

A space-charge problem including trapped positive charge. A case of solar cell

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Manuscript received 1998.04.17, revised version 2002.06.17

A theoretical description of an interior of a solid for space charge is continued. Carrier generation and recombination conditions are considered. On this basis it is found that the metal-solid-metal system can act as a perfect blocking diode. A solar cell problem is discussed.

Keywords: space charge, trapping levels, solid, electric field

1. INTRODUCTION

The fundamental difficulty for a bipolar space charge problem is to define the interior and the boundaries of the metal-solid-metal system. In the works [1–6], using allowed electron transitions in the band gap, the divergence of the electric field distributions has been determined. In this paper, that problem will be continued and developed.

The purpose of this work is to develop a space-charge problem and to determine new electric field distributions in a solid.

2. NEW MATHEMATICAL MODEL

From numerous investigations (Fig. 1) [7–18] it follows that electrical properties of the metal-solid-metal system correspond to a crystalline structure of a solid and to the electrode processes. Taking into account this fact, for our mathematical considerations, we will consider a solid in which the system of atoms is very chaotic and the different structural defects (pollutants and impurities, the Frenkl defects) and dislocations occur. This property will be characterised by the Zeeman internal effect (the splitting of the total energy of an orbital electron). For an orbital electron in the given atom, we will assume that the total energy (the sum of the kinetic energy and the negative potential energy of the electric field of the positive nucleus) is negative and that the zero reference level is at a finite distance from the nucleus. In this paper, we will consider a special case of electric-magnetic force interactions between atoms in a solid. To this end, let us take into account

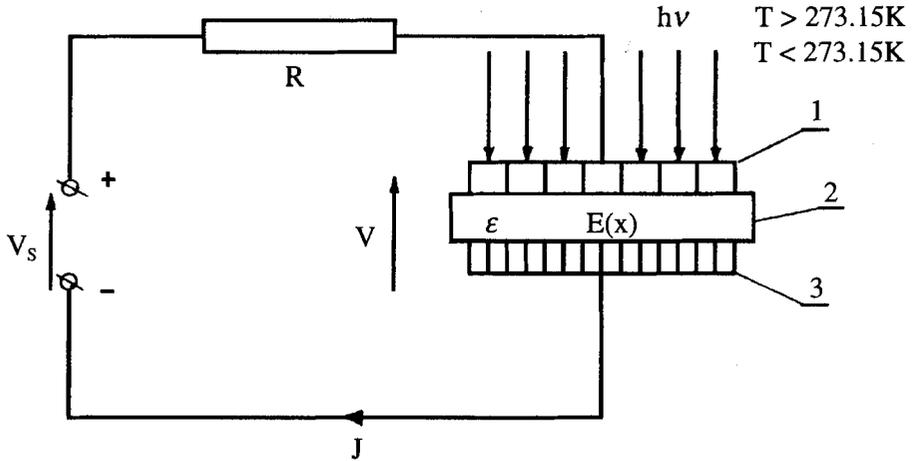


Fig. 1. The technique realising a space charge phenomenon: 1 — semitransparent anode; 2 — bulk; 3 — cathode; $h\nu$ — the energy of an incident photon (the γ or X radiation, the thermal radiation, the LASER light etc); R — external resistance; V_s — source voltage; T — temperature. In this system, we have two general cases: (a) when R is very small, a function $J = J(V)$ is found — a case of the voltage source, (b) when R is very great, a function $V = V(J)$ is found — a case of the current source

the two adjacent atoms and let us denote them as the A atom and as the B atom. For the valence electron in the A atom, let us suppose that the total energy is equal to W_v . Similarly, for the valence electron in the B atom (the given pollutant or impurity), let us assume that the total energy is equal to W_v^B and $W_v^B > W_v$. Now, let us take into consideration the two orbital electrons in the A atom and in the B atom. Next, in the A atom, the valence electron with the spin number $-1/2$ will be denoted by $-e_A$. Analogously, in the B atom the orbital electron with the spin number $+1/2$ will be denoted by $+e_A$. Thus, we will assume that many discrete energy states are available (the trapping levels) for the $-e_A$ electron and the $+e_B$ electron in the A atom as well as in the B atom, respectively (this property corresponds to the Zeeman internal effect). When the $-e_A$ electrons are permanently bombarded by photons and phonons, these valence electrons can pass to the zero reference level in which the electrons become free. In this case, in the valence level, the empty energy states, which are left by these electrons, represent the vacancies with the positive charge (holes). Thus, under these conditions, the hole-electron pair generation occurs. When the additional kinetic energy is given to the free electron by an external electric field, the current flow with the mobility μ_n is observed. In the case when a great portion of the kinetic energy is given by an external electric field to the valence ($-e_A$ or $+e_B$) electron of an adjacent atom, this electron can occupy the empty energy state (the hole) in the valence level and the hole flow with the mobility μ_p occurs. For the free electron, it is possible to have a case in which this electron can lose a very great portion of the kinetic energy (this is caused by the Coulomb force between the electron and the positive nucleus of the A or B atom) and allowed electron transition from the conduction band *via* the

trapping levels to the valence level (in the A or B atom) occurs. In this case, the electron-hole recombination is observed. Now, let us limit our attention only to the B atom (impurity or pollutant). When the additional kinetic energy is given by photons and phonons to the $+e_B$ electron with the total energy W_v^B , this electron can pass to the zero reference level. In this case, the empty energy state, which is left by the valence electron of the B atom, represents the trapped hole in the band gap. For the $+e_B$ electrons with the total energy $W < W_v$, we can assume that these electrons can occupy the valence level which is characterised by the total energy W_v (for example, for the B atoms such, Cu^+ , Ag^+ , Au^+ and Cu^{+2} , Ag^{+2} , Au^{+3} and others, this is possible when a sufficiently great portion of the kinetic energy is given to these electrons by an external electric field and by photons and phonons). Additionally, when the $+e_B$ electrons are permanently bombarded by photons and phonons, the higher (trapping) energy levels can be occupied by these electrons. In the particular case, the $+e_B$ trapped electrons can fill the empty energy states (the trapped holes) in the highest trapping level and the empty energy states, which are left by these electrons in the lower trapping level, define the other trapped holes in the lower trapping level. Moreover, these new trapped holes can be filled by the $+e_B$ electrons occupying the valence level. In this case the unfilled energy states, which are left by these electrons, represent the free holes. Such allowed electron transitions in the B atom define the electron-hole pair generation. For the B atom, the inverse case can occur. The $+e_B$ trapped electrons can lose a portion of the kinetic energy because of the Coulomb force between these trapped electrons and the positive nucleus of the B atom. These electrons can fill the free holes in the valence level. However, now, the empty energy states, which are left by these trapped electrons, represent the trapped holes in the higher energy level. The same situation occurs in the case of the $+e_B$ trapped electrons occupying the highest trapping levels. Additionally the free electrons can fall into the B atom and lose a portion of the kinetic energy. In this case, in the highest trapping level the trapped holes can be filled by the free electrons. For such allowed electron transitions in the B atom, the hole-electron recombination occurs.

