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The Effect of Generation-Recombination Processes and of Injection Mechanisms on Current-Voltage Dependences in the Metal-Dielectric-Metal System
II. Carrier Recombination

by

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Summary. A dielectric model in which electrons and holes are the charge carriers was studied. The space charge is formed by free electrons and holes as well as by trapped electrons. Carrier recombination with two trapping states in steady state and at constant voltage was considered in the assumed dielectric model. The considerations refer to the flat capacitor system. The charge injection mechanisms were described by the function of current density and field intensity at the electrodes. The effect of injection mechsnisms in conditions of carrier recombination on current-voltage relations was investigated.

1. Introduction. Continuing the considerations in Part I of this work, we will analyse the steady state of bipolar electric conductivity with the participation of trapping states in the metal-dielectric-metal system. The stationary state of conductivity in the flat capacitor system is described by the equations [2]

$$j = e_0 \mu_p pE + e_0 \mu_n nE; \quad j = \text{const} \tag{1}$$

$$\frac{\varepsilon}{e_0} \frac{dE}{dx} = p - n - n_{t1} - n_{t2}, \tag{2}$$

$$\frac{d}{dx}(\mu_n nE) = C_n n - v_1 n_{t1}, \tag{3}$$

$$C_n n - v_1 n_{t1} = C_{12} n_{t1} - C_{21} n_{t2}, \tag{4}$$

$$C_{12} n_{t1} - C_{21} n_{t2} = C_p pn_{t2} - v_2 N_{t2}, \tag{5}$$

$$\int_0^L E dx = U; \quad U = \text{const}; \quad U > 0. \tag{6}$$

We intend to determine the current-voltage dependence $j = j(U)$ or $U = U(j)$ in conditions of carrier recombination and generation. To arrive at the current-voltage dependence we need additional information about boundary conditions. In this work the boundary conditions are determined on the basis of the mechanisms of charge injection from the electrodes into the dielectric. The charge injection mechsnisms were described by the function of current density and electric field intensity at the electrodes.

2. Problem solution. In this work we consider cases of dielectric interior leading to an analytic determination of the current-voltage characteristic.

2.1. *Carrier recombination.* In this case we assume in equations (3)–(5) zero values of coefficients $v_1 = v_2 = C_{21} = 0$. After introducing new variables A_1, A_2, A_3 and A_4 described by the formulas

(7) $$A_1 = \frac{e_0 \mu_n pE}{j}; \quad A_2 = \frac{e_0 \mu_n nE}{j}; \quad A_3 = \frac{e_0 \mu_n n_{t1} E}{j};$$

$$A_4 = \frac{e_0 \mu_n n_{t2} E}{j},$$

the equations (1)–(5) take the following form in conditions of carrier recombination

(8) $$rA_1 + A_2 = 1; \quad r = \mu_p/\mu_n,$$

(9) $$\frac{\varepsilon \mu_n E}{j}\frac{dE}{dx} = A_1 - A_2 - A_3 - A_4,$$

(10) $$\mu_n E \frac{dA_2}{dx} = C_n A_2,$$

(11) $$C_n A_2 = C_{12} A_3,$$

(12) $$C_p A_1 A_4 = \frac{C_{12} e_0 \mu_n E}{j} A_3.$$

Hence, after transformations, we obtain the firs-order differential equation

(13) $$-\frac{\varepsilon C_n}{rj}\frac{dE}{dA_1} = \frac{A_1}{A_2} - 1 - \frac{C_n}{C_{12}} - \frac{C_n e_0 \mu_n E}{C_p j A_1}.$$

The determination of the distribution of electric field intensity $E(x)$ from equation (13) is a fairly complex problem. It is worth noting that the last component of the r.h.s. of this equation equals $c_n N_{t1}/[C_p A_1 (rp+n)]$. Assuming [1, 3]

(i) the equality of coefficients $C_n = C_{12}$; $c_n = C_p$,

(ii) the same order of concentrations n and p (i.e. A_1/A_2 being of the order of unity),

