} < 0; \quad \text{for } (x, t) \in \Omega. \quad (24)$$

Hence, on the basis of (15c), we ascertain that the ordinary conduction occurs, that is

$$j(t) < 0 \quad \text{for } t \geq 0. \quad (25)$$

Next, using the phase plane method to (19b) and referring to the Picard theorem for (19a), we obtain

$$\lim_{t \rightarrow \infty} Q_2(x(t), t) = Q_2^0 = -1 \wedge \lim_{t \rightarrow \infty} D(x, t) = 0; \quad \text{for } 0 \leq x \leq 1 \quad (26)$$

and

$$\lim_{t \rightarrow \infty} j(t) = 0. \quad (26a)$$

The properties (26) and (26a) denote that the system goes back to the equilibrium conditions. Now, we shall determine the shape of the  $D(x, t)|_{t=\text{const}}$  curve. From (14) and (24) it follows that there exists a point  $x_0 = x_0(t) \in <0, 1>$  such that

$$D(x_0(t), t) = 0 \quad \text{and} \quad j(t) \equiv \frac{\partial D(x_0, t)}{\partial t}. \quad (27)$$

Combining (27), we have

$$\frac{\partial D}{\partial t} + \frac{\partial D}{\partial x_0} \cdot \frac{dx_0}{dt} \equiv 0, \quad (27a)$$

therefore

$$\frac{dx_0}{dt} = -\frac{j(t)}{Q(x_0, t)} < 0; \quad Q(x, t) = Q_1(x, t) + Q_2(x, t) = \frac{\partial D(x, t)}{\partial x}. \quad (27b)$$

Here,  $Q(x, t)$  is the space charge density. Since

$$\frac{\partial^2 D}{\partial x^2} = \frac{1 + Q_2^2}{Q_2^2} \cdot \frac{\partial Q_2}{\partial x}$$

thus, on the basis of (24), (27b) and (26a), we can write the following properties

$$Q(x, t) < 0 \wedge \frac{\partial Q(x, t)}{\partial x} < 0 \wedge \frac{\partial^2 D(x, t)}{\partial x^2} < 0 \quad \text{for } (x, t) \in \Omega \quad (28)$$

and

$$\frac{j(t)}{Q(x_0, t)} > \frac{j(t)}{Q(1, t)} \quad \text{for } t \geq 0, \quad (28a)$$

which leads to

$$0 < \int_0^\infty \frac{j(s)}{Q(1, s)} ds < \int_0^\infty \frac{j(s)}{Q(x_0, s)} ds < x_0(0) - x_0(\infty) < 1. \quad (28b)$$



Next, according to (26) and (26a), we obtain an asymptotic behaviour of  $j(t)$

$$\lim_{t \rightarrow \infty} \frac{j(t)}{Q(1, t)} = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} Q(1, t) = 0. \tag{28c}$$

Here,  $|Q(1, t)|$  denotes the maximum value. Now, let us define a set of  $x_0(t)$ . To this end, (28) is written as

$$\left| \frac{\partial D(x', t)}{\partial x'} \right| < \left| \frac{\partial D(x'', t)}{\partial x''} \right| \quad \text{for} \quad 0 \leq x' \leq x_0; \quad x_0 < x'' \leq 1 \tag{29}$$

and the voltage condition (14) can take the form

$$\int_0^1 D(x, t) dx = \int_0^{x_0} D(x', t) dx' + \int_{x_0}^1 D(x'', t) dx'' = I_1 + I_2 = 0, \tag{30}$$

where

$$I_1 = \int_0^{x_0} D(x', t) dx' = \int_0^{x_0} x' \left| \frac{\partial D(x', t)}{\partial x'} \right| dx'; \tag{30a}$$

$$I_2 = \int_{x_0}^1 D(x'', t) dx'' = \int_{x_0}^1 (1 - x'') \frac{\partial D(x'', t)}{\partial x''} dx''.$$

Next, substituting  $z = 1 - x''$  and  $x_1 = 1 - x_0$  into  $I_2$ , we obtain

$$\frac{\partial D(z, t)}{\partial z} = \left| \frac{\partial D(x'', t)}{\partial x''} \right|; \quad I_2 = \int_{x_1}^0 z \left( - \frac{\partial D(z, t)}{\partial z} \right) (-dz) = - \int_0^{x_1} z \left| \frac{\partial D(x'', t)}{\partial x''} \right| dz. \tag{30b}$$

Therefore, according to (30), the condition  $I_1 = -I_2$  becomes

$$\int_0^{x_0} x' \left| \frac{\partial D(x', t)}{\partial x'} \right| dx' = \int_0^{x_1} z \left| \frac{\partial D(x'', t)}{\partial x''} \right| dz. \tag{30c}$$

Finally, on the basis of (29), we ascertain that (30c) is satisfied only when

$$1 > x_0 > x_1 = 1 - x_0 \quad \text{that is} \quad \frac{1}{2} < x_0 < 1 \quad \text{for} \quad t \geq 0. \tag{31}$$

Now, using (28) and (31), we shall find a relation between the boundary values  $D(0, t)$  and  $D(1, t)$ . First, let us notice that a function  $D(x)$  for  $t = \text{const}$  is convex upward. This convexity can be expressed by

$$I_1 > S_1 = \frac{1}{2} D(0, t) x_0 \quad \text{and} \quad -I_2 < S_2 = \frac{1}{2} (1 - x_0) |D(1, t)|. \tag{31a}$$

Here,  $S_1$  and  $S_2$  are the areas of the convenient triangles. Since the voltage condition (30) is equivalent to  $I_1 = -I_2$ , therefore from (31a) we have  $S_1 < S_2$ , that is

$$0 < D(0, t)x_0 < (1 - x_0)|D(1, t)| \quad \text{for } t \geq 0. \quad (31b)$$

Thus, if  $D(0, t) \geq |D(1, t)|$ , then from (31) we could have  $S_1 > S_2$ . This denotes that (31b) is not satisfied. Finally, we have

$$|D(1, t)| > D(0, t) > 0 \quad \text{for } t \geq 0. \quad (32)$$

The final step is to define the signs of the derivatives of the functions  $D(0, t)$  and  $D(1, t)$ , taking into account (24) and (25). Since

$$j(t) = \frac{dD(0, t)}{dt} + [Q_1(0, t) - Q_2(0, t)]D(0, t) \quad \text{and} \quad D(0, t) > 0 \quad (33)$$

as well as  $Q_1 - Q_2 > 0$ , therefore on the basis of (25), we see that a function  $D(0, t)$  is decreasing, that is  $\frac{dD(0, t)}{dt} < 0$ . Now, let us find the sign of the derivative of  $D(1, t)$ . The function  $j(t)$  can be written as

$$\begin{aligned} j(t) &= \frac{dD(1, t)}{dt} + [Q_1(1, t) - Q_2(1, t)]D(1, t) \equiv \\ &\equiv \int_0^1 [Q_1(x, t) - Q_2(x, t)]D(x, t)dx. \end{aligned} \quad (34)$$