Generally, for such allowed electron transitions, the number of the generation-recombination parameters becomes very great. For our mathematical considerations, in order to avoid this difficulty, the trapping levels will be grouped into the four permissible energy levels. To this end, the so-called effective parameters such as the frequency parameters c_{21} and c_{12} as well as the recombination parameters c_{12} and c_{21} will be used. For the trapped electrons, the concentrations of traps in the first and second trapping level will be represented by N_{t1} and N_{t2} , respectively. Analogously, for the trapped holes, the concentrations of traps in the first and second trapping level will be equal to P_{t1} and P_{t2} , respectively. The system of atoms will be treated as an unlimited reservoir of traps, that is $P_{t1} \gg p_{t1}$; $P_{t2} \gg p_{t2}$; $N_{t1} \gg n_{t1}$ and $N_{t2} \gg n_{t2}$. According to Fig. 1, the metal-solid-metal system will be represented by a planar capacitor system with the anode $x = 0$ and the cathode $x = L$. Also, L denotes the distance between the electrodes. Moreover, we will assume that the contact capacitance at the anode and cathode as well as the diffusion current are negligible [19–20]. For a solid, we will assume that the polarisation effect is characterised by the dielectric constant ϵ and that the mobilities of free carriers are independent of the electric field intensity E . Under these conditions, the basic equations such

as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral are written as follows:

$$\frac{\varepsilon}{q} \frac{\partial E(x,t)}{\partial x} = p(x,t) + p_{t1}(x,t) + p_{t2}(x,t) - (n(x,t) + n_{t1}(x,t) + n_{t2}(x,t)), \quad (1)$$

$$\begin{aligned} \frac{\partial}{\partial x} \left\{ [\mu_n n(x,t) + \mu_p p(x,t)] E(x,t) \right\} + \frac{\partial p(x,t)}{\partial t} + \frac{\partial p_{t1}(x,t)}{\partial t} + \frac{\partial p_{t2}(x,t)}{\partial t} + \\ - \frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial n(x,t)}{\partial t} = v_{t1} P_{t1} + v_{n1} n_{t1}(x,t) - c_n N_{t1} n(x,t) - C_t n(x,t) p_{t1}(x,t) + \\ + \frac{\partial}{\partial x} \left\{ [\mu_n n(x,t)] E(x,t) \right\}, \end{aligned} \quad (3)$$

$$\frac{\partial p_{t1}(x,t)}{\partial t} = v_{t1} P_{t1} + c_{t21} P_{t1} p_{t2}(x,t) - c_{t12} P_{t2} p_{t1}(x,t) - C_t n(x,t) p_{t1}(x,t), \quad (4)$$

$$\frac{\partial p_{t2}(x,t)}{\partial t} = c_{t12} P_{t2} p_{t1}(x,t) - c_{t21} P_{t1} p_{t2}(x,t) - v_{t2} P_{t2}(x,t) + c_{t2} P_{t2} p(x,t), \quad (5)$$

$$\frac{\partial n_{t1}(x,t)}{\partial t} = c_{21} N_{t1} n_{t2}(x,t) - v_{n1} n_{t1}(x,t) - c_{12} N_{t2} n_{t1}(x,t) + c_n N_{t1} n(x,t), \quad (6)$$

$$\frac{\partial n_{t2}(x,t)}{\partial t} = v_{p2} N_{t2} + c_{12} N_{t2} n_{t1}(x,t) - c_{21} N_{t1} n_{t2}(x,t) - C_p p(x,t) n_{t2}(x,t), \quad (7)$$

$$\int_0^L E(x,t) dx = V; \quad V = \text{const} > 0, \quad (8)$$

where q is the electric charge, x — distance from the electrode, t — time, n and p are the free hole and electron concentrations, respectively, n_{t1} ; n_{t2} ; p_{t1} ; p_{t2} are the concentrations of the trapped holes and electrons, respectively, v_{p2} ; v_{n1} ; v_{t1} ; v_{t2} denote the frequency parameters, c_n ; C_p ; C_t ; c_{t2} denote the recombination parameters and V is the applied voltage between the electrodes (Fig. 1). For such internal processes we shall define the stationary state and we shall find different current-voltage characteristics.

3. THE STATIONARY STATE

Photo-electrical properties of the metal-solid-metal system (Fig. 1) can be expressed by a current-voltage function. From (1)–(8) it follows that the steady state is described by the following equations

$$\frac{\varepsilon}{q} \frac{dE(x)}{dx} = p(x) + p_{i1}(x) + p_{i2}(x) - (n(x) + n_{i1}(x) + n_{i2}(x)), \quad (1a)$$

$$J = qE(x) [\mu_n n(x) + \mu_p p(x)]; \quad J = \text{const}, \quad (2a)$$

$$v_{i1} P_{i1} + v_{n1} n_{i1}(x) - c_n N_{i1} n(x) - C_i n(x) p_{i1}(x) + \frac{d}{dx} [\mu_n n(x) E(x)] = 0, \quad (3a)$$

$$v_{i1} P_{i1} + c_{i21} P_{i1} p_{i2}(x) - c_{i12} P_{i2} p_{i1}(x) - C_i n(x) p_{i1}(x) = 0, \quad (4a)$$

$$c_{i12} P_{i2} p_{i1}(x) - c_{i21} P_{i1} p_{i2}(x) - v_{i2} p_{i2}(x) + c_{i2} P_{i2} p(x) = 0, \quad (5a)$$

$$c_{21} N_{i1} n_{i2}(x) - v_{n1} n_{i1}(x) - c_{i2} N_{i2} n_{i1}(x) + c_n N_{i1} n(x) = 0, \quad (6a)$$

$$v_{p2} N_{i2} + c_{i2} N_{i2} n_{i1}(x) - c_{21} N_{i1} n_{i2}(x) - C_p p(x) n_{i2}(x) = 0, \quad (7a)$$

$$\int_0^L E(x) dx = V, \quad (8a)$$

where J is the current density. Thus, referring to Fig. 1, the metal-solid-metal system will be characterised by a current-voltage function in the form $J = J(V)$ or $V = V(J)$. In order to find these functions, we have to give the two boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$ describing the mechanisms of carrier injection from the anode $x = 0$ and the cathode $x = L$ into a solid, respectively [21–26]. In what follows, we will consider some possible interactions between carriers.

3.1. The carrier flow under conditions of $C_i = C_p = 0$

In this section, we will take into account a case in which the carrier generation processes are still dominant. For such internal processes, in order to determine a function $E(x)$, let us first introduce the new following symbols:

$$\lambda = \frac{q\mu_n n E}{J}; \quad r = \frac{\mu_n}{\mu_p}; \quad \gamma_1 = 1 + \frac{c_{i2} P_{i2}}{v_{i2}} + \frac{c_{i21} P_{i1} c_{i2}}{v_{i2} c_{i12}}; \quad \gamma_2 = 1 + \frac{c_n N_{i1}}{v_{n1}} + \frac{c_{i2} N_{i2} c_n}{v_{n1} c_{21}}; \quad (9)$$

$$n_{r1,0} = \frac{v_{p2}N_{r2}}{v_{n1}}; n_{r2,0} = \frac{v_{p2}N_{r2}(v_{n1} + c_{12}N_{r2})}{v_{n1}c_{21}N_{r1}}; p_{r1,0} = \frac{v_{r1}P_{r1}(v_{r2} + c_{r21}P_{r1})}{v_{r2}c_{r12}P_{r2}}; p_{r2,0} = \frac{v_{r1}P_{r1}}{v_{r2}}.$$

