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ELECTROMAGNETIC THEORY

Bipolar Problem of Space – Charge in Dielectric. II. System with Trapping

by

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Summary. A model of dielectric with bipolar injection of charge and with trapping taken into account is presented. Charge transport was described by partial differential equations in the system of planeparallel condenser. The space – time distributions of charge and induction as well as time courses of resorption current were determined. Initial values were found on the basis of stationary state of absorption. In analytical and numerical consideration the theory of partial quasi – linear differential equations was used.

1. Introduction. The problem of bipolar charge without the effect of trapping taken into consideration was discussed in [1]. In view of the general equations of solid state electronics, the phenomenon of transport in solids is greatly dependent on various defects in their structure and it is therefore to be expected that in particular the presence of trapping states will have an important effect of space – time charge distributions. Developing the idea and methodology assumed in [1] it was attempted to solve the same problems but with the effect of trapping states taken into account.

2. Equation formulation. The description of bipolar charge transport in dielectric is performed with the use of the continuity equation, the Poisson equation, the law of mass action, the equation describing the dynamics of carrier trapping, the material equation and the boundary condition.

We shall adopt the following simplifying assumptions:

—the diffusion currents are omnited in the continuity equation,

—delayed polarization is omitted,

—the electron - hole mechanism of conductance with electron trapping taken into account is assumed [2].

In such conditions the charge transport in dielectric in the field of a planeparallel condenser is described by the equations

$$\frac{\partial}{\partial x}\{\mu_p\, p(x,t)\, E(x,t)+\mu_n\, n(x,t)\, E(x,t)\}+\frac{\partial p(x,t)}{\partial t}-\frac{\partial n(x,t)}{\partial t}-\sum_{i=1}^{m}\frac{\partial n_{ti}(x,t)}{\partial t}=0, \tag{1}$$

$$\frac{\partial D}{\partial x}(x,t)=e_0\left\{(p(x,t)-p_0)-\sum_{i=1}^{m}\left(n_{ti}(x,t)-n_{t_{i,0}}\right)-(n(x,t)-n_0)\right\}, \tag{2}$$

$$n(x,t)\, p(x,t)=n_0\, p_0=K \tag{3}$$

$$\frac{\partial n_{ti}}{\partial t}(x,t)=C_{ni}\left\{n(x,t)\left(N_{ti}-n_{ti}(x,t)\right)-\Theta_{0i}\left(N_{ti}-n_{t_{i,0}}\right)n_{ti}(x,t)\right\} \quad i=1,\ldots,m \tag{4}$$

$$D(x,t)=\varepsilon_0\,\varepsilon E(x,t) \tag{5}$$

$$\int_0^L E(x,t)\,dx=0 \tag{6}$$

where: p, n—nonequilibrium concentrations of free holes and electrons; p_o, n_o—equilibrium concentrations of free holes and electrons; μ_n, μ_p—mobility of electrons and holes; n_{ti}—nonequilibrium concentration of trapped electrons in the "i - th" group of trapping levels; $n_{t_{i,o}}$—equilibrium concentration of trapped electrons in the "i - th" group of trapping levels; N_{ti}—concentration of the "i - th" group of trapping levels, C_{ni}—probability of trapping electrons in the "i - th" group of trapping levels; $\theta_{oi}=\frac{n_o}{n_{t_{i,o}}}$; L—distance between electrodes; K—constant, ε—relative electric permeability; $\varepsilon_o = 8.85\cdot 10^{-12}$ F/m; $e_o = 1.6\cdot 10^{-19}$ C. The density of resorption current $j_{(t)}$ is determined on the basis of (1) and (2)

$$j_{(t)}=\frac{\partial D}{\partial t}(x,t)+e_o\,\mu_p\, p(x,t)\, E(x,t)+e_o\,\mu_n\, n(x,t)\, E(x,t). \tag{7}$$

The aim of the present work is the determination of the time course of current density $j_{(t)}$, of space – time distributions of charges $^{+}q_{v}(x, t) = e_{o}\, p(x, t)$, $^{-}q_{v}(x, t) = -e_{o}\, n(x, t)$, $^{-}q_{vt}(x, t) = -e_{o}\, n_{t}(x, t)$, and of the space – time distributions of induction $D(x, t)$. In terms of [1] this would correspond to the solution of the initial problem. The analytical determination of the density of resorption current $j_{(t)}$ is very difficult. The solution of this problem is greatly simplified for equal mobilities of electrons and holes $\mu_{n} = \mu_{p}$ and zero probabilities of capture $(n_{ti} = n_{ti,o})$. In the analysis of charge transport in dielectrics it is necessary to know boundary and initial conditions.