Next, using the mean-value theorem, we can write

$$j(t) = [Q_1(c, t) - Q_2(c, t)]D(c, t) \quad \text{and} \quad c \in (0, 1). \quad (34a)$$

According to (13), (24) and to (28), we have

$$D(1, t) < D(c, t) < 0 \quad \text{and} \quad x_0 < c < 1. \quad (34b)$$

and

$$\frac{\partial}{\partial x}(Q_1(x, t) - Q_2(x, t)) > 0 \quad \text{for } 0 \leq x \leq 1. \quad (34c)$$

Thus, finally, we can note

$$-j(t) = [Q_1(c, t) - Q_2(c, t)]|D(c, t)| < [Q_1(1, t) - Q_2(1, t)]|D(1, t)|, \quad (34d)$$

that is

$$j(t) > [Q_1(1, t) - Q_2(1, t)]D(1, t). \quad (35)$$

Hence, on the basis of (34), it follows that a function  $D(1, t)$  is increasing, that is  $\frac{dD(1, t)}{dt} > 0$ .

All these above considerations are illustrated in Fig. 2 and Fig. 3. A function  $j(t)$  and the derivative  $\frac{\partial Q_2}{\partial t}$  have been found by a numerical algorithm. The shapes of the functions  $j(t)$  and  $Q_2(x, t)_{t=\text{const}}$  are presented in Fig. 4.

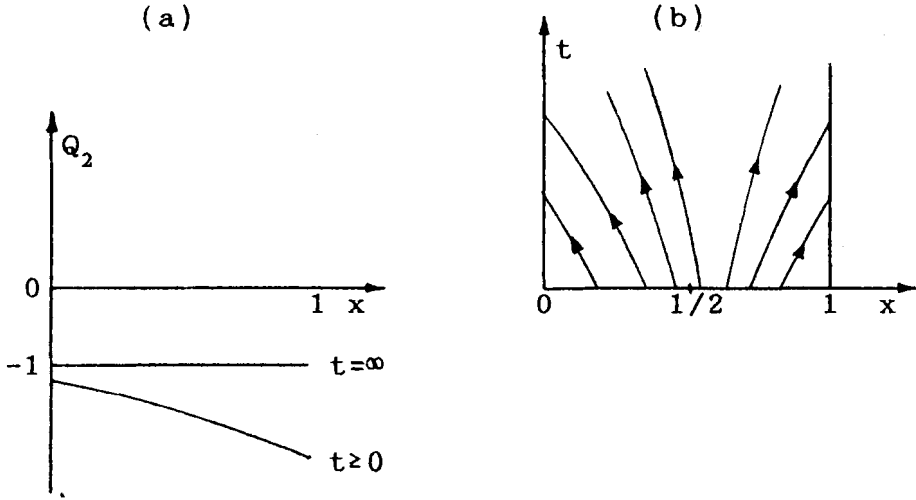


Fig. 2. The current flow curves obtained by the analytical considerations in the case when  $Q_{2i}(x, t) \equiv 0$ : a) the distribution of the free negative charge density; b) the characteristics  $x(t)$  (the flow curves)

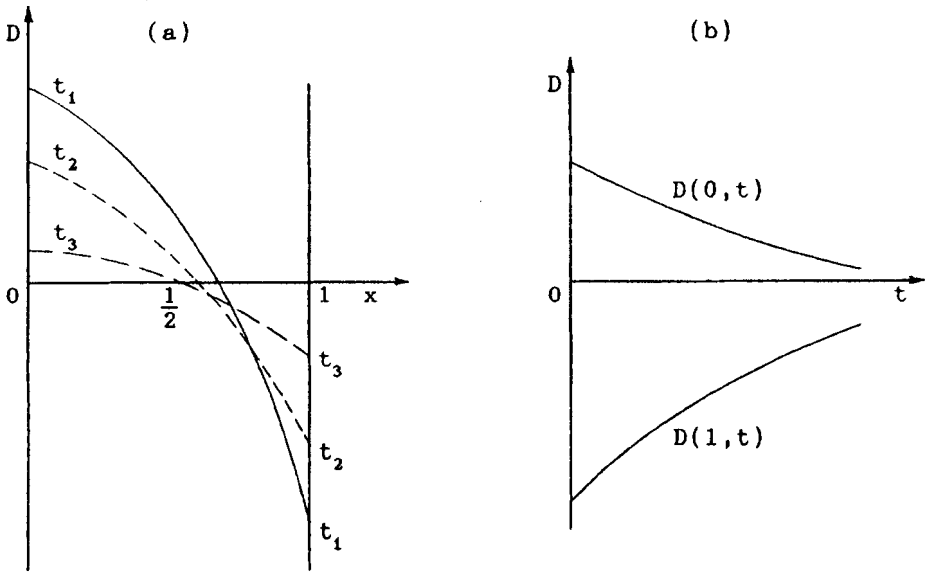


Fig. 3. The electric field distributions  $D(x, t)$  obtained by the analytical considerations in the case when  $Q_{2i}(x, t) \equiv 0$ : a) the convexity of the electric field distributions  $D(x)$  for the different times  $0 < t_1 < t_2 < t_3$ ; b) the shapes of the boundary values of the electric field  $D(0, t)$  and  $D(1, t)$ . (Also, in the case of shallow traps, these similar curves are obtained by a numerical method)

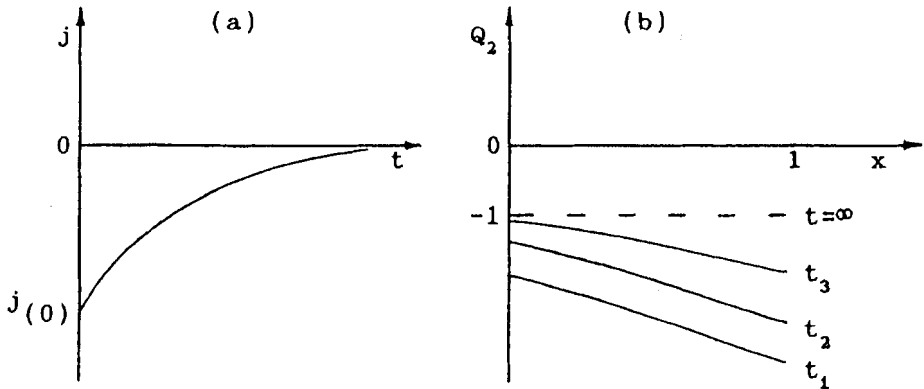


Fig. 4. The shapes of the current and charge curves obtained by a numerical method in the case when  $Q_{2i}(x, t) \equiv 0$ : a) the shape of the  $j(t)$  curve; b) the free negative charge distributions  $Q_2(x)$  for the different times  $0 \leq t_1 < t_2 < t_3$

Returning to (16)–(18), in the case when the initial charge is positive in the bulk, that is  $\frac{dD_s(x)}{dx} > 0$  and  $1 < Q_1(x, 0)$  for  $0 \leq x \leq 1$ , we have the similar considerations. In this case of the current flow, omitting mathematical details, we ascertain that the ordinary conduction expressed by  $j(t) < 0$  occurs (Fig. 4a). In the case when the initial space charge is equal to zero (that is, the electric field distribution defined by (16)–(18) is uniform), we have  $Q_2(x, t) \equiv Q_2^0 = -1$  and  $Q_1(x, t) \equiv Q_1^0 = +1$  and  $j(t) \equiv 0$ . In the next section, we consider the conditions in which the current flow can be anomalous (Fig. 1).