Using these symbols and combining (1a)–(7a), we obtain

$$\frac{\epsilon\mu_n E}{J} \frac{dE}{dx} = r\gamma_1(1-\lambda) - \gamma_2\lambda + (p_{r1,0} + p_{r2,0} - n_{r1,0} - n_{r2,0}) \frac{\mu_n q E}{J}, \quad (10)$$

$$\frac{d\lambda}{dx} = -\frac{q}{J}(v_{r1}P_{r1} + v_{p2}N_{r2}). \quad (11)$$

From (11) it follows that a function $\lambda = \lambda(x)$ is linear. Thus, the electric field distribution $E(x)$ is described by the following differential equation

$$E \frac{dE}{dx} = \alpha(x - x_0) + \alpha_1 E \quad (12)$$

or by

$$\frac{dE}{dx} = \frac{\alpha(x - x_0)}{E} + \alpha_1, \quad (12a)$$

where

$$\alpha = \frac{q(r\gamma_1 + \gamma_2)(v_{r1}P_{r1} + v_{p2}N_{r2})}{\epsilon\mu_n}; \alpha_1 = \frac{q}{\epsilon}(p_{r1,0} + p_{r2,0} - n_{r1,0} - n_{r2,0}) \quad (13)$$

and x_0 is a constant of integration. According to (12), the general integral $E(x)$ is of the form

$$E(x) = \sqrt{\alpha(x - x_0)^2 + K} \quad \text{for } \alpha_1 = 0 \quad (14)$$

and

$$K = E^2(0) - \alpha x_0^2; \quad x_0 = \frac{1}{2}L - \frac{E^2(L) - E^2(0)}{2\alpha L}, \quad (14a)$$

where K is a new constant of integration. We can show that the general integral of (12a) results in (14) when $4\alpha \gg \alpha_1^2$ (that is, carriers are not too mobile). Relation (12a) is the homogenous equation whose singular solutions are

$$E(x) = z_1(x - x_0) \quad \text{and} \quad z_1 > 0 \quad \text{or} \quad E(x) = z_2(x - x_0) \quad \text{and} \quad z_2 < 0, \quad (15)$$

where

$$z_{1,2} = \frac{1}{2} \left(\alpha_1 \pm \sqrt{\alpha_1^2 + 4\alpha} \right) \quad \text{for } \alpha_1 > 0 \tag{15a}$$

or

$$-z_{1,2} = \frac{1}{2} \left(|\alpha_1| \pm \sqrt{\alpha_1^2 + 4\alpha} \right) \quad \text{for } \alpha_1 < 0. \tag{15b}$$

In the case when $\alpha_1 = 0$, the singular solutions (15) take the form

$$E(x) = -\sqrt{\alpha}(x - x_0) \quad \text{or} \quad E(x) = \sqrt{\alpha}(x - x_0). \tag{15c}$$

According to (14) and (14a), the constants of integration K and x_0 depend on the boundary values $E(L)$ and $E(0)$. For the general integral (14), we will consider electric conduction when a low level of carrier injection occurs. To this end, we will assume that the two boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$, which describe the mechanisms of carrier injection from the anode $x = 0$ and the cathode $x = L$ into the bulk, are identical

$f_0[E(0)] \equiv f_L[E(L)]$ (that is $E(0) = E(L)$). This denotes that $x_0 = \frac{1}{2}L$. Next, referring to

(14), we notice that the space charge density $q_v = \epsilon \frac{dE}{dx}$ is negative for $0 \leq x < \frac{1}{2}L$ and

positive for $\frac{1}{2}L < x \leq L$. Thus, we see that the metal-solid-metal system acts as an n - p

junction (Fig. 2). In this case, substituting (14) and (14a) into (8a), we ascertain that the current-voltage characteristic $V = V(J)$ has the following parametric form

$$V = \frac{1}{2\sqrt{\alpha}} \left(E^2(L) - \frac{\alpha L^2}{4} \right) \ln \left| \frac{E(L) + \frac{1}{2}L\sqrt{\alpha}}{E(L) - \frac{1}{2}L\sqrt{\alpha}} \right| + \frac{1}{2}LE(L) \quad \text{and} \quad J = f_L[E(L)]. \tag{16}$$

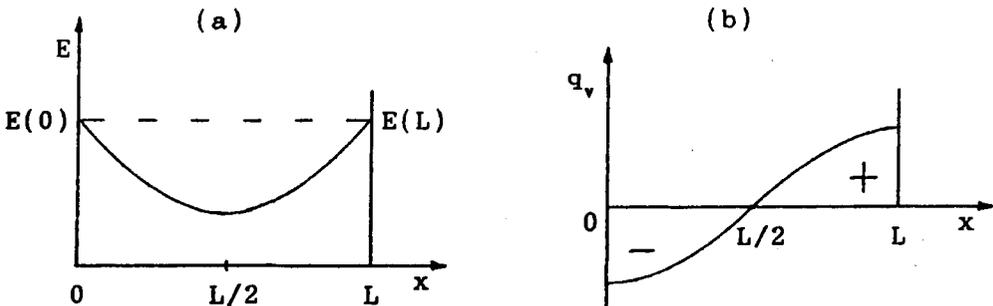


Fig. 2. The shapes of the curves determined by (14) and (14a) when $E(0) = E(L)$: (a) the electric field distribution $E(x)$, (b) the space charge density distribution. The scale is arbitrary for clarity

We can show that the current-voltage relation (16) is defined and differentiable for all values of $V \geq 0$ when the boundary function $J = f_L[E(L)]$ is differentiable and increasing. Function (16) has two characteristic parameters

$$V_0 = \frac{1}{4}L^2\sqrt{\alpha} \quad \text{and} \quad J_0 = f_L(2V_0/L), \quad (16a)$$

for which the derivative dJ/dV is equal to zero (Fig. 3). Referring to the singular solutions (15) and (15c), the current-voltage characteristic $J(V)$ is determined by the boundary function $J = f_L[E(L)]$ or $J = f_0[E(0)]$ describing the mechanism of carrier injection from the anode or the cathode into the bulk. Thus, for the mechanism of electron injection, which is described by $J = f_L[E(L)]$, we have

$$J = f_L[(V - V_{02})/L] \quad \text{for} \quad V \geq V_{02} = |z_2|L^2/2 \quad \text{and} \quad x_0 > L. \quad (17)$$

For the mechanism of hole injection, which is described by $J = f_0[E(0)]$, a current-voltage function becomes

$$J = f_0[(V - V_{01})/L] \quad \text{for} \quad V \geq V_{01} = |z_1|L^2/2 \quad \text{and} \quad x_0 < 0. \quad (18)$$

From (15c) it follows that the voltage parameters (the thresholds voltages) V_{01} and V_{02} are identical $V_{01} = V_{02}$.

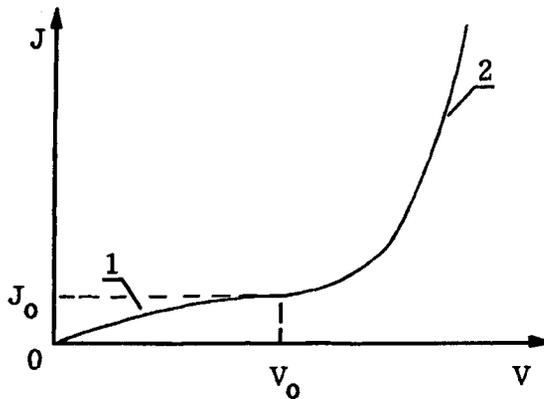


Fig. 3. The shapes of the current-voltage characteristic (16) determined by an increasing and differentiable boundary function $J = f_L[E(L)]$: 1 — low voltage conditions $V < V_0$, 2 — high voltage conditions

$$V \gg V_0, \quad \frac{dJ}{dV} = 0 \quad \text{for} \quad V = V_0. \quad \text{The scale is arbitrary for clarity}$$

3.2. Recombination conditions and a case of strong asymmetric double injection

In this part we shall consider a space charge problem when allowed electron transition from the conduction band to the valence level *via* the trapping levels and hole transition

from the valence level to the conduction band *via* the trapping levels are dominant. This electrical property of a solid is characterised by $v_{i1} = c_{i12} = v_{i2} = 0$ and $v_{n1} = c_{21} = v_{p2} = 0$ (a case of low temperature conditions and the absence of photons). In this case, we additionally assume that the internal parameters satisfy the following conditions:

$$\mu_n = \mu_p = \mu; \quad c_{i2}P_{i2} = c_nN_{i1} = \tau^{-1}; \quad C_i = C_p. \quad (19)$$

Under these conditions, equations (1a)–(8a) will be solved by the use of the following variables

$$\lambda_1 = \frac{q\mu p E}{J}; \quad \lambda_2 = \frac{q\mu n E}{J}; \quad \lambda_3 = \frac{q\mu p_{i1} E}{J}; \quad \lambda_4 = \frac{q\mu n_{i2} E}{J}. \quad (19a)$$