The initial values of charges $^{+}q_{v}(x, o)$ and $^{-}q_{v}(x, o)$, $^{-}q_{vt}(x, o)$ are determined on the basis of the steady state of absorption current

(8) $$j_{a} = e_{o}\, \mu_{p}\, p(x)\, E(x) + e_{o}\, \mu_{n}\, n(x)\, E(x); \quad j_{a} = \text{const.}$$

(9) $$\frac{dD}{dx}(x) = e_{o}\left\{(p(x) - p_{o}) - \sum_{i=1}^{m}(n_{ti}(x) - n_{ti,o}) - (n(x) - n_{o})\right\}$$

(10) $$n_{ti}(x) = \frac{N_{ti}\, n(x)}{n(x) + n_{1}}; \quad n_{1} = \theta_{oi}(N_{ti} - n_{t_{i},o}) \quad i = 1, \ldots, m$$

(11) $$n(x)\, p(x) = K$$

(12) $$D(x) = \varepsilon_{o}\, \varepsilon E(x)$$

(13) $$\int_{o}^{L} E(x)\, dx = U.$$

Hence, the initial values of charges are described by the formulae

(14) $$^{+}q_{v}(x, 0) = e_{o}\, p(x), \quad ^{-}q_{v}(x, 0) = -e_{o}\, n(x), \quad ^{-}q_{v_{t}}(x, 0) = -e_{o}\, n_{t}(x).$$

The subsequent considerations are based on the above way of determining the initial values.

3. Solution of the problem. The problem of determining space – time distributions of charge and induction as well as of the density of resorption current may be solved numerically. Analytical considerations are possible in the case of equal mobilities of electrons and holes $\mu_{u} = \mu_{p} = \mu$ and zero probability of capture $(n_{ti} = n_{t_{i},o})$. These assumptions were adopted in subsequent considerations in this paper. The analysis of resorption state was performed in the system of normalized variables $^{+}Q_{v}$, $^{-}Q_{v}$, $^{-}Q_{v_{t}}$, D', x', t', j'

(15) $$^{+}Q_{v} = \frac{p}{K^{1/2}}; \quad ^{-}Q_{v} = -\frac{n}{K^{1/2}}; \quad ^{-}Q_{vt_{i},o} = -\frac{n_{t_{i},o}}{K^{1/2}},$$

(15) (cont.)

$$^{+}Q_{v,o}=\frac{p_o}{K^{1/2}};\qquad {}^{-}Q_{v,o}=-\frac{n_o}{K^{1/2}};\qquad x'=\frac{x}{L},$$

$$D'=\frac{D}{e_o LK^{1/2}};\qquad t'=\frac{e_o\mu K^{1/2}}{\varepsilon_o\varepsilon}t;\qquad j'=\frac{\varepsilon_o\varepsilon}{e_o^2\mu LK}j.$$

After omitting primes in the symbols and taking into consideration the equations $\mu_n=\mu_p$ and $n_{ti}=n_{t_{i,o}}$, the equations describing resorption state in the system of dimensionless variables may be written in the form

(16) $$\frac{\partial\,^{+}Q_v(x,t)}{\partial t}+\frac{\partial\,^{-}Q_v}{\partial t}(x,t)+\frac{\partial}{\partial x}\{[^{+}Q_v(x,t)-{}^{-}Q_v(x,t)]\,D(x,t)\}=0,$$

(17) $$\frac{\partial D}{\partial x}(x,t)=\left(^{+}Q_v(x,t)-{}^{+}Q_{v,o}\right)+\left(^{-}Q_v(x,t)-{}^{-}Q_{v,o}\right),$$

(18) $$^{+}Q_v(x,t)\;^{-}Q_v(x,t)=-1,$$

(19) $$\int_o^1 D(x,t)\,dx=0,$$

Moreover, the following conditions are satisfied

(20) $$^{+}Q_{v,o}\;^{-}Q_{v,o}=-1;\qquad ^{+}Q_{v,o}+{}^{-}Q_{v,o}+\sum_{i=1}^{m}Q_{vti,o}=0.$$

