BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
TECHNICAL SCIENCES
Vol. 38, No. 1-12, 1990

VARIA

Electron-Ion Conductance in the Metal-Dielectric-Metal System with Two Given Boundary Conditions
I. Carrier Generation with Three Mobilities

by

Bronisław ŚWISTACZ

Presented by T. ŚLIWIŃSKI on July 11, 1989

Summary. A dielectric model is considered in which space charge is formed by free carriers of positive and negative charge and trapped negative charge carriers. It was found that a characteristic feature of carriers generation is the switching of the current-voltage characteristic.

1. Introduction. One of the conceptions in the „theory of space charge" in a solid is the analysis of stationary state of electric conductivity in the metal-dielectric-metal system at constant voltage. The aim of this analysis is to find current-voltage characteristics which characterize processes ocurring inside the dielectric and at the metal-dielectric junction. In studies of bipolar charge transport in the metal-dielectic-metal system it was assumed that the space charge is formed by mobile postive and negative charge carriers [1–3] with mobilities μ_p and μ_n, respectively, and trapped negative charge carriers [2–4]. In this approach, the electric field intensity E in the flat capacitor system is a function of the position of point x and depends on the integration constants C_1 and C_2, i.e. $E = E(x, C_1, C_2)$. This function satisfies the voltage condition

$$U = \int_0^L E(x, C_1, C_2)\,dx = U(C_1, C_2); \qquad U = \text{const} \tag{1}$$

where L — distance between electrodes, U — voltage applied to the electrode.

The values of integration constants are determined by boundary values $E(0)$ and $E(L)$ of electric field intensity at electrodes $x = 0$ and $x = L$, respectively.

$$E(0) = E(x = 0, C_1, C_2) \text{ and } E(L) = E(L, C_1, C_2). \tag{2}$$

In this approach, from (1) and (2) we get the dependence between voltage value and the boundary values in the form

$$U[(E(0); E(L)].$$

The form of function $E(x, C_1, C_2)$ and the resultant dependence $U = U[E(0); E(L)]$ are determined by the dielecric's interior, while boundary values $E(0)$ and $E(L)$ depend on mechanism of carrier injection into the dielectric from electrodes $x = 0$ and $x = L$, respectively. The mechanisms of carrier injection are described using the function of current density j and electric field intensity next to the electrode in the form $j = f_0[E(0)]$ and $j = f_L[E(L)]$ [2,3]. Hence, the current-voltage dependence $j = j(U)$ or $U = U(j)$ is of the form

$$\text{(3)} \qquad \begin{cases} U = U[E(0); E(L)] \\ j = f_0[E(0)]; \quad j = f_L[E(L)]. \end{cases}$$

Function $U = U(j)$ or $j = j(U)$ characterizes the dielectric interior and the metal-dielectric junctions.

Till now, in analysis of bipolar charge transport it was assumed that the trapped carriers of negative charge are immobile. However, it may be concluded from the general tenets of the "theory of solids" that to trapped carriers there may be assigned mobility μ_t which usually depends on electric field intensity, $\mu_t = \mu_t(E)$. The parameter μ_t is a feature characterizing amorphous bodies and strongly doped crystalline structures. The introduction of this additional parameter μ_t requires the formulation of three boundary conditions. In strongly amorphous bodies and strongly doped crystalline structures we may assume zero value of one free carrier mobilities, i.e. $\mu_n = 0$ or $\mu_p = 0$. In this case, the metal-dielectric-metal system is described by function (3).

In this work we study the steady state of electric conductivity in amorphous bodies or strongly-doped crystalline structures described by function $j = j(U)$ which in its turn is described by (3).

2. Dielectric model. In the bipolar space charge conception, the concentration of free positive charge carriers p and the concentration of free negative charge carriers n are related by the continuity equation, the Gauss equation, and equation describing generation-recombination processes [2, 3]. The generation-recombination of a pair of free carriers occurs with the participation of discretely distributed trapping states [3]. The generation-recombination of free carriers may be accompanied by processes such as

— the passage of carriers between trapping levels [3],

— the interaction of free carriers with any of the trapping levels [1],

— mobility of trapped carriers on the given trapping level characterized by current density j_t.