Moreover, we introduce into our analysis the two time constants:

$$\tau_p^{-1} = c_{i21}P_{i1}; \quad \tau_n^{-1} = c_{i2}N_{i2}. \quad (19b)$$

Thus, we can write

$$\lambda_1 + \lambda_2 = 1; \quad \lambda_3 = \frac{q\mu E}{\tau C_i J} \cdot \frac{\lambda_1}{\lambda_2}; \quad \lambda_4 = \frac{q\mu E}{\tau C_p J} \cdot \frac{\lambda_2}{\lambda_1}, \quad (20)$$

$$\frac{\varepsilon \mu E}{J} \frac{dE}{dx} = \left(1 + \frac{\tau_p}{\tau}\right) \lambda_1 - \left(1 + \frac{\tau_n}{\tau}\right) \lambda_2 + \lambda_3 - \lambda_4, \quad (21)$$

$$\frac{d\lambda_2}{dx} = \frac{1}{\tau \mu E}. \quad (22)$$

Hence, on the basis (19), we get

$$\frac{\varepsilon}{\tau J} \frac{dE}{d\lambda_2} = 1 + \frac{\tau_p}{\tau} - \left(2 + \frac{\tau_p + \tau_n}{\tau}\right) \lambda_2 + \frac{q\mu E}{\tau C_p J} \cdot \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1 \lambda_2}. \quad (23)$$

In what follows, we will find the general integral of (23) when $\lambda_2 \ll 1$ and $\tau_n + \tau_p \leq \tau$ (a case of strong asymmetric double injection). Since

$$\frac{\lambda_1^2 - \lambda_2^2}{\lambda_1 \lambda_2} = \frac{1 - 2\lambda_2}{\lambda_2(1 - \lambda_2)} = \frac{(1 - 2\lambda_2)(1 + \lambda_2 + \lambda_2^2 + \dots)}{\lambda_2} \approx \frac{1 + \lambda_2 - 2\lambda_2}{\lambda_2} \approx \frac{1}{\lambda_2} \quad (23a)$$

therefore (23) becomes

$$\frac{dE}{d\lambda_2} = \frac{q\mu}{\epsilon C_p} \cdot \frac{E}{\lambda_2}, \quad (23b)$$

from which it follows that the general integral has the form

$$E = K_1 \lambda_2^m; \quad m = \frac{q\mu}{\epsilon C_p}, \quad (24)$$

where K_1 is a constant of integration. Next, substituting (24) into (22), we have

$$\lambda_2(x) = \left(K_2 + \frac{m+1}{\tau\mu K_1} \cdot x \right)^{\frac{1}{m+1}} \quad (25)$$

where K_2 is the second constant of integration. Hence, on the basis (24), we ascertain that the electric field distribution is of the form

$$E(x) = K_1 \left(K_2 + \frac{m+1}{\tau\mu K_1} \cdot x \right)^{\frac{m}{m+1}}. \quad (26)$$

In the case when $\lambda_1 \ll 1$ (another case of strong asymmetric double injection), the function $E(x)$ takes the other possible form

$$E(x) = K_1 \left(K_2 - \frac{m+1}{\tau\mu K_1} \cdot x \right)^{\frac{m}{m+1}}. \quad (27)$$

Using the voltage condition (8a) to (26) and (27), we get the current-voltage characteristic in the following parametric form

$$V = \frac{(m+1)LE(0)}{(2m+1)} \cdot \frac{(\rho^{i+1} - 1)}{(\rho^i - 1)}; \quad \rho = E(L)/E(0); \quad i = \frac{m+1}{m} \quad (28)$$

and $J = f_0[E(0)]$ and $J = f_L[E(L)]$;

where $J = f_0[E(0)]$ and $J = f_L[E(L)]$ are the boundary functions describing the mechanisms of carrier injection from the electrodes into the bulk.

For example, when both boundary functions are linear or quadratic, the current-voltage dependence is $J \propto V$ or $J \propto V^2$, respectively. In the case when both boundary functions describe the Pool effect $J = J_0 \exp(b_0 E(0))$ and $J = J_0 \exp(b_L E(L))$, where J_0 ; $b_0 \neq b_L$ are the boundary parameters, we have $J \propto \exp(\text{constant} \times V)$. In the last case, replacing $E(0)$ and $E(L)$ by $\sqrt{E(0)}$ and $\sqrt{E(L)}$ (the Schottky effect), the current-voltage charac-

teristic is the Schottky type function $J \propto \exp(\text{constant} \times \sqrt{V})$ (Fig. 4). In particular case, when the cathode injects an infinite amount of electrons $E(L) \rightarrow 0$, (28) is reduced to

$$J = f_0\left(\frac{\chi V}{L}\right); \quad \chi = \frac{2m+1}{m+1}. \tag{28a}$$

Similarly, in the case when $E(L) \rightarrow 0$, (28) results in

$$J = f_L\left(\frac{\chi V}{L}\right); \quad \chi = \frac{2m+1}{m+1}. \tag{28b}$$

The functions (28a) and (28b) are illustrated in Fig. 4.

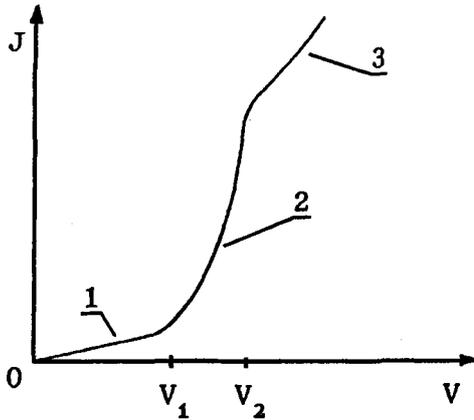


Fig. 4. The current-voltage dependence obtained from (28)–(28b): 1 — low voltage conditions (linear function), 2 — mean voltage conditions (Pool or Schottky function), 3 — high voltage conditions Fowler-Nordheim or quadratic function). The characteristic parameters V_1 and V_2 satisfy the continuity condition for $J = J(V)$. The scale is arbitrary for clarity

3.3. Perturbation of carrier generation

In this section we shall take into consideration a case when allowed hole transition from the conduction band to the valence level *via* the trapping levels occurs and the valence electrons are localised in the deep trapping states. This property of a solid is characterised by $c_{i2} = c_{i21} = C_i = c_n = 0$ and $v_{n1} = 0$. Under these conditions, we have

$$P_{r1} = p_{r1,0} = \frac{v_{i1}P_{i1}}{c_{i12}P_{i2}}; \quad P_{i2} = p_{i2,0} = \frac{v_{i1}P_{i1}}{v_{i2}}; \quad n_{i1} = \frac{c_{21}N_{i1}n_{i2}}{c_{i2}N_{i2}}; \quad pn_{i2} = \frac{v_{p2}N_{i2}}{C_p}. \tag{29}$$

Additionally, in order to avoid a large number of parameters, we will assume that the time constants $(c_{21}N_{i1})^{-1}$ and $(c_{12}N_{i2})^{-1}$ are of the same order of quantities, that is $n_{i1} = n_{i2}$. Next, making use of the new symbols

$$B_1 = \frac{q\mu_n p E}{J}; \quad B_2 = \frac{q\mu_n p_{i1} E}{J}; \quad B_3 = \frac{q\mu_n p_{i2} E}{J}; \quad B_4 = \frac{q\mu_n n E}{J}; \quad B_5 = \frac{q\mu_n n_{i1} E}{J} \quad (30)$$

equations (1a)–(7a) are written as follows

$$B_4 + rB_1 = 1; \quad r^{-1} = \frac{\mu_n}{\mu_p}, \quad (30a)$$

$$\frac{\epsilon\mu_n E}{J} \frac{dE}{dx} = B_1 + B_2 + B_3 - (B_4 + 2B_5), \quad (30b)$$

$$B_4 = -\frac{qv_{i1}P_{i1}}{J} \cdot x + C'; \quad B_5 = \frac{v_{p2}N_{i2}}{C_p} \left(\frac{q\mu_n E}{J} \right)^2 \cdot \frac{1}{B_1}. \quad (30c)$$

where C' is a constant of integration. Since $B_4 < 1$, thus, additionally, we can assume that $B_4 \ll 1$ (a case of strong asymmetric double injection). With this assumption there is

$$\frac{1}{B_1} = \frac{r}{1 - B_4} \approx r(1 + B_4) \quad \text{and} \quad 2B_5 = \frac{2rv_{p2}N_{i2}}{C_p} \left(\frac{q\mu_n E}{J} \right)^2 (1 + B_4). \quad (30d)$$