(iii) the inequality of trap and carrier concentrations $N_{t1} \gg n$ and $N_{t1} \gg p$, we may replace (13) by a differential equation with separated variables

(14) $$\frac{\varepsilon}{r}\frac{dE}{dA_1} = \frac{e_0 \mu_n E}{C_p A_1},$$

the solution of which is of the form

(15) $$E = K_1 A_1^{\chi}; \quad \chi = \frac{e_0 \mu_p}{\varepsilon C_p},$$

where K_1 is the integration constant. After substituting (15) in equation (10) and taking (8) into consideration we get

(16) $$\frac{\mu_n K_1 (1-A_2)^{\chi}}{r^{\chi} A_2} dA_2 = C_n dx.$$

The subsequent considerations pertain to natural values of parameter χ. In this case the general integral of equation (16) is of the form

(17) $$\ln A_2 + \sum_{i=1}^{\chi} \frac{1}{i}\binom{\chi}{i}(-A_2)^i = \frac{r^{\chi} C_n x}{\mu_n K_1} + K'; \quad K' = \text{const}.$$

It results from equation (8) that the function $\ln A_2$ may be developed into the power series

(18) $$\ln A_2 = \ln(1 - rA_1) = -\sum_{i=1}^{\infty} \frac{(rA_1)^i}{i}.$$

Limiting ourselves to the power $\chi+1$, we may write (18) as

(19) $$\ln A_2 = -\sum_{i=1}^{\chi} \frac{(1-A_2)^i}{i} - \frac{(rA_1)^{\chi+1}}{\chi+1}.$$

Thus, relation (17) takes the form

(20) $$-\sum_{i=1}^{\chi}\left[\frac{(1-A_2)^i}{i} - \frac{1}{i}\binom{\chi}{i}(-A_2)^i\right] - \frac{(rA_1)^{\chi+1}}{\chi+1} = \frac{r^{\chi} C_n x}{\mu_n K_1} + K'.$$

One must note that the sum of terms containing the power $(-A_2)^k$ in the expression $\sum_{i=1}^{\chi} (1-A_2)^i/i$, where $k = 1, 2, \ldots, \chi$, takes the value

(21) $$\frac{(-A_2)^k}{k} + \frac{(-A_2)^k \binom{k+1}{k}}{k+1} + \frac{(-A_2)^k \binom{k+2}{k}}{k+2} + \ldots \frac{(-A_2)^k \binom{\chi}{k}}{\chi} =$$

$$= (-A_2)^k \left[\frac{(k-1)!}{k!} + \frac{k!}{k!\,1!} + \frac{(k+1)!}{k!\,2!} + \ldots \frac{(\chi-1)!}{k!\,(\chi-k)!}\right] =$$

(21) (cont.)

$$= (-A_2)^k \frac{1}{k}\left[1+\binom{k}{1}+\binom{k+1}{2}+\binom{k+2}{3}+\ldots\binom{\chi-1}{\chi-k}\right]= (-A_2)^k \frac{1}{k}\binom{\chi}{k}.$$

In the last sum of (21) use was made of the well-known properties of Newton's symbol

$$\binom{n}{k}+\binom{n}{k+1}=\binom{n+1}{k+1}; \quad \binom{n}{k}=\binom{n}{n-k}.$$

Accordingly, relation (20) simplifies to the form

$$-\frac{(rA_1)^{\chi+1}}{\chi+1}-\sum_{i=1}^{\chi}\frac{1}{i}=\frac{r^{\chi}C_n x}{\mu_n K_1}+K'. \tag{22}$$

Hence results the formula describing the value of A_1

$$A_1=\frac{1}{r}\left(K_2-\frac{(\chi+1)\,r^{\chi}C_n x}{\mu_n K_1}\right)^{\frac{1}{\chi+1}}; \quad K_2=-(\chi+1)\left(K'+\sum_{i=1}^{\chi}\frac{1}{i}\right). \tag{23}$$