### 3.2. The current flow through a solid with deep traps $Q_{2i}(x, t) \equiv Q_{2i}^0$ .

In this part of the paper we consider a solid in which the deep trapping levels exist. In this case, we will assume that the trapped electron concentration is uniform (according to (12) and (16), formally, this condition can be characterised by  $\tau_g \ll \tau_n$ ) and this concentration is independent of the free electron concentration. Under these conditions, in (16) we have  $\theta = 1$  and  $Q_1^0 + Q_2^0 = -Q_{2i}^0 \neq 0$ , where  $Q_{2i}^0$  is independent of  $Q_2^0 = -1/Q_1^0$ . Theoretically, the simplest case occurs when  $r = 1$ . With this assumption, (10)–(13) result in

$$(1 + Q_2^0) \frac{\partial Q_2}{\partial t} + D(1 - Q_2^0) \frac{\partial Q_2}{\partial x} - (1 + Q_2^0)(Q_2 - Q_1^0)(Q_2 - Q_2^0) = 0, \quad (36)$$

from which we obtain the equations of characteristics in the form

$$\frac{dx}{dt} = D \frac{1 - Q_2^0}{1 + Q_2^0} \quad (36a)$$

and

$$\frac{dQ_2}{dt} = (Q_2 - Q_1^0)(Q_2 - Q_2^0). \tag{36b}$$

According to (16)–(18), the initial values  $Q_2(x, 0)$  are described by

$$Q_2(x, 0) = -\frac{j_s + \sqrt{j_s^2 - 4D_s^2(x)}}{2D_s(x)}; \quad D_s(x) > 0 \quad \text{for } 0 \leq x \leq 1, \tag{37}$$

where

$$D_s(x) \frac{dD_s(x)}{dx} = -\sqrt{j_s^2 - 4D_s^2(x)} - (Q_1^0 + Q_2^0) D_s(x). \tag{37a}$$

On this basis, we can write the following property

$$Q_2(x, 0) < -1 \wedge \frac{\partial Q_2(x, 0)}{\partial x} < 0 \wedge \frac{\partial D(x, 0)}{\partial x} < 0 \quad \text{for } 0 \leq x \leq 1. \tag{38}$$

Next, referring to (15c), we have

$$j(0+) < 0. \tag{38a}$$

Using the Picard theorem to (36a) and (36) and, as well, using the phase plane method to (36b), we ascertain that there exist the following limits

$$\lim_{t \rightarrow \infty} Q_2(x(t), t) = Q_2^0 \quad \text{and} \quad \lim_{t \rightarrow \infty} Q_1(x(t), t) = Q_1^0 \quad \text{for } x = x(t) \in ]0, 1[ \tag{39}$$

and consequently

$$\lim_{t \rightarrow \infty} D(x, t) = 0 \quad \text{for } 0 \leq x \leq 1 \quad \text{and} \quad \lim_{t \rightarrow \infty} j(t) = 0, \tag{39a}$$

where  $-1 < Q_2^0 < 0$  and  $Q_1^0 > 1$ . Taking into account (39), (37) and (37a), we can show that there exist the values  $t_1$  and  $t_2 (t_1 < t_2)$  such that (Fig. 5)

$$\left\{ \begin{array}{l} \left. \frac{dx}{dt} \right|_{x=0} < 0; \quad 0 \leq t < t_1 \\ \left. \frac{dx}{dt} \right|_{x=0} = 0; \quad t = t_1 \\ \left. \frac{dx}{dt} \right|_{x=0} > 0; \quad t > t_1 \end{array} \right. \quad \text{and} \quad \left\{ \begin{array}{l} \left. \frac{dx}{dt} \right|_{x=1} > 0; \quad 0 \leq t < t_2 \\ \left. \frac{dx}{dt} \right|_{x=1} = 0; \quad t = t_2 \\ \left. \frac{dx}{dt} \right|_{x=1} < 0; \quad t > t_2 \end{array} \right. \tag{40}$$

In other words, the set  $\Omega$  is the sum of the sets  $\Omega_1$  and  $\Omega_2$  (Fig. 5), that is  $\Omega = \Omega_1 \cup \Omega_2$ , where in the set  $\Omega_1$  we have the initial characteristics

$$x(t) = \int_0^t D(x(s), s) \frac{1 - Q_2^2(x(s), s)}{1 + Q_2^2(x(s), s)} ds + x(0) \quad \text{for } t \geq 0 \tag{40a}$$

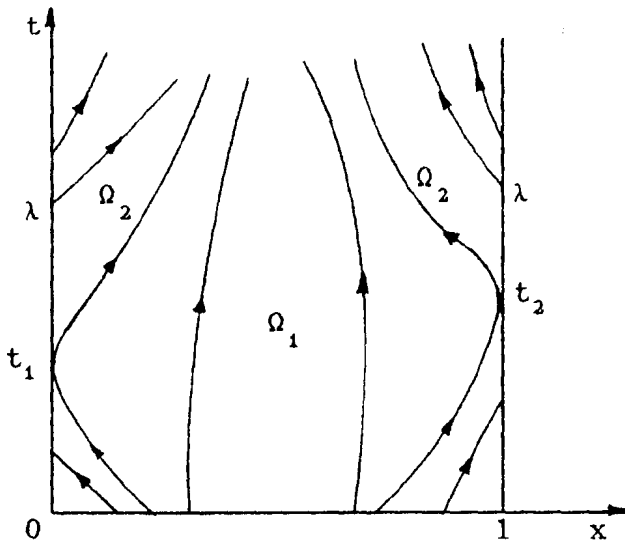


Fig. 5. The boundary (set  $\Omega_2$ ) and initial (set  $\Omega_1$ ) characteristics in the case when  $Q_{2i}(x, t) \equiv Q_2^0$ ,

and in the set  $\Omega_2$  there are only the boundary characteristics

$$x(t) = \int_{\lambda}^t D(x(s), s) \frac{1 - Q_2^2(x(s), s)}{1 + Q_2^2(x(s), s)} ds \quad \text{for } t \geq \lambda \geq t_1 \quad (40b)$$

$$x(t) = \int_{\lambda}^t D(x(s), s) \frac{1 - Q_2^2(x(s), s)}{1 + Q_2^2(x(s), s)} ds + 1 \quad \text{for } t \geq \lambda \geq t_2. \quad (40c)$$