Completing (30a)–(30d), we obtain

$$E \frac{dE}{dx} = \alpha'(x - x_0) + \alpha_1 E - \alpha_3(1 + C')E^2 + \alpha_2 x E^2, \quad (31)$$

where

$$\begin{aligned} \alpha' &= \frac{(\mu_n + \mu_p)qv_{i1}P_{i1}}{\epsilon\mu_n\mu_p}; \quad \alpha_1 = (p_{i1,0} + p_{i2,0})\frac{q}{\epsilon}; \\ \alpha_2 &= \frac{2q^3\mu_p v_{p2}N_{i2}v_{i1}P_{i1}}{\epsilon C_p J^2}; \quad \alpha_3 = \frac{2q^2\mu_p v_{p2}N_{i2}}{\epsilon C_p J}; \quad 0 \leq C' \ll 1 \end{aligned} \quad (31a)$$

and x_0 depends on C' . Let us notice that (31) results in (12) for $\alpha_2 \rightarrow 0$ and $\alpha_3 \rightarrow 0$. In what follows, we will consider a case when the carrier generation conditions are not too strongly perturbed, that is $-\alpha'x_0 \gg -\alpha_3 E^2$. Moreover, we will assume that the hole generation processes are very quick, that is $v_{i1} \ll v_{i2}$ and $v_{i1}P_{i1} \ll c_{i2}P_{i2}$ (this denotes that

the parameter α'_1 is $\alpha'_1 \rightarrow 0$). In this case, instead of (31), we have a Bernoulli type — differential equation in the form

$$\frac{dE}{dx} - \alpha_2 x E = \alpha'(x - x_0)E^{-1}. \tag{32}$$

Combining (32), we can write

$$\frac{dY_0}{dx} - 2\alpha_2 x Y_0 = 2\alpha'(x - x_0); \quad Y_0 = E^2. \tag{32a}$$

Therefore

$$Y_0 = C_1 \exp(\alpha_2 x^2) + \exp(\alpha_2 x^2) \int 2\alpha'(x - x_0) \exp(-\alpha_2 x^2) dx, \tag{32b}$$

where C_1 is a constant of integration. For (32b), the most interesting case is when $\alpha_2 L^2 \ll 1$.

With this assumption, making use of a property $\exp(\pm \alpha_2 x^2) \approx 1 \pm \alpha_2 x^2$, the general integral $Y_0(x)$ becomes

$$Y_0 = C_1(1 + \alpha_2 x^2) + C_2 \left(x + \frac{2}{3} \alpha_2 x^3 \right) - \frac{\alpha'}{\alpha_2} \tag{32c}$$

and $C_2 = -2\alpha'x_0$ denotes the second constant of integration. Next, limiting (32c) only to the second power, finally, a function $E(x)$ has the form

$$E^2(x) = \alpha_2 C_1 x^2 + C_2 x + C_1 - \frac{\alpha'}{\alpha_2}. \tag{33}$$

For (33), it should be noted that the boundary conditions $E(L) = E(0) = 0$ are not possible. In what follows, we shall consider two special cases of the boundary conditions.

The first case occurs when the anode injects an infinite quantity of holes, that is $E(0) \rightarrow 0$ (this property corresponds to $B_4 \ll 1$). Under these conditions, the particular integral of (33) is

$$E^2(x) = \alpha'x^2 + C_2x; \quad C_2 = \frac{E^2(L) - \alpha'L^2}{L}, \tag{33a}$$

for which the voltage condition (8a) leads to

$$V = \frac{(E^2(L) + \alpha'L^2)E(L)}{4\alpha'L} - \frac{(E^2(L) - \alpha'L^2)^2}{8\alpha'^{3/2}L^2} \ln \left| \frac{E(L) + L\sqrt{\alpha'}}{E(L) - L\sqrt{\alpha'}} \right|; \quad J = f_L[E(L)], \tag{34}$$

where $J = f_L[E(L)]$; $J = f_L[E(L)]$ is the boundary function describing electron injection from the cathode into the bulk. Let us investigate a function (34). Expressing V and $E(L)$ by the normalised values of Y and of w in the form

$$Y = \frac{V}{L^2 \sqrt{\alpha'}}; \quad w = \frac{E(L)}{L \sqrt{\alpha'}} \quad (34a)$$

a function $V[E(L)]$ is written as follows

$$Y(w) = \frac{w^3 + w}{4} - \frac{(w^2 - 1)^2}{8} \ln \left| \frac{1+w}{1-w} \right|. \quad (34ab)$$

Next, expanding the logarithm in the power series, we have

$$Y(w) = \frac{w^3 + w}{4} - \frac{(w^2 - 1)^2}{8} \cdot 2 \left(w + \frac{w^3}{3} + \frac{w^5}{5} + \dots \right); \quad \text{for } 0 \leq w \ll 1 \quad (34ac)$$

or

$$Y(w) = \frac{w^3 + w}{4} - \frac{(w^2 - 1)^2}{8} \cdot 2 \left(\frac{1}{w} + \frac{1}{3w^3} + \frac{1}{5w^5} + \dots \right); \quad \text{for } w \gg 1 \quad (34ad)$$

Thus, we see that the function $Y(w)$ can be reduced to

$$Y(w) = \begin{cases} \frac{2}{3} w^3; & 0 \leq w \ll 1 \\ \frac{2}{3} w; & w \gg 1. \end{cases} \quad (34ae)$$

Similarly, proceeding with the derivative $Z(w) = \frac{dY}{dw}$, which has the form

$$Z(w) = w^2 - \frac{w(w^2 - 1)}{2} \ln \left| \frac{1+w}{1-w} \right| \quad (34b)$$

we ascertain that this derivative $Z(w)$ can be expressed by

$$Z(w) = \begin{cases} 2w^2; & 0 \leq w \ll 1 \\ \frac{2}{3}; & w \gg 1. \end{cases} \quad (34ba)$$

According to (34b), some typical points of the derivative $Z(w)$ are

$$\begin{aligned} Z(0) = 0; \quad Z(0.5) = 0.326; \quad Z(1) = 1; \quad Z(2) = 0.704; \\ Z(5) = 0.675; \quad Z(8) = 0.6687; \quad Z(10) = 0.668; \quad Z(100) = 0.66(6). \end{aligned} \quad (34b)$$

Taking into account (34b)–(34bb), we ascertain that the shape of a $Z(w)$ curve is determined. This function is illustrated in Fig. 5. On this basis, we can estimate the shape of a function $V[E(L)]$ and the shape of the current-voltage characteristic (34) (Fig. 6).