After substituting (23) in (15) we get the electric field intensity distribution

$$E(x)=K_3\left(K_2-\frac{(\chi+1)\,C_n x}{\mu_n K_3}\right)^{\frac{\chi}{\chi+1}}; \quad K_3=\frac{K_1}{r^{\chi}}. \tag{24}$$

Substituting the function $E(x)$ in the integral (6) we get

$$U=\frac{\mu_n}{(2\chi+1)\,C_n K_3^{1/\chi}}\left(E(0)^{\frac{2\chi+1}{\chi}}-E(L)^{\frac{2\chi+1}{\chi}}\right). \tag{25}$$

The integration constant K_3 is determined on the basis of boundary conditions

$$K_3^{1/\chi}=\frac{\mu_n}{(\chi+1)\,C_n L}\left(E(0)^{\frac{\chi+1}{\chi}}-E(L)^{\frac{\chi+1}{\chi}}\right). \tag{26}$$

Hence, on the basis of (25), we obtain the current-voltage dependence in parametric form

$$U=\frac{(\chi+1)\,L}{(2\chi+1)}\cdot\frac{\left(E(0)^{\frac{2\chi+1}{\chi}}-E(L)^{\frac{2\chi+1}{\chi}}\right)}{\left(E(0)^{\frac{\chi+1}{\chi}}-E(L)^{\frac{\chi+1}{\chi}}\right)} \tag{27}$$

and $j=f_0\,[E(0)]$ and $j=f_L\,[E(L)]$, where f_0 and f_L are functions describing the mechanism of charge injection from electrode $x=0$ and $x=L$, respectively, into the dielectric. For example when f_0 and f_L describe the tunnel effect $j=a_0\,E^2(0)$ and $j=a_L\,E^2(L)$ $(a_L>a_0)$, the dependence $j=j(U)$ is a quadratic function $j\sim U^2$. When parameter χ is a rational number $\chi=m/n$, we make in equation (16) the substitution

$$z^n=1-A_2=rA_1; \quad dA_2=(-n)\,z^{n-1}\,dz, \tag{28}$$

and (16) takes the form

$$\frac{(-n)\,z^m z^{n-1}}{1-z^n}\,dz=\frac{r^{m/n}C_n}{\mu_n K_1}\,dx. \tag{29}$$

Hence, after integration we get

$$z^m\ln(1-z^n)-m\int_0^z z^{m-1}\ln(1-z^n)\,dz=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'; \quad K'=\text{const}. \tag{30}$$

Since $z<1$, the function $\ln(1-z^n)$ may be developed into a power series. After transformations, solution (30) takes the equivalent form

$$-z^m\left(z^n+\frac{z^{2n}}{2}+\frac{z^{3n}}{3}+\ldots\right)+m\left(\frac{z^{m+n}}{m+n}+\frac{z^{m+2n}}{2\,(m+2n)}+\frac{z^{m+3n}}{3\,(m+3n)}+\ldots\right)=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'. \tag{31}$$

Limiting ourselves to the firs approximation of the terms in parentheses, relation (31) is simplified to the form

$$-\frac{nz^{m+n}}{m+n}=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'. \tag{32}$$

After taking into consideration substitution (28), we get the relation describing the value of A_1

$$-\frac{n}{m+n}(rA_1)^{\frac{m+n}{n}}=\frac{r^{m/n}C_n x}{\mu_n K_1}+K'. \tag{33}$$

Since $\chi+1=(m+n)/n$, the form of A_1 is identical with that of solution (23) for natural values of

$$A_1=\frac{1}{r}\left(K_2-\frac{(\chi+1)\,r^{\chi}C_n x}{\mu_n K_1}\right)^{\frac{1}{\chi+1}}; \quad K_2=-K'(\chi+1). \tag{34}$$

We claim on this basis that the current-voltage dependence (27) is satisfied for all rational values of χ.