The time course of resorption current density is described by the formula

(21) $$j_{(t)}=-\int_o^1 F\,\frac{(1-{}^{-}Q_v^2)}{^{-}Q_v^2}\,\frac{\partial\,^{-}Q_v}{\partial x}\,dx;\qquad F=\int_o^x D(x,t)\,dx,$$

The above dependence was used to determine the shape of the curve of $j_{(t)}$. After transforming equations (16)-(18) we obtain the system of characteristic equations

(22) $$\frac{dx}{dt}=D\,\frac{1-{}^{-}Q_v^2}{1+{}^{-}Q_v^2},$$

and

$$\frac{d\,^{-}Q_v}{dt}=(^{-}Q_v-{}^{-}Q_{v,o})\,(^{-}Q_v-{}^{+}Q_{v,o}),$$

The initial values of charge $^{-}Q_v(x,0)$ is determined on the basis of steady state of absorption current. In steady state the values of induction D_u and density of current j_a satisfy the equation

(24) $$D_u\frac{dD_u}{dx}=-\sqrt{j_a^2-4D_u^2}-(Q_{v,o}+{}^{-}Q_{v,o})\,D_u,\qquad D_u>0,$$

with

(25) $$^{-}Q_v(x, o) = -\frac{j_a + \sqrt{j_a^2 - 4D_u^2}}{2D_u},$$

From equations (19), (20), (22), (24) and (25) it results [3] that there exist moments t_1 and t_2 $(t_1 < t_2)$ in which the gradient of the characteristics dx/dt changes its sign next to the electrodes. The shape of characteristics $x(t)$ are shown in Fig. 1. In region Ω_1 one needs to know initial conditions, and in region Ω_2—the boundary conditions. The solution of equation (22) in region Ω_1 has the form

(26) $$x_{(t)} = \int_o^t D \frac{1 - {}^{-}Q_v^2}{1 + {}^{-}Q_v^2} dt + x(o) \qquad t > 0,$$

and in region Ω_2

(27) $$x_{(t)} = \int_\lambda^t D \frac{1 - {}^{-}Q_v^2}{1 + {}^{-}Q_v^2} dt + 1 \qquad t \geqslant \lambda \geqslant t_2,$$

and

(28) $$x_{(t)} = \int_\lambda^t D \frac{1 - {}^{-}Q_v^2}{1 + {}^{-}Q_2^2} dt \qquad t \geqslant \lambda \geqslant t_1.$$

In the further considerations it was assumed that the solution of equation (22) is unique. Hence we have

(29) $$\left.\frac{dx_{(t)}}{dx_{(o)}}\right|_{t=\text{const.}} > 0 \qquad \left.\frac{dx_{(t)}}{d\lambda}\right|_{t=\text{const.}} > 0.$$

Inequality (29) was used to evaluate the monotonicity of the distribution of charge ${}^{-}Q_v(x, t)$, of induction $D(x, t)$, and to determine the shape of the curve of $j_{(t)}$. The change of concentration of negative charge ${}^{-}Q_v$

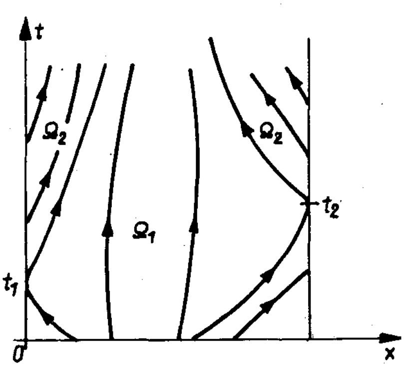

Fig. 1. Exemplary characteristics of resorption state with trapping taken into account

in region Ω_1 may be written as

$$\left.\frac{\partial\ {}^-Q_v(x,t)}{dx}\right|_{x=x(t)} = \left.\frac{\partial\ {}^-Q_v(x,t)}{\partial x_{(o)}}\right|_{x=x_{(t)}} \cdot \frac{1}{\left.\frac{dx_{(t)}}{dx_{(o)}}\right|_{t=\text{const.}}} \tag{30}$$

and in region Ω_2

$$\left.\frac{\partial\ {}^-Q_v(x,t)}{\partial x}\right|_{x=x_{(t)}} = \left.\frac{\partial\ {}^-Q_v(x,t)}{\partial \lambda}\right|_{x=x_{(t)}} \cdot \frac{1}{\left.\frac{dx_{(t)}}{d\lambda}\right|_{t=\text{const.}}} \tag{31}$$