In such conditions, the generation-recombination equations in the flat capacitor system take the form

$$\text{(4)} \quad \frac{\partial}{\partial t}\sum_{i=1}^{m} n_{t_i} = \sum_{i=1}^{m}\left\{C_{n_i} n(N_{t_i} - n_{t_i}) - v_{n_i} n_{t_i} + v_{p_i}(N_{t_i} - n_{t_i}) - C_{p_i} p n_{t_i} + \frac{\partial}{\partial x}(j_{t_i}/e_0)\right\}$$

$$\text{(5)} \qquad \frac{\partial n}{\partial t} = \frac{\partial}{\partial x}(\mu_n n E) + \sum_{i=1}^{m}[v_{n_i} n_{t_i} - C_{n_i} n(N_{t_i} - n_{t_i})]$$

where $e_0 = 1.6 \cdot 10^{-19} C$; n_{t_i} — concentration of trapped negative charge carriers; N_{t_i} — concentration of traps, i — number of given trapping level, m — total number of trapping levels, t — time; C_{n_i}, v_{n_i}, C_{p_i}, v_{p_i} — material coefficients. Current density j_{t_i} of trapped carriers may have the form of Ohm current density

$$\text{(6)} \qquad j_{t_i} = e_0 \cdot \mu_{t_i} \cdot n_{t_i} \cdot E; \qquad \mu_{t_i} = \text{const}$$

or may be described by Poole's function

$$\text{(7)} \qquad j_{t_i} = a_i n_{t_i} \cdot \exp(b_i E); \qquad a_i = \text{const}, \ b_i = \text{const}.$$

In this work we will consider the function $j_{t_i} = j_{t_i}(n_{t_i}, E)$ described by formula (6). In what follows we will assume identical values of carrier mobility in trapping states $\mu_{t_i} = \mu_t = \text{const}$ ($i = 1, 2, \ldots, m$), and the dielectric will be treated as an infinite reservoir of trapping states, i.e. $N_{t_i} \gg n_{t_i}$. We will be considering isotropic dielectrics, i.e. materials whose electric induciton $D = \varepsilon E$, where ε is high-frequency electric permeability. Given these assumptions, the stationary state of electric conductivity in the flat capacitor system is described by

$$\text{(8)} \qquad \frac{\varepsilon}{e_0}\frac{dE}{dx} = p - n - n_t; \qquad n_t = \sum_{i=1}^{m} n_{t_i}$$

$$\text{(9)} \qquad j = e_0 \mu_p p E + e_0 \mu_n n E + e_0 \mu_t n_t E; \qquad j = \text{const}$$

$$\text{(10)} \qquad \sum_{i=1}^{m}\{C_{n_i} n N_{t_i} + v_{p_i} N_{t_i} - v_{n_i} n_{t_i} - C_{p_i} p n_{t_i}\} + \frac{d}{dx}(\mu_t n_t E) = 0$$

$$\text{(11)} \qquad \sum_{i=1}^{m}\{v_{n_i} \cdot n_{n_i} - C_{n_i} \cdot n N_{t_i}\} + \frac{d}{dx}(\mu_n n E) = 0$$

with the given integration condition

$$\text{(12)} \qquad \int_0^L E\,dx = U; \qquad U = \text{const}.$$

We get equations (10) and (11) from equations (4) and (5). Dependence (9) results from the law of charge conservation, and (8) is Gauss's equation written with the assumption that equilibrium concentrations p_0, n_0 and $n_{t_i,0}$ satisfy the neutrality condition

$$p_0 = n_0 + \sum_{i=1}^{m} n_{t_i,0}. \tag{13}$$

Our objective is to determine the current-voltage dependences $j = j(U)$ or $U = U(j)$ described by (3) in the metal-dielectric-metal system. We will be considering amorphous bodies or strongly-doped (high concentration of traps) crystalline structures in which current density j_t of trapped carriers is described by function (6) as presented in (8)–(11).

3. Solution of the problem. The following cases of electric charge transport are possible during electric conductivity in a solid with a large number of traps:

— predominance of generation processes with three mobilities μ_p, μ_n, μ_t,

— there exist generation-recombination processes with two mobilities μ_p and μ_t.