2.2. *Carrier generation-recombination with immobile positive charge* $\mu_p=0$. This case is solved with the additional assumption of coefficient equality $C_{12}=C_{21}$. Taking into consideration the identical values of coefficients C_{12}

and C_{21} and after introducting variables A_1, A_2, A_3 and A_4 described by formulas (7), the following relations result from equations (1)-(5)

$$(35)\qquad A_2 = 1;\quad A_3 = A_4 = \frac{C_n}{v_1} = \frac{1}{\theta_0};\quad A_1 = \frac{(e_0\,\mu_n)^2\,v_2\,N_{t2}\,\theta_0\,E^2}{C_p\,j^2}.$$

Hence, basing on (7) and (2) we get the differential equation describing the distribution of electric field intensity $E(x)$

$$(36)\qquad \frac{\varepsilon\mu_n\,E}{j}\,\frac{dE}{dx} = \frac{(e_0\,\mu_n)^2\,v_2\,N_{t2}\,\theta_0}{C_p\,j^2}\,E^2 - \left(1+\frac{2}{\theta_0}\right).$$

The general solution of equation (36) is of the form

$$(37)\qquad E = \frac{j}{\beta^{1/2}}\left\{\theta + C\exp\left(\frac{2\beta x}{\varepsilon\mu_n\,j}\right)\right\}^{1/2};\quad \theta = 1+\frac{2}{\theta_0};\qquad \beta = \frac{(e_0\,\mu_n)^2\,v_2\,N_{t2}\,\theta_0}{C_p},$$

where C is the integration constant, determined on the basis of the boundary condition $x = L$

$$(38)\qquad C = \left(\frac{\beta E^2(L)}{j^2} - \theta\right)\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right).$$

The substitution of the function $E(x)$ described by (37) in the integral (6) leads to

$$(39)\qquad U = \frac{\varepsilon\mu_n\,j^2}{\beta^{3/2}}\left\{\frac{\beta^{1/2}}{j}(E(L)-E(0)) + \frac{\theta^{1/2}}{2}\ln\left|\frac{(\beta^{1/2}E(L)-j\theta^{1/2})(\beta^{1/2}E(0)+j\theta^{1/2})}{(\beta^{1/2}E(L)+j\theta^{1/2})(\beta^{1/2}E(0)-j\theta^{1/2})}\right|\right\}.$$

From (37) and (38) we obtain the relation between boundary values $E(0)$, $E(L)$ and current density j

$$(40)\qquad E(0) = \frac{j}{\beta^{1/2}}\left\{\theta\left[1-\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right)\right] + \frac{\beta E^2(L)}{j^2}\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right)\right\}^{1/2}.$$

It results from formulas (39) and (40) that the current-voltage dependence is described parametrically by the system of functions $U = U(E(L), j)$ and $j = f_L[E(L)]$, where f_L is the function describing the mechanism of charge injection from electrode $x = L$ into the dielectric. It is easily seen that in the case of Schottky's phenomenon, $j = a\exp(bE^{1/2}(L))$ or the tunnel effect, $j = a_1\,E^2(L)\exp\left(\frac{-b_1}{E(L)}\right) \approx a_1\,E^2(L)$, the current-voltage relation may be

represented in the overt form $U = U(j)$. In the most simple case when the charge injection mechanism is described by the function $j = \left(\frac{\beta}{\theta}\right)^{1/2} E(L)$, we obtain, on the basis of (37), (38) and (6), the Ohm law

$$(41)\qquad j = \sigma\frac{U}{L};\quad \sigma = \frac{e_0\,\mu_n\,(v_2\,N_{t2}\,\theta_0)^{1/2}}{C_p^{1/2}\left(1+\frac{2}{\theta_0}\right)^{1/2}}.$$