After solving equation (23) and taking into consideration (24), (25) and (29) one infers that in region Ω_1 the distribution of charge ${}^-Q_v$ is decreasing $\frac{\partial\ {}^-Q_v}{\partial x} < 0$. Moreover, we know that at an arbitrary moment $t \geqslant t_1$ (Fig. 1) we need to know the boundary conditions. In the present work it was assumed that next to the electrodes $x=0$ and $x=1$ the charge ${}^-Q_v$ satisfies the equation

$$\frac{d\ {}^-Q_v}{d\lambda} = ({}^-Q_v - {}^-Q_{v,o})\,({}^-Q_v - {}^+Q_{v,o}), \tag{32}$$

with initial values

$${}^-Q_v(0, \lambda=0) = -1; \qquad {}^-Q_v(1, \lambda=0) = -1, \tag{33}$$

Given such boundary conditions the distribution of charge ${}^-Q_v(x,t)$ is continuous in the entire dielectric, and in region Ω_2 next to the electrodes

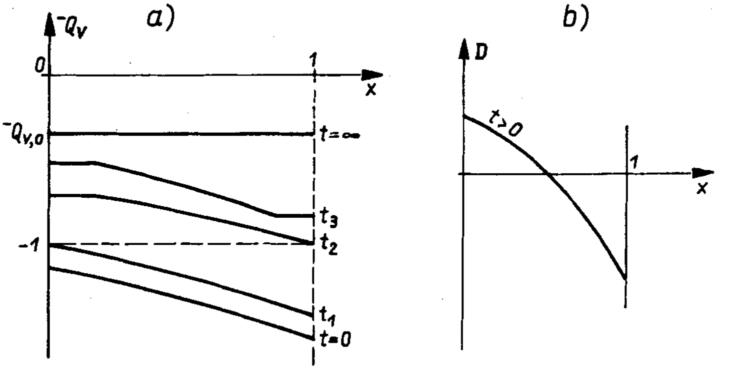

Fig. 2. Exemplary resorption curves with trapping taken into account
a—distribution of negative charge ${}^-Q_v$ in arbitrary moments $t_1 < t_2 < t_3$;
b—distribution of induction at an arbitrary moment $t > 0$

it is uniform $\frac{\partial\ {}^{-}Q_v}{\partial x} = 0$ and constantly increasing $\frac{\partial\ {}^{-}Q_v}{\partial t} > 0$ to the equilibrium value ${}^{-}Q_{v,0}$.

Moreover, in the entire dielectric there is a negative excess charge $\frac{\partial D}{\partial x} < 0$. The obtained results are illustrated in Fig. 2.

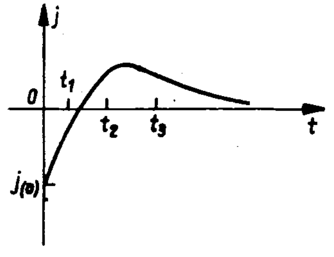

Fig. 3. Example of time distribution of resorption current density with trapping taken into account

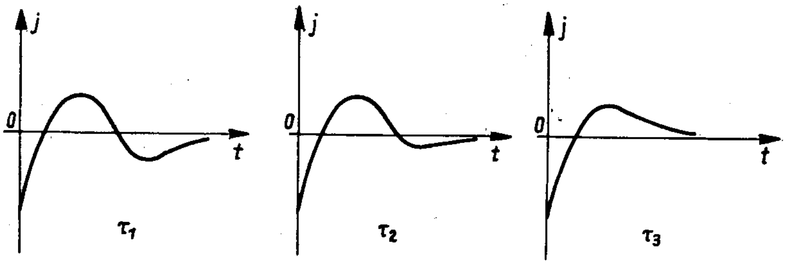

Fig. 4. Examples of resorption current density time courses for various trapping time constants $\tau_1 > \tau_2 > \tau_3$