According to (36b), in the set  $\Omega_1$ , the free negative charge density is of the form

$$Q_2(x(t), t) = \frac{Q_1^0 A e^{-(Q_1^0 - Q_2^0)t} - Q_2^0}{A e^{-(Q_1^0 - Q_2^0)t} - 1}; \quad A = \frac{Q_2(x(0), 0) - Q_2^0}{Q_2(x(0), 0) - Q_1^0} \quad \text{for } t \geq 0. \quad (41)$$

Similarly, proceeding with (36b), in the set  $\Omega_2$ , the free negative charge density takes the form

$$Q_2(x(t), t) = \frac{Q_1^0 B e^{-(Q_1^0 - Q_2^0)(t-\lambda)} - Q_2^0}{B e^{-(Q_1^0 - Q_2^0)(t-\lambda)} - 1}, \quad (42)$$

where

$$B = \frac{Q_2(0, \lambda) - Q_2^0}{Q_2(0, \lambda) - Q_1^0}; \quad t_1 \leq \lambda \leq t \quad \text{or} \quad B = \frac{Q_2(1, \lambda) - Q_2^0}{Q_2(1, \lambda) - Q_1^0}; \quad t_2 \leq \lambda \leq t. \quad (42a)$$

In what follows, in the set  $\Omega_1$ , the uniqueness condition (21a) is satisfied. In this set we have

$$\frac{\partial Q_2(x(t), t)}{\partial x(t)} = \frac{\partial Q_2(x(t), t)}{\partial x(0)} \cdot \frac{1}{\frac{dx(t)}{dx(0)}} \tag{43}$$

$$\frac{\partial Q_2(x(t), t)}{\partial x(0)} = -\frac{(Q_1^0 - Q_2^0)}{(Y_1 - 1)^2} \cdot e^{-(\varrho_1^0 - \varrho_2^0)t} \cdot \frac{dA}{dx(0)}; \quad Y_1 = Ae^{-(\varrho_1^0 - \varrho_2^0)t} \tag{43a}$$

and

$$\frac{dA}{dx(0)} = -\frac{(Q_1^0 - Q_2^0)}{(Q_2(x(0), 0) - Q_1^0)^2} \cdot \frac{\partial Q_2(x(0), 0)}{\partial x(0)}. \tag{43b}$$

Hence, on the basis of (21a), we ascertain that the free negative charge distribution is permanently decreasing in the set  $\Omega_1$ , that is

$$\frac{\partial Q_2(x, t)}{\partial x} < 0; \quad (x, t) \in \Omega_1. \tag{44}$$

Next, using (37) and (37a) as well as (41), we see that, along the initial characteristics (40a), the free negative charge is  $Q_2(x(t), t) < Q_2^0$ . Therefore, along the initial characteristics, we have

$$\frac{dQ_2(x(t), t)}{dt} > 0; \quad (x(t), t) \in \Omega_1. \tag{45}$$

Since there exists the following equality  $\frac{\partial D}{\partial x} = \frac{1}{Q_2} \cdot \frac{dQ_2}{dt}$ , thus, in the set  $\Omega_1$  the space charge is negative, that is

$$\frac{\partial D(x, t)}{\partial x} < 0; \quad (x, t) \in \Omega_1. \tag{46}$$

Completing (44), (46), (37a) and (37), we can note

$$\frac{\partial Q_2(x, t)}{\partial x} < 0 \wedge \frac{\partial D(x, t)}{\partial x} < 0 \wedge Q_2(x, t) < -1; \quad 0 \leq x \leq 1; \quad 0 \leq t \leq t_1 \tag{47}$$

and

$$Q_2(x, t) = -1 \quad \text{for } x = 0 \quad \text{and } t = t_1. \tag{47a}$$

Hence, on the basis of (15c), we ascertain that the ordinary electrical transport

$$j(t) < 0 \quad \text{for} \quad 0 \leq t \leq t_1 \quad (48)$$

occurs.

Now, let us limit our attention only to the set  $\Omega_2$  (Fig. 5). Here, the uniqueness condition takes the form

$$\left. \frac{dx(t)}{d\lambda} \right|_{t = \text{const}} \neq 0. \quad (49)$$

From (40) it follows that the boundary conditions corresponding to (39) must be given. For this problem (that is, the transient current conditions in which the system can go back to the equilibrium conditions characterised by  $Q_2^0 Q_1^0 = -1$ ), the boundary conditions are described by [31–32]

$$\begin{cases} \frac{dQ_2(0, \lambda)}{d\lambda} = (Q_2(0, \lambda) - Q_2^0)(Q_2(0, \lambda) - Q_1^0) \\ \frac{dQ_2(1, \lambda)}{d\lambda} = (Q_2(1, \lambda) - Q_2^0)(Q_2(1, \lambda) - Q_1^0) \end{cases} \quad (50)$$

and

$$Q_2(0, \lambda = t_1) = -1; \quad Q_2(1, \lambda = t_2) = -1. \quad (50a)$$

With (50) and (50a), we will determine a distribution of the negative charge density in the set  $\Omega_2$ . To this end, we make use of (42) and (42a) in order to obtain

$$\frac{\partial Q_2(x(t), t)}{\partial x(t)} = - \frac{(Q_1^0 - Q_2^0)}{(Y_2 - 1)^2} \cdot \frac{dY_2}{d\lambda} \cdot \frac{1}{\frac{dx(t)}{d\lambda}}; \quad Y_2 = B e^{-(Q_1^0 - Q_2^0)(t-\lambda)} \quad (51)$$

and

$$\frac{dY_2}{d\lambda} = e^{-(Q_1^0 - Q_2^0)(t-\lambda)} \cdot \frac{dB}{d\lambda} + B(Q_1^0 - Q_2^0) e^{-(Q_1^0 - Q_2^0)(t-\lambda)}. \quad (51a)$$

Therefore

$$\frac{dY_2}{d\lambda} = - \frac{(Q_1^0 - Q_2^0) e^{-(Q_1^0 - Q_2^0)(t-\lambda)}}{(Q_2(0, \lambda) - Q_1^0)^2} \left[ \frac{dQ_2(0, \lambda)}{d\lambda} - (Q_2(0, \lambda) - Q_2^0)(Q_2(0, \lambda) - Q_1^0) \right] \quad (52)$$

or

$$\frac{dY_2}{d\lambda} = - \frac{(Q_1^0 - Q_2^0) e^{-(Q_1^0 - Q_2^0)(t-\lambda)}}{(Q_2(1, \lambda) - Q_1^0)^2} \left[ \frac{dQ_2(1, \lambda)}{d\lambda} - (Q_2(1, \lambda) - Q_2^0)(Q_2(1, \lambda) - Q_1^0) \right] \quad (53)$$



Hence, on the basis of (50), we notice that the free negative charge densities are uniform in the electrode regions (Fig. 5, the set  $\Omega_2$ ), that is

$$\frac{\partial Q_2(x, t)}{\partial x} \equiv 0; \quad (x, t) \in \Omega_2. \quad (54)$$