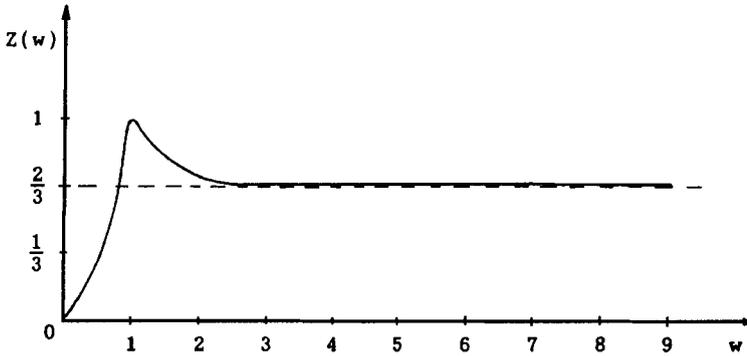


Fig. 5. The normalised derivative $Z(w)$ defined by (34b)–(34bb). This function is typical for a Zener diodes

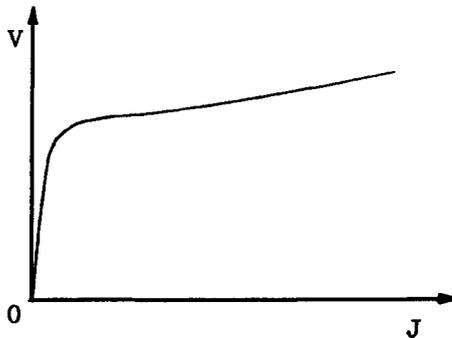


Fig. 6. The shape of the current-voltage characteristic (34) corresponding to the derivative $Z(w)$ (from Fig. 5) and to increasing and non-linear boundary function $J = f_L[E(L)]$. This scale is arbitrary for clarity

The second case is when $E(0) = E(L) > 0$ (this denotes that the contacts $x = 0$ and $x = L$ are identical). For these boundary conditions, our assumption $B_4 \ll 1$ is possible when the trapped electron concentration (this value is stimulated by the parameter $v_{p2}N_D$) is sufficiently great in the anode region. Thus, according to (33), we have

$$E^2(x) = \alpha_2 C_1 x^2 - \alpha_2 C_1 Lx + E^2(L); \quad C_1 = E^2(L) + \frac{\alpha'}{\alpha_2} \quad (35)$$

or

$$E^2(x) = \alpha_2 C_1 \left\{ \left(x - \frac{1}{2}L \right)^2 - \frac{\Delta}{4(\alpha_2 C_1)^2} \right\}; \quad \Delta = (\alpha_2 C_1 L)^2 - 4\alpha_2 C_1 E^2(L). \quad (35a)$$

Function (35a) is defined when the parameter Δ is negative, that is

$$0 < C_1 \leq 4E^2(L)/(\alpha_2 L^2) \quad \text{and} \quad \alpha_2 L^2 \ll 1 \quad \text{and} \quad -\alpha_2 C_1 L \gg -2\alpha_3 E^2(L). \quad (35aa)$$

This denotes that the constant of integration C_1 is $0 < C_1 < \frac{4\alpha'}{3\alpha_2}$ and $2 > \frac{qV_{t1}P_{t1}L}{J}$. Thus,

the space charge density $q_v = \epsilon dE/dx$ is determined by

$$q_v(x) = \frac{\epsilon \alpha_2 C_1 (x - L/2)}{E(x)}; \quad E(x) > 0. \quad (35b)$$

From (35b) it follows that a negative charge is distributed in the anode region $0 < x < \frac{L}{2}$

and a positive charge is in the cathode region $\frac{L}{2} < x \leq L$. Therefore, the metal-solid-metal

system acts as an $n-p$ junction (Fig. 2). Referring to (8a) and (35), we see that this electrical property can be characterised by a current-voltage function $V = V(J)$ in the following parametric form

$$V = \frac{1}{2\sqrt{\alpha_2 C_1}} \left(E^2(L) - \frac{\alpha_2 C_1 L^2}{4} \right) \ln \left| \frac{E(L) + \frac{1}{2}L\sqrt{\alpha_2 C_1}}{E(L) - \frac{1}{2}L\sqrt{\alpha_2 C_1}} \right| + \frac{1}{2}LE(L); \quad (36)$$

$$C_1 = E^2(L) + \frac{\alpha'}{\alpha_2}; \quad J = f_L[E(L)].$$

Now, let us determine the behaviour of a function (36). To this end, introducing the new symbol X into (36), we have

$$V = \frac{L^2}{4} \sqrt{\alpha_2 C_1} \cdot F(X); \quad F(X) = X + \frac{X^2 - 1}{2} \ln \left| \frac{X + 1}{X - 1} \right|; \quad E(L) = X \cdot \frac{1}{2}L\sqrt{\alpha_2 C_1}. \quad (37)$$

The function $F(X)$ has the following properties

$$F(X) = \begin{cases} \frac{2}{3}X^3; & 0 \leq X \ll 1 \\ 2X; & X \gg 1. \end{cases} \quad F(0) = 0; \quad F(1) = 1. \quad (37)$$

Moreover, the derivative dF/dX is positive

$$\frac{dF}{dX} = X \ln \left| \frac{X+1}{X-1} \right|; \quad \lim_{X \rightarrow 1} \frac{dF}{dX} = \infty; \quad \lim_{X \rightarrow 0} \frac{dF}{dX} = 0. \quad (37ab)$$

Thus, the derivative

$$\frac{dV}{dE(L)} = \frac{L^2 \alpha_2 E(L)}{4 \sqrt{\alpha_2 C_1}} F(X) + \frac{L^2}{4} \sqrt{\alpha_2 C_1} \cdot \frac{dF}{dX} \cdot \frac{dX}{dE(L)}; \quad \frac{dX}{dE(L)} = \frac{2\alpha'}{L(\alpha_2 C_1)^{3/2}}. \quad (37b)$$

is positive and

$$\lim_{X \rightarrow 1} \frac{dV}{dE(L)} = \infty. \quad (37c)$$

On this basis, we ascertain that the derivative dJ/dV , which can be expressed by

$$\frac{dJ}{dV} = \frac{dJ}{dE(L)} \cdot \frac{1}{\frac{dV}{dE(L)}} > 0 \quad \text{for } X \neq 1, \quad (37d)$$

is defined and positive when the boundary function $J = f_L[E(L)]$ is differentiable and increasing. From (37c) and (37d) it follows that this derivative is equal to zero

$$\frac{dJ}{dV} = 0 \quad \text{for } X = 1. \quad (37e)$$

Next, referring to Fig. 3, we ascertain that the perturbation of carrier generation defines a set of the characteristic voltages V_0 . This set is determined by the condition $X = 1$, that is

$$E^2(L) = \frac{1}{4} L^2 (\alpha_2 E^2(L) + \alpha') \quad \text{and} \quad \alpha_2 L^2 \ll 1. \quad (38)$$

Making use of the normalised parameter $w = \frac{E(L)}{L\sqrt{\alpha'}}$, the condition (38) results in

$$w \ll \frac{\sqrt{3}}{3}. \quad (38a)$$

The characteristic voltage parameter V_0 (Fig. 3), which will now be denoted by V'_0 , is of the form

$$V'_0 = \frac{L^2}{4} \sqrt{\alpha_2 C_1} = \frac{L^2 \sqrt{\alpha'}}{4} \sqrt{\alpha_2 L^2 w^2 + 1} \approx \frac{L^2}{4} \sqrt{\alpha'}. \quad (38b)$$

Since

$$J = f_L[E(L)] \quad \text{and} \quad E(L) = wL\sqrt{\alpha'} = w\frac{4V'_0}{L}, \quad (38c)$$

therefore the characteristic current density J_0 (Fig. 3) depends on the small parameter w in the form

$$J_0 = f_L(4wV'_0/L); \quad w \ll \frac{\sqrt{3}}{3}. \quad (39)$$

In particular, the condition (38a) is satisfied when the small parameter is infinitesimal $w \rightarrow 0$. This denotes that the values of J_0 becomes infinitesimal $J_0 \rightarrow 0$ when the boundary function satisfies the condition $f_L(0) = 0$ (for example, the linear function, the Fowler-Nordheim function, the quadratic function). Finally, we ascertain that the current-voltage characteristic (36) has the following form

$$J = \begin{cases} f_L(V/L) & \text{for } V \gg V'_0 \\ 0 & \text{for } 0 \leq V \leq V'_0. \end{cases} \quad (40)$$

Next, referring to (35aa), we see that the function (40) exists and that this function describes the current-voltage dependence of an n - p perfect blocking diode (Fig. 7).