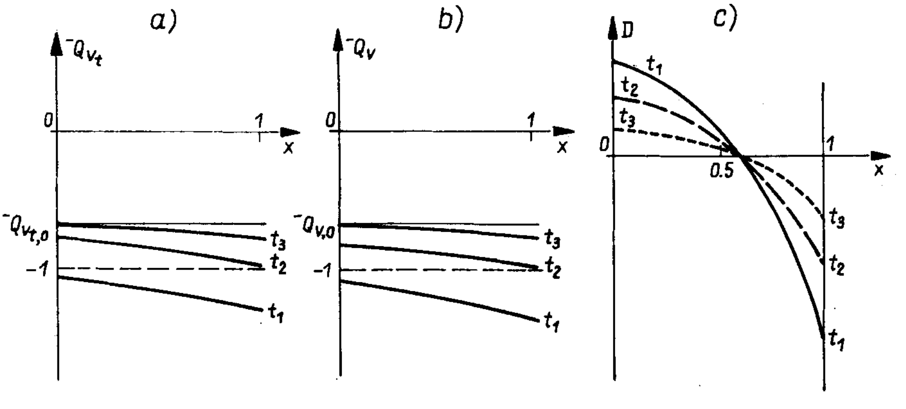

Fig. 5. Examples of resorption curves corresponding to trapping time τ_3 from Fig. 4

a—distribution of trapped negative charge at different moments $t_1 < t_2 < t_3$; b—distribution of free negative charge at different moments $t_1 < t_2 < t_3$; c—distribution of electric induction at different moments $t_1 < t_2 < t_3$

From Fig. 2 and equation (21) it results that at any moment $t: 0 \leqslant t \leqslant t_1$, the density of current $j_{(t)}$ is negative, while at an arbitrary moment $t \geqslant t_2$ the density of current $j_{(t)}$ is positive. In [3] it was demonstrated that the current vanishes to zero through positive values. Thus the time course of $j_{(t)}$ may be of the shape depicted in Fig. 3.

The problem of resorption current for the case of finite values of capture probability was solved numerically. The dielectric was described by an additional parameter called the time constant of trapping

$$T_t = [C_n (N_t - n_{t,o})]^{-1} \tag{34}$$

The constant T_t is normalized according to the formula (15) and denoted by τ. In numerical calculations a dielectric with one group of trapping levels was considered. The performed calculations demonstrates the effect of the trapping constant on the $j_{(t)}$ curve. Moreover it is inferred that if the total excess charge $\partial D/\partial x$ at an arbitrary point of the dielectric is constantly negative and if the distribution of the negative free charge $^{-}Q_v(x, t)$ is monotonic, then the time course of resorption current density changes its sign at least once. Examples of resorption curves obtained with the use of a computer are shown in Figs 4 and 5.

4. Conclusions. In the performed experiments the dielectric sample is kept for an extended period in an electric field under constant voltage, and then resorption state is attained. In view of this it may be assumed that the initial values of resorption state are equal to the values of charge in stationary state at constant voltage. Given this assumption it is necessary to know boundary conditions to be able to solve the resorption current problem. In the case of negative excess charge $\frac{\partial D}{\partial x} < 0$ and infinite times of trapping T_t and given such boundary conditions that at certain moments t_1 and t_2 $(t_1 \neq t_2)$ the distribution of concentrations $n(x, t)$ becomes uniform $\frac{\partial n}{\partial x} = 0$ in regions next to the electrodes, the time course of density of resorption current $j_{(t)}$ is not a monotonic function. Moreover, the resorption current changes its direction of flow at least once.

In the case of negative excess charge and finite value of trapping time, the shape of the time course of resorption current density depends on trapping time. The time course of $j_{(t)}$ is not monotonic and the resorption current changes its direction of flow at least once.

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REFERENCES

[1] B. Świstacz, *Bipolar problem of space – charge in dielectric. I. System without trapping*, Bull. Ac. Pol.: Tech., this issue, pp. 427–436.

[2] A. Many, G. Rakavy, *Theory of transient space – charge – limited current in solids in the presence of trapping*, Phys. Rev., **126** (1962), 1980–1988.

[3] B. Świstacz, *Wpływ bipolarnego ładunku przestrzennego i procesu pułapkowania na przebiegi relaksacyjne prądu resorpcji*, Doctor's Thesis, IPEiE, PWr, I – 7/P – 125/82.

Б. Свистач, **Задача носителей обоих знаков, инжектированных в диэлектрик. Часть II. Диэлектрик с ловушками**

В настоящей статье представляется модель двойной инжекции в диэлектрик с ловушками в системе плоского конденсатора. Транспорт заряда описывается дифференциальными уравнениями с частными производными. Определяются пространственно-переходные характеристики заряда, электрической индукции и перехододные характеристики тока резорбции. Начальные величины установливаются по стационарному состоянию абсорбции. В аналитических и вычислительных решениях используется теория квазилинейных дифференциальных уравнений с частными производными.