Now, let us find the signs of the derivatives  $\partial D/\partial x$  and  $\partial Q_2/\partial t$  in order to define the shape of a function  $j(t)$ . First, referring to (50) and (50a), let us notice that the boundary values are  $Q_2(0, \lambda) < Q_2^0$  and  $Q_2(1, \lambda) < Q_2^0$  for  $\lambda \geq t_1$  and  $\lambda \geq t_2$ , respectively. Thus, along the boundary characteristics  $x(t)$ , we have

$$\frac{dQ_2(x(t), t)}{dt} > 0; \quad (x(t), t) \in \Omega_2. \quad (55)$$

Taking into account the equality  $\frac{\partial D}{\partial x} \equiv \frac{1}{Q_2} \cdot \frac{dQ_2}{dt}$  (which we can easily show) and (46), we see that the negative space charge

$$\frac{\partial D(x, t)}{\partial x} < 0 \quad \text{for } (x, t) \in \Omega \quad (56)$$

is distributed in the bulk. Similarly, making use of (54) and of the other equality

$$\frac{dQ_2}{dt} \equiv \frac{\partial Q_2}{\partial t} + \frac{\partial Q_2}{\partial x} \cdot \frac{dx}{dt}$$

we get

$$\frac{\partial Q_2(x, t)}{\partial t} > 0; \quad (x, t) \in \Omega_2. \quad (57)$$

Completing (54)–(57) and (50a), we can write the following property

$$-1 \leq Q_2(x, t) < 0 \wedge \frac{\partial D(x, t)}{\partial x} < 0 \quad \text{for } 0 \leq x \leq 1; \quad t \geq t_2. \quad (58)$$

Hence, on the basis of (44), (54), (57), (15d) and (39a), we see that an anomalous current flow occurs, that is

$$j(t) > 0 \quad \text{for } t \geq t_2 \quad \text{and} \quad \lim_{t \rightarrow \infty} j(t) = 0+. \quad (59)$$

Since a function  $j(t)$  is continuous, therefore from (48) it follows that a values  $t_0$  exists for which we have  $j(t_0) = 0$ . In other words, on the basis of (48) and (59), we ascertain that the direction of the current flow is changed at the time  $t = t_0$ . With

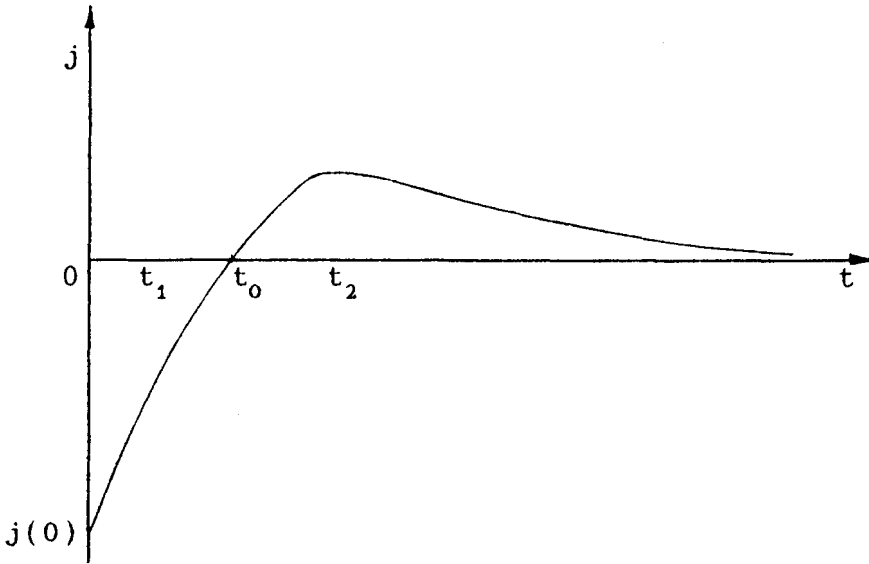


Fig. 6. The possible shape of the  $j(t)$  curve in the case when  $Q_{2t}(x, t) \equiv Q_{2t}^0$

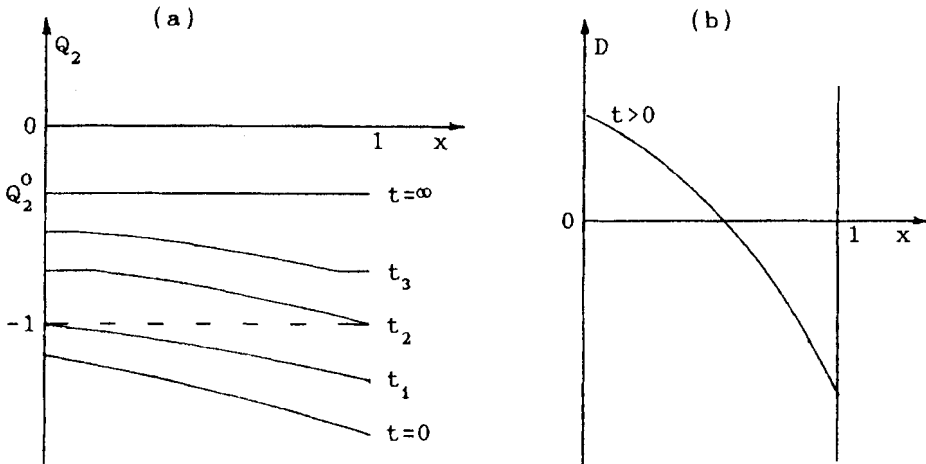


Fig. 7. The shapes of the curves obtained by the analytical considerations in the case when  $Q_{2t}(x, t) \equiv Q_{2t}^0$ : a) the free negative charge distributions for the different times  $0 < t_1 < t_2 < t_3$ ; b) the electric field distribution (also, in the case of shallow traps, a similar curve is obtained by a numerical method)

(48) and (59), a possible shape of a current function  $j(t)$  is presented in Fig. 6. With the boundary conditions (50) and (50a), the free negative charge distributions  $Q_2(x, t)$  are continuous. In Fig. 7 the properties (44), (54), (56) and (57) are illustrated. In the next section we consider the other conditions in which an anomalous current flow occurs.