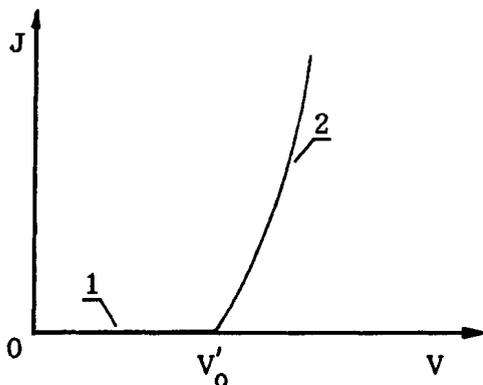


Fig. 7. The shape of the current-voltage characteristic (40) (a case of a perfect Zener diode): 1 — $J(V) \equiv 0$ corresponding to (49a), 2 — $J = f_L(V/L)$ corresponding to (49). This scale is arbitrary for clarity

4. DISCUSSION

In this section we shall define the physical importance of the internal parameters, and on this basis, we shall interpret our results. To this end, let us return to the second part of this paper and let us take into account electric-magnetic interactions between the adjacent

A and B atoms. Here, the A atom is treated as a basic atom of the given crystalline structure and the B atom is treated as an impurity or as a pollutant (for example, the GaAs structure as the A atoms with impurities Ag, Cu and Au as the B atoms). Thus, for our space charge problem, the general form of the Gauss equation is

$$\varepsilon \frac{\partial E}{\partial x} = q \left\{ (p - p_0) + \sum_{j=1}^{m_p} (p_{ij} - p_{ij,0}) - (n - n_0) - \sum_{i=1}^{m_n} (n_{ii} - n_{ii,0}) \right\}, \quad (41)$$

where

$$p_0 + \sum_{j=1}^{m_p} p_{ij,0} = n_0 + \sum_{i=1}^{m_n} n_{ii,0} \quad (41a)$$

are the equilibrium concentrations characterising the bulk at the temperature $T > 0$ K, p_{ij} and n_{ii} are the concentrations of the trapped holes and electrons in the j th and i th trapping levels, respectively, m_p and m_n denote the numbers of the trapping levels for the trapped holes and the trapped electrons, respectively. Allowed electron transition between the i th (lower) energy level and the i' th (higher) energy level is characterised by the rate of change of concentration n_{ii} in the form [1]

$$\frac{\partial n_{ii}}{\partial t} = c_{i'i} N_{ii'} n_{ii'} - c_{ii'} N_{ii'} n_{ii}; \quad i, i' = 1, 2, \dots, m_n, \quad (42)$$

where

$$c_{i'} N_{ii'} = v_{i0} \exp(-W_{ii'} / kT); \quad W_{ii'} = W_{i'} - W_i > 0 \quad \text{and} \quad c_{ii'} = \frac{(Z_i q^2)^2}{4(2kT)^{3/2} \pi \varepsilon^2 m_e^{1/2}} \quad (42a)$$

where k is the Boltzmann constant, m_e is the electron mass, $v_{i0} \approx 10^{12} \text{ s}^{-1}$, Z_i denotes the atomic number corresponding to the i th trapping level $W_{ii'}$ is the energy separation in the band gap, N_{ii} and $N_{ii'}$ are the concentrations of traps in the i th and i' th trapping level, respectively. Under these conditions, the frequency parameters and the recombination parameters are expressed by

$$v_{p2} = v_{p0} \exp((W_v - W_2) / kT); \quad v_{n1} = v_{n0} \exp(W_1 / kT); \quad c_{21} N_{i1} = \langle c_{ii'} N_{ii'} \rangle; \\ c_n = \frac{(Z_1 q^2)^2}{4(2kT)^{3/2} \pi \varepsilon^2 m_e^{1/2}}; \quad c_{12} N_{i2} = \langle c_{ii'} N_{ii'} \rangle; \quad C_p = \frac{(Z_v q^2)^2}{4(2kT)^{3/2} \pi \varepsilon^2 m_e^{1/2}}; \quad (43)$$

where $v_{p0} \approx v_{n0} \approx 10^{12} \text{ s}^{-1}$, $\langle \rangle$ denotes the mean value, Z_1 and Z_v are the atomic numbers corresponding to the first trapping level and to the valence level, respectively. Analo-

gously, considering allowed hole transition between the j th (higher) trapping level and the j' th (lower) trapping level, we can write

$$\frac{\partial p_{ij}}{\partial t} = c_{ij'j} P_{ij} P_{ij'} - c_{ijj'} P_{ij'} P_{ij} \quad (44)$$

and

$$c_{ijj'} P_{ij'} = v_{j0} \exp(-W_{ij'} / kT); \quad W_{ij'} = W_j - W_{j'} > 0 \quad \text{and} \quad c_{ijj} = \frac{(Z_{j'} q^2)^2}{4(2kT)^{3/2} \pi \epsilon^2 m_e^{1/2}}, \quad (44a)$$

where $v_{j0} \approx 10^{12} \text{ s}^{-1}$, $Z_{j'}$ is the atomic number corresponding to the j' th trapping level, $P_{ij'}$ and P_{ij} are the concentrations of traps in the j th and j' th trapping level, respectively. Thus, the generation-recombination parameters are defined by

$$\begin{aligned} v_{i1} &= v_{01} \exp(W_{i1} / kT); \quad c_{i12} P_{i2} = \langle c_{ijj'} P_{ij'} \rangle; \quad v_{i2} = v_{02} \exp((W_v - W_{i2}) / kT); \\ C_i &= \frac{(Z_{i1} q^2)^2}{4(2kT)^{3/2} \pi \epsilon^2 m_e^{1/2}}; \quad c_{i21} P_{i1} = \langle c_{ijj} P_{ij} \rangle; \quad c_{i2} = \frac{(Z_{iv} q^2)^2}{4(2kT)^{3/2} \pi \epsilon^2 m_e^{1/2}}, \end{aligned} \quad (45)$$

where $v_{01} \approx v_{02} \approx 10^{12} \text{ s}^{-1}$, Z_{i1} and Z_{iv} are the atomic numbers corresponding to the first trapping level and to the valence level, respectively. In the case of an insulator, when the $-e_A$ and $+e_B$ electrons pass from the highest to lowest energy level, a very great portion of energy is emitted. This maximum energy, which can be lost by an electron, is

$$-W_v = \sum W_{ji} > 2eV; \quad W_{ji} = W_j - W_i = h\nu_{ji}, \quad (46)$$

where h is the Planck constant and $h\nu_{ji}$ denotes a photon energy which is emitted by the $-e_A$ or $+e_B$ electron when allowed electron transition from the j th (higher) to i th (lower) energy level occurs. In this work, we assumed that all the photons are absorbed by the other trapped and valence electrons, so that the generation processes occur. In a particular case, a part of the energy $-W_v$ can be emitted by the bulk, giving the luminescence effect. According to Fig. 1, this property denotes that the additional portion of the kinetic energy, which is given to the trapped electrons by incident photons and by phonons (rotators and oscillators), is still sufficiently small.

The inverse case is considered in section 3.1. Here, we assumed that the additional kinetic energy, which is given to the $-e_A$ or $+e_B$ trapped and valence electrons by incident photons and by phonons, is in a very great portion. In this case the metal-solid-metal system can act as a solar cell. The characteristic voltage parameter of a solar cell is denoted by V_0 (Fig. 3) or by V'_0 (Fig. 7).

In section 3.2, taking into consideration low temperature conditions and the absence of photons, we have considered electric conduction in the metal-solid-metal system. Here,

referring to Fig. 1, this denotes that (for example) $T = 77$ K, $T = 20$ K or $T = 4$ K as well as $h\nu = 0$ (that is, a solid is not bombarded by any incident photons) [27–28]. Under these conditions, the given material structure becomes a perfect insulator (Fig. 4).