An interesting case is that of electrode $x = L$ injecting infinitely many electrons $E(L)\to 0$. Then from (39) and (40) we have

$$(42)\qquad U = \frac{\varepsilon\mu_n\,j^2}{\beta^{3/2}}\left\{\frac{\theta^{1/2}}{2}\ln\left|\frac{1+\left[1-\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right)\right]^{1/2}}{1-\left[1-\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right)\right]^{1/2}}\right| + {} - \theta^{1/2}\left[1-\exp\left(\frac{-2\beta L}{\varepsilon\mu_n\,j}\right)\right]^{1/2}\right\}.$$

For sufficiently large values of current density j, relation (42) may be written in the form

$$(43)\qquad U = \frac{\varepsilon\mu_n\,j^2}{\beta^{3/2}}\left\{\frac{\theta^{1/2}}{2}\ln\frac{1+w}{1-w} - \theta^{1/2}\,w\right\};\quad w = \left(\frac{2\beta L}{\varepsilon\mu_n\,j}\right)^{1/2}.$$

Function $\ln\frac{1+w}{1-w}$ has the following development into a power series

$$(44)\qquad \ln\frac{1+w}{1-w} = 2\left(w+\frac{w^3}{3}+\frac{w^5}{5}+\ldots\right);\quad w<1.$$

Limiting ourselves to the second approximation we get

$$(45)\qquad U = \frac{\varepsilon\mu_n\,j^2}{\beta^{3/2}}\left\{\theta^{1/2}\left(w+\frac{w^3}{3}\right) - \theta^{1/2}\,w\right\} = \frac{\varepsilon\mu_n\,\theta^{1/2}\,j^2}{3\beta^{3/2}}\,w^3,$$

that is to say

$$(46)\qquad U = \frac{1}{3}\left(\frac{8L^3\,\theta j}{\varepsilon\mu_n}\right)^{1/2}.$$

It results from this that the current-voltage dependence $j = j(U)$ is the Child law

$$(47)\qquad j = \frac{9}{8}\,\varepsilon\mu_n\,\theta^{-1}\,\frac{U^2}{L^3}.$$

The parameter $\theta = 1+\frac{2C_n}{\nu_1}$ in (47) takes into account the effect of the first trapping level on the conduction current density value.

3. Conclusions. Referring to Part I of this work, we find that the current--voltage dependence is a feature characterizing the metal-dielectric-metal system. In the case of carrier recombination and the presence of the tunnel effect at both contacts, the current-voltage dependence may be the quadratic function $j \sim U^2$. In the case of carrier generation-recombination, immobile positive charge carriers and identical values of coefficients $C_{12} = C_{21}$, the dependence $j = j(U)$ may have the form of:

(i) the Ohm law $j = \sigma U/L$ if the electron-injecting electrode introduces carriers into the dielectric in such a manner that current density is expressed by a linear dependence on electric field intensity at the electrode, $j = \sigma_L E(L)$, and parameter σ_L takes the value $\sigma_L = \sigma$ described by (41);

(ii) the Child law

$$j = \frac{9}{8}\varepsilon\mu_n \theta^{-1}\frac{U^2}{L^3},$$

if the electrode injecting electrons introduces infinitely many carriers. In this case the trapping level closest to the conductance band is the only level affecting the current density value.

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Б. Свистач, Влияние процессов генерации-рекомбинации и механизмов инжекции на вольтамперные характеристики соотношения металл-диэлектрик-металл, II. Рекомбинация носителей

В статье исследуется модель диэлектрика, в которой содержатся свободные электроны и дырки. Объемный заряд определяется концентрацией свободных носителей и захваченных электронов. В предложенной модели диэлектрика анализируется рекомбинация носителей в случае двух уровней прилипания, в стационарном сосотянии, при постоянном напряжении. Решения получены в системе плоского конденсатора. Механизмы инжекции заряда описываются по функции плотности тока и напряженности электрического поля на контактах электрод-диэлектрик. Изучается влияние механизмов инжекции в условиях рекомбинации носителей на вольтамперные характеристики.