3.3. The current flow through a solid with shallow traps  $\tau_g = \tau_n = \tau < \infty$ .

In this part of the work we consider a solid in which the trapped negative charge distribution is not uniform. In this case we take into account the shallow trapping states in which the generation-recombination parameters are  $\tau_n = \tau_g$  (this denotes that  $\theta = 2$  and  $Q_2^0 = Q_{2t}^0 = \frac{-1}{\sqrt{2}}$  in (16)). Additionally, we assume that  $r = 1$ , that is  $\mu_n = \mu_p$ . This space charge problem is solved by a numerical method basing on the theory of characteristics. To this end, according to (10)–(14), the electrical transport in the discharging capacitor system is described by

$$\begin{cases} \frac{dx}{dt} = D \frac{1 - Q_2^2}{1 + Q_2^2} \\ \frac{dQ_2}{dt} = \frac{-\frac{Q_2^2}{\tau}(Q_2 - Q_{2t}) + (1 + Q_2^2)(-1 + Q_2^2 + Q_2 Q_{2t})}{1 + Q_2^2} \end{cases} \tag{60}$$

$$\frac{\partial Q_{2t}(x, t)}{\partial t} = \frac{1}{\tau}(Q_2(x, t) - Q_{2t}(x, t)), \tag{61}$$

$$Q(x, t) = Q_1(x, t) + Q_2(x, t) + Q_{2t}(x, t); \quad Q_1(x, t) = \frac{-1}{Q_2(x, t)}, \tag{62}$$

$$D(0, t) = - \int_0^1 (1 - x) Q(x, t) dx, \tag{63}$$

$$D(x, t) = D(0, t) + \int_0^x Q(x, t) dx; \quad 0 \leq x \leq 1, \tag{64}$$

$$j(t) = \frac{1}{2}[D^2(1, t) - D^2(0, t)] - 2 \int_0^1 Q_2(x, t) D(x, t) dx - \int_0^1 Q_{2t}(x, t) D(x, t) dx. \tag{65}$$

The initial conditions  $Q_{2t}(x, 0) \equiv Q_2(x, 0)$  (a case of the shallow traps) are determined by (16)–(18). For numerical calculations we have taken into consideration the following time constants:  $\tau = 100$ ;  $\tau = 1$ ;  $\tau = 0.1$  (also, the other values can be considered). Under these conditions a function  $j(t)$  characterising an anomalous electric conduction between the two electrodes is determined. Some numerical results are illustrated in Fig. 8–11. From our numerical calculations it follows that the negative space charge is distributed in the bulk, that is  $\frac{\partial D(x, t)}{\partial x} < 0$

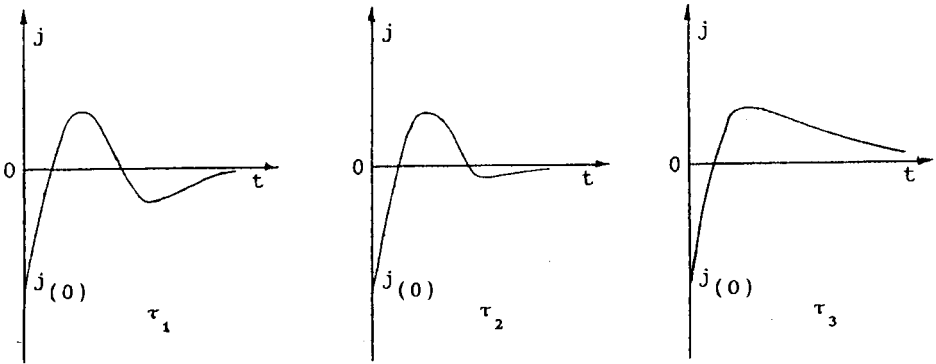


Fig. 8. The shapes of the  $j(t)$  curve obtained by a numerical method in the case of the finite time constants  $\tau_1 = 100$ ;  $\tau_2 = 1$ ;  $\tau_3 = 0.1$

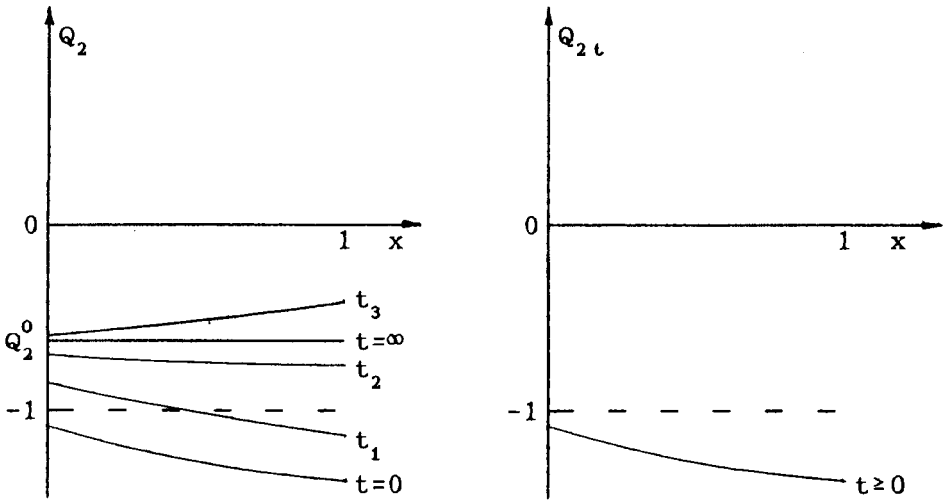


Fig. 9. The free (left figure) and trapped (right figure) negative charge distributions obtained by a numerical method in the case of a finite time constant  $\tau = 100$  ( $0 < t_1 < t_2 < t_3$ )

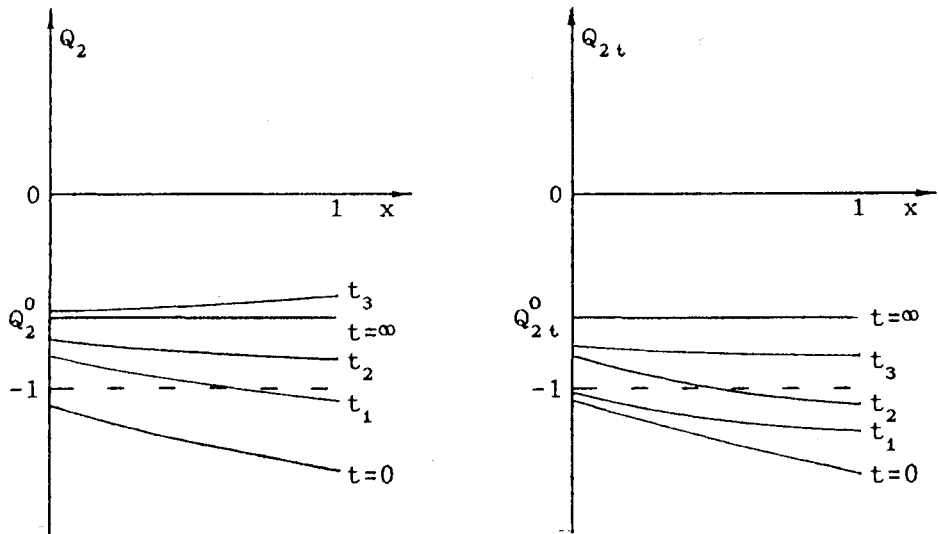


Fig. 10. The free (left figure) and trapped (right figure) negative charge distributions obtained by a numerical method in the case of a finite time constant  $\tau = 1$  ( $0 < t_1 < t_2 < t_3$ )