As a special case of allowed electron transitions, under conditions of an external electric field, we have assumed that, in the valence level, the empty energy states (holes) can be filled by the $-e_A$ or $+e_B$ valence electrons. Such allowed electron transition is characterised by the hole mobility μ_p . Here, we have supposed that the new permissible energy states are available for phonons (rotators and oscillators). In other words, we assumed that electric-magnetic interactions between the two adjacent phonons are stimulated by the electric field E . In this case, a portion of an additional kinetic energy, which is given by the electric field E to a phonon, is

$$\frac{hc}{\lambda_e} = c\sqrt{2qm_eEd}, \quad (47)$$

where $c = 10^8$ m/s, d is a microscopic distance and λ_e is the length of an electron wave.

While considering electron passage in the conduction band, we notice that interactions between two adjacent phonons can be stimulated by the electric field E . For allowed electron transitions, we assumed that the A and B atoms correspond to microscopic regions in which the negative potential energy of the electric field of the positive nucleus is condensed. In these microscopic regions, the electric field vector \mathbf{E}_m and the magnetic vector \mathbf{H}_m determine the Zeeman internal effect (the splitting of the energy levels in the given A or B atom). These vectors are expressed by a scalar potential φ and a vector potential \mathbf{A} in the form

$$\mathbf{E}_m = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial\tau'} \quad \text{and} \quad \mu_0\mathbf{H}_m = \nabla \times \mathbf{A}, \quad (48)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m, ∇ is the nabla operator and τ' denotes the time. The formula (48) (in particular, for every atom and for every crystalline lattice) is universally valid. Thus, we ascertain that allowed electron transition from the conduction level *via* the trapping levels to the valence level can occur when the potential electric field $-\nabla\varphi$ in the given microscopic region (the A or B atom) is dominant. The Zeeman internal effect is

described in terms of the electric field $-\frac{\partial\mathbf{A}}{\partial\tau'}$. Under these conditions, we can define the change of the electric-magnetic field energy in the given microscopic volume G (the A or B atom) enclosed by the closed surface $S(G)$. A portion of the kinetic energy, which can be absorbed or emitted by the closed surface $S(G)$ in a small time interval $\Delta\tau'$, is

$$\Delta\tau' \cdot \oint_{S(G)} (\mathbf{E}_m \times \mathbf{H}_m) d\mathbf{S} = \sum_{\lambda_{ph}} \frac{hc}{\lambda_{ph}} + \sum_{\lambda_e} \frac{hc}{\lambda_e}, \quad (49)$$

where λ_{ph} is the length of a photon wave and Σ denotes an algebraic sum. The formula (49) denotes that many permissible energy levels for phonons are available. In other words,

we assume that a configuration of atoms exists for which the change of the electric-magnetic field energy in the given A and B atom is possible. With this assumption, allowed electron transitions can occur. The formula (49) can be treated as a general form of the Compton effect (for a phonon). For a model (1)–(8) and our physical interpretations, the formulas (47)–(49) are fundamental.

Now, let us return to section 3.1 (Fig. 2 and Fig. 3). From (14) it follows that the electric field distribution $E(x)$ does not exist when the contacts $x = 0$ and $x = L$ are identical $E(0) = E(L)$ (a case of low level of carrier injection, Fig. 2) and the voltage V is V_0 (Fig. 3). However, a function (16) is defined for all values of $V \geq 0$. This mathematical property defines a singularity of electric conduction in the metal-solid-metal system when the orbital electrons are permanently bombarded by incident photons and phonons. In this case, (also, referring to section 3.3), for $V < V_0$ the electric field $E(x)$ and the dielectric constant ϵ cannot be known. Only, on the basis of the Gauss equation and of the formulas (47)–(49), this property can be explain. First, let us notice that the electric field $E(x)$ is the sum of the external electric field $E_{ex}(x) \equiv V/L$ and of the internal electric field $E_{in}(x)$ which

is determined by the space charge density $q_v = q_v(x)$ in the form $\frac{dE_{in}}{dx} = \frac{q_v(x)}{\epsilon}$. The space

charge density corresponds to the total energy $W \geq W_v$ of the electron. Under conditions of carrier generations, also, a very great portion of the kinetic energy is given by incident photons to the phonons (rotators and oscillators), and the higher energy levels are occupied by phonons. In other words, the system of atoms becomes very chaotic. Thus, a portion of the kinetic energy cannot be given by the internal electric field E_{in} and by the external electric field E_{ex} to the orbital electrons when the applied voltage V is still too small ($V < V_0$) and a low level of carrier injection $E(0) = E(L)$ occurs. Therefore, in the microscopic regions (the A or B atoms) with the negative potential energy (48), the electron waves (47) do not exist and the polarisation effect does not occur. The waves (47) can exist in the valence level and in the conduction band (a microscopic region with the positive total energy of the electron), and the current flow can be observed. However, under these conditions, for a phonon, the Compton effect (49) is

$$\Delta\tau' \cdot \oint_{S(G)} (\mathbf{E}_m \times \mathbf{H}_m) d\mathbf{S} = \sum_{\lambda_{ph}} \frac{hc}{\lambda_{ph}}, \quad (49a)$$

which denotes that the metal-solid-metal system acts as a reservoir of solar energy (a case of solar cell) for $V < V_0$ (Fig. 3 or Fig. 7). From Fig. 7 it follows that the waves (47) do not exist in the valence level and in the conduction band. Also, using the singular solutions (15) and the displaced functions (17) and (18), we can show that the electron waves (47) do not exist when the applied voltage V is

$$V < \frac{V_{01}V_{02}}{V_{01} + V_{02}}. \quad (50)$$

According to Fig. 3 and Fig. 7, we see that a microscopic property (49a) corresponds to a macroscopic property (38a) and (39) or (50). From Fig. 3 and (16a) it follows that the maximum power P_{\max} of a solar cell can be expressed by

$$P_{\max} = SV_0 f_L(2V_0 / L), \quad (51)$$

where S is the surface of the electrode. For example, when the boundary function $f_L[E(L)]$ is linear or quadratic, the maximum power becomes $P_{\max} \propto V_0^2$ or $P_{\max} \propto V_0^3$, respectively. Referring to section 3.3 and to (34), we see that the metal-solid-metal system can act as a reservoir for solar energy when high level of carrier injection $E(0) = 0$ occurs (Fig. 5 and Fig. 6). Moreover, in this case, the system acts as the voltage stabiliser (a case of a Zener diode). The current-voltage characteristics, which are obtained from the model (1)–(8), are typical for solids based on the material structures such as Se (amorphous structure), Si, Ge, KCl, ZnS, CdS, GaAs, Al_2O_3 , TiO_2 , SiO_2 , SiC (different hexagonal configurations, amorphous structure), C_2F_4 (Teflon) and others.

5. CONCLUSIONS

According to [1–4], under conditions of carrier generation, we ascertain that the electric field (14) is universally valid. In this case, for every level of carrier injection, the metal-solid-metal system can act as a solar cell. The maximum power of a solar cell is expressed by (51). Also, taking into account the singular solutions (15), we ascertain that the metal-solid-metal system acts as a solar cell when the applied voltage satisfies the condition (50). In the case when the contacts are identical and carrier generation conditions are perturbed by recombination processes, which occur in the deep trapping levels, the metal-solid-metal system can act as a perfect blocking diode (Fig. 7). For carrier generation conditions, when a high level of carrier injection occurs, the metal-solid-metal system can act as a Zener diode (a case of a voltage stabiliser). Under low temperature conditions (a case of perfect insulator) and the absence of photons, the current-voltage characteristic is of the form (28)–(28b). Referring to the singular solutions (15), we can show that the current-voltage dependence can be discontinuous and a switching effect of the current-voltage characteristic can occur [2].

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