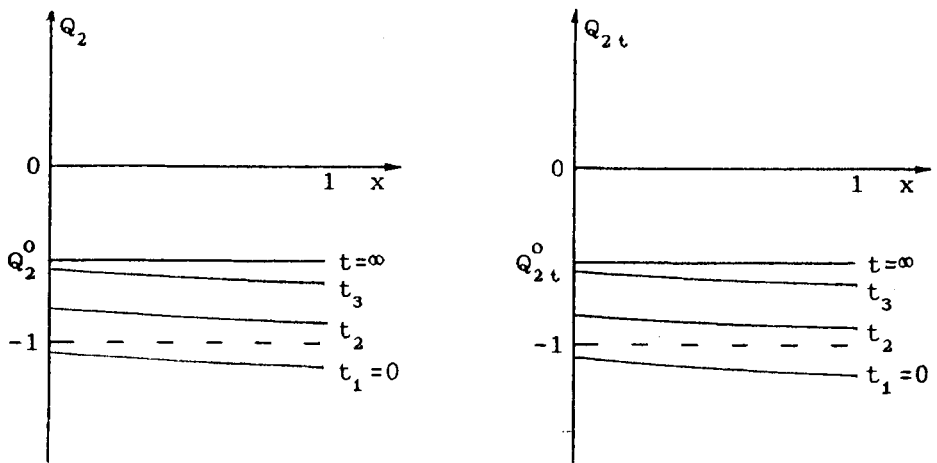


Fig. 11. The free (left figure) and trapped (right figure) negative charge distributions obtained by a numerical method in the case of a finite time constant  $\tau = 0.1$  ( $0 = t_1 < t_2 < t_3$ )

for  $x \in \langle 0, 1 \rangle$  and  $t \geq 0$ . Also, for the bulk with shallow traps the electric field distributions obtained by our numerical algorithm and the  $D(x)$  curves presented in Fig. 3 and Fig. 7b are similar.

#### 4. CONCLUSIONS

In this section, we will determine the physical interpretation and define the importance of the above for the technology. According to (10)–(14) and (6), we have assumed that all the functions are continuous. Moreover, for the transient current — discharging capacitor problem, we assumed that the initial conditions can be determined by the steady state of solid conduction (since, an experiment concerning a charging and discharging capacitor system is easily realised). For our further discussion we must define the form of the generation-recombination parameters (7). Referring to [26], these parameters can be written as

$$C_n = \frac{(Zq^2)^2 N_i}{4(2kT)^{3/2} \pi \epsilon^2 m_e^{1/2}}; \quad v_n = v_{n0} \exp(-W_n/kT) \quad (66)$$

where  $v_{n0} \approx 10^{12} s^{-1}$ ,  $k$  is the Boltzmann constant,  $m_e$  is the electron mass,  $T$  is the temperature,  $Z$  is the atomic number and  $W_n$  denotes the activation energy of the electron. Here, the parameters such as  $W_n$ ,  $N_i$  and  $Z$  are of the form of mean values. The recombination parameter  $C_n$  (or the time constant  $\tau_n$  expressed by (9)) corresponds to the Coulomb force between the positive nucleus and the electron. Similarly, the generation parameter  $v_n$  (or the time parameter  $\tau_g$  expressed by (9)) corresponds to the kinetic energy of phonon and photon. Thus, referring to (10)–(14), we notice that the properties of a current function (6) can be interpreted in terms of the Coulomb force between the positive and negative charge carriers and in terms of the crystalline lattice vibrations. According to the law of mass action (4) or (13), we take into consideration a solid in which the trapping energy states exist (these energy states are caused by the different defects and by impurities and pollutants) and we assume that allowed electron transitions between the valence level and the conduction level via trapping energy states are very quick. Under these conditions, for a current function  $j(t)$  characterising a discharging capacitor system, we can make some general conclusions:

1) When the atomic number is sufficiently great and the additional kinetic energy is given to an orbital electron in a small portion (the activation energy  $W_n$  is sufficiently great in the given constant temperature conditions), the deep trapping levels can exist in which the trapped electron concentration is uniform  $Q_{2i}(x, t) \equiv Q_{2i}^0$ , and the value  $Q_{2i}^0 = -(Q_1^0 + Q_2^0)$  is independent of  $Q_2^0 = -1/Q_1^0$  (here, we can easily show that  $-1 < Q_2^0 < 0$ ). In the particular case, when the negative space charge is distributed in the bulk (Fig. 7b), in the electrode regions (Fig. 5, the set  $\Omega_2$ ) the free electron concentrations can become uniform  $\frac{\partial Q_2(x, t)}{\partial x} \equiv 0$  and the direction of the current flow is changed (Fig. 6). For  $t \leq t_1$  (Fig. 5, and Fig. 7a) the free electron concentration is greater than the free hole

concentration  $n'(x', t') > p'(x', t')$  in the bulk. Under these conditions, from the interior to the boundaries the free electron flow occurs because of the Coulomb force between the free negative charge carriers, and the ordinary current flow is observed  $j(t) < 0$ . After a time  $t_2$  (Fig. 5 and Fig. 7a) the free hole concentration is greater than the free electron concentration  $p'(x', t') > n'(x', t')$  in the bulk. In this case, the Coulomb force between the free positive and negative charge carriers is dominant, so that the free electron flow from the electrodes into the interior occurs (Fig. 5, the set  $\Omega_2$ ) and the anomalous current flow is observed  $j(t) > 0$  (Fig. 6).

2) In the case when in a solid there exist the additional shallow trapping levels in which the same time constants are  $\tau_n = \tau_p = \tau < \infty$  and the negative space charge is distributed, that is  $\frac{\partial D}{\partial x} < 0$  for  $0 \leq x \leq 1$  (Fig. 3 and Fig. 7b), also the anomalous current flow is observed (Fig. 8). From Fig. 8 it follows that the shape of the  $j(t)$  curve depends on the time constant  $\tau$ . Here, we can distinguish the three cases of the current flow. The first case is when  $\tau = 100$ . This denotes that the trapping levels are still sufficiently deep, that is (Fig. 9, the  $t \geq 0$  curve)

$$|Q_2(x, t) - Q_{2t}(x, t)| \ll \tau; \quad Q_{2t}(x, t) \approx Q_{2t}(x, 0) < -1; \tag{67}$$

$$\frac{\partial Q_{2t}(x, 0)}{\partial x} < 0 \quad \text{for } 0 \leq x \leq 1$$

From Fig. 8 (the  $\tau_1$  curve) it follows that there exist the two values  $t_{01}$  and  $t_{02}$  ( $t_{01} < t_{02}$ ) such that  $j(t) < 0$  for  $0 \leq t \leq t_{01}$  or  $t > t_{02}$ , that is, the ordinary current flow is observed. For  $t_{01} < t < t_{02}$  we have  $j(t) > 0$ , that is, the anomalous current flow occurs. Referring to Fig. 9 (the left figure), the parameters  $t_{01}$  and  $t_{02}$  are defined by  $Q_2(x = 0, t = t_{01}) = -1$  and  $Q_2(x = 1, t = t_{02}) = -1$  (these parameters are not shown in Fig. 9). From our numerical calculations it follows that the free electron concentration is greater than the free hole concentration in the bulk when  $0 \leq t \leq t_{01} < t_1$  (Fig. 9). Also, referring to Fig. 9 (the  $t_1$  curve), after a time  $t = t_{01} < t_1$ , the free hole concentration becomes greater than the free electron concentration at the electrode  $x = 0$ . Next, after a time  $t = t_{02} < t_2$ , the free hole concentration is greater than the free electron concentration in the bulk (Fig. 9, the  $t_2$  curve). The behaviour of the  $j(t)$  curves can be explained in terms of the Coulomb force. Thus, according to (67) (or to Fig. 9), when the free negative charge is dominant (that is, when  $0 \leq t \leq t_{01} < t_1$ ), the free electron flow from the interior to the electrodes  $x = 0$  and  $x = 1$  is dominant because of the Coulomb force between the free and trapped electrons. In the case when  $t_1 < t < t_2$ , (referring to (67) and to Fig. 9), from the electrode  $x = 0$  to the electrode  $x = 1$  the free hole flow becomes dominant because of the Coulomb force between the free holes and the trapped electrons. In this case we have  $j(t) > 0$  (Fig. 8, the  $\tau_1$  curve). After a time  $t_2$  we have  $p'(x', t') > n'(x', t')$  and  $\partial p' / \partial x' > 0$  in the bulk (Fig. 9, the  $t_3$  curve). However, the space charge is negative in the bulk (Fig. 3 and Fig. 7b). In this situation, the electric field of the free positive space charge is neutralised by the electric field of the trapped negative space charge, and the Coulomb force between the free electrons and the free holes is

greater than the Coulomb force between the free and trapped electrons. Under these conditions, the free electron flow from the electrode  $x = 0$  to the electrode  $x = 1$  is dominant, that is  $j(t) < 0$ .

3) In the case when  $\tau = 1$  (traps are sufficiently shallow, Fig. 10), the similar property of the current flow occurs (Fig. 8, the  $\tau_2$  curve). However, in this case the minimum of the function  $j(t)$  is greater than the convenient minimum of  $j = j(t)$  for  $\tau_1 = 100$  (Fig. 8, the  $\tau_1$  and  $\tau_2$  curves).

4) In the case when the electrons are localised in the shallow trapping levels (this is expressed by  $\tau = 0.1$  and by  $\theta = 2$ ), the negative space charge can be distributed (Fig. 3 and Fig. 7b) and the anomalous current flow is observed (Fig. 8, the  $\tau_3$  curve). Moreover, from our numerical calculations it follows that the system goes back to the equilibrium conditions characterised by  $Q_1^0 = +\sqrt{\theta} = +\sqrt{2}$  and by

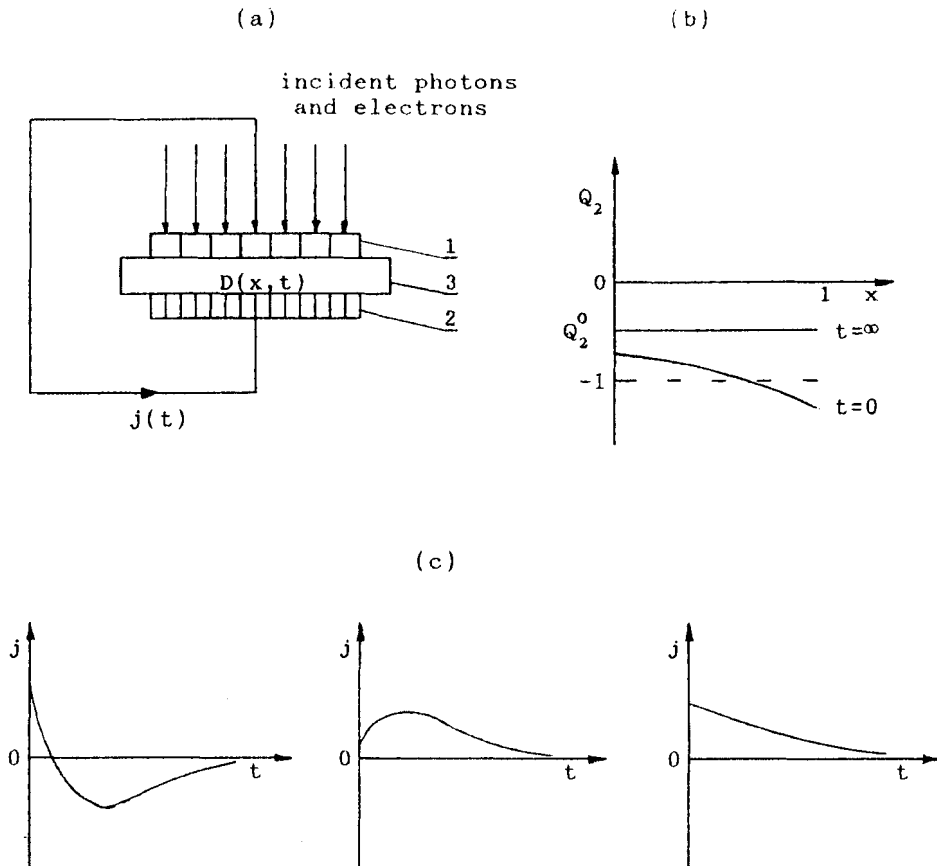


Fig. 12. Technical application of the theoretical considerations [6-8, 23, 25, 27, 33, 34]: a) the electron-photon detector system (or a solar cell): 1 — the semitransparent electrode treated as an injecting cathode, 2 — the electrode, 3 — the bulk. Here, the current flow  $j(t)$  is anomalous; b) possible initial distribution of the free negative charge in the electron-photon detector system; c) the possible shapes of the  $j(t)$  curves in the electron-photon detector system



$Q_1^0 = Q_{2t}^0 = \frac{-1}{\sqrt{\theta}} = \frac{-1}{\sqrt{2}}$  (Fig. 11). Referring to Fig. 6 and Fig. 8 (the  $\tau_3$  curve), we ascertain that the electrical properties of the bulk with deep traps and the electrical properties of the bulk with shallow traps are similar.

5) In the case when the electrons are not localised in the additional trapping levels (this is characterised by  $Q_{2t}(x, t) \equiv 0$ ), the ordinary current flow characterised by  $j(t) < 0$  occurs. In this case, the whole energy of the system is given back, and the system goes back to the equilibrium conditions defined by  $-Q_1^0 = Q_2^0 = -1$  (Fig. 2–4).

6) Generally, from Fig. 6 and Fig. 8 it follows that the current flow is stimulated by the trapped electrons.

7) Generally, from Fig. 5, 6 and Fig. 8 it follows that the additional energy is given by both the contacts (this property is expressed by  $j(t) > 0$ ). This physical property denotes that the system acts as a reservoir of electrical energy. Practically, this electrical phenomenon can occur when the additional and finite portion of energy is given to the system by incident light and electrons (Fig. 12).

8) Using the technique presented in Fig. 12a and making use of the  $j(t)$  curves showed in Fig. 12c, for a solar cell, the bulk can be tested.

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