In this part of the work we will consider the case of positive and negative carriers generation with transport of carriers in trapping states. In this case we must assume $C_{n_i} = C_{p_i} = 0$ in equations (10) and (11). We also make an additional assumption concerning generation parameters, i.e. $v_{n_i} \ll v_{p_i}$. The following auxiliary notation is introduced:

$$\lambda_1 = \frac{e_0 \mu_n p E}{j}; \quad \lambda_2 = \frac{e_0 \mu_n n_t E}{j}; \quad r_1 = \frac{\mu_p}{\mu_n}; \quad r_2 = \frac{\mu_t}{\mu_n}. \tag{14}$$

When $C_{n_i} = C_{p_i} = 0$ and $v_{n_i} \leqslant v_{p_i}$ and the relation $N_{t_i} \gg n_{t_i}$ are taken into consideration in (10) and (11), the parameters λ_1 and λ_2 and electric field intensity $E(x)$ satisfy the following equations

$$\frac{\varepsilon \mu_n E}{j} \frac{dE}{dx} = (r_1 + 1)\lambda_1 - (1 - r_2)\lambda_2 - 1 \tag{15}$$

$$\frac{e_0 \sum\limits_{i=1}^{m} v_{p_i} \cdot N_{t_i}}{r_1 j} - \frac{d\lambda_1}{dx} = 0 \tag{16}$$

$$\frac{e_0 \sum\limits_{i=1}^{m} v_{p_i} \cdot N_{t_i}}{r_2 j} + \frac{d\lambda_2}{dx} = 0. \tag{17}$$

From (16) and (17) we get functions $\lambda_1(x)$ and $\lambda_2(x)$

$$\lambda_1 = \frac{e_0}{r_1 j}\left(\sum_{i=1}^{m} v_{p_i} \cdot N_{t_i}\right) x + C_1; \quad C_1 = \text{const} \tag{16a}$$

$$\lambda_2 = -\frac{e_0}{r_2 j}\left(\sum_{i=1}^{m} v_{p_i} \cdot N_{t_i}\right) x + C_2; \quad C_2 = \text{const}. \tag{17a}$$

Hence, basing on (15), we get the distribution of electric field intensity $E(x)$

$$E\frac{dE}{dx} = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \frac{e_0}{\varepsilon \mu_n} \left(\sum_{i=1}^{m} v_{p_i} \cdot N_{t_i}\right) \cdot x + \frac{C_3}{2}; \quad C_3 = \text{const} \tag{18}$$

$$E^2(x) = \alpha x^2 + C_3 x + C_4; \quad \alpha = \left(\frac{1}{\mu_p} + \frac{1}{\mu_t}\right) \frac{e_0}{\varepsilon} \sum_{i=1}^{m} v_{p_i} \cdot N_{t_i}; \quad C_4 = \text{const} \tag{19}$$

i.e.

$$E(x) = \sqrt{\alpha\left(x + \frac{C_3}{2\alpha}\right)^2 + K}; \quad K = \frac{4\alpha C_4 - C_3^2}{4\alpha}. \tag{20}$$

After substituting function $E(x)$ into the integral (12) and performing Euler substitutions under the integral, we get

$$U = \frac{1}{2\sqrt{\alpha}} \left\{ z_1 \cdot E(L) - z_0 \cdot E(0) + K \ln \left| \frac{z_1 + E(L)}{z_0 + E(0)} \right| \right\} \tag{21}$$

$$z_0 = \frac{C_3}{2\sqrt{\alpha}}; \quad z_1 = \sqrt{\alpha} L + z_0; \quad C_3 = \frac{E^2_{(L)} - E^2(0) - \alpha L^2}{L} \tag{22}$$

$$K = = \frac{4\alpha E^2(0) - C_3^2}{4\alpha} = E^2(0) - z_0^2.$$

From (20) it results that function $E(x)$ is given in the interval $0 \leqslant x \leqslant L$ when the condition $K \geqslant -\frac{C_3^2}{4\alpha}$ is satisfied. Hence, it follows from (22) that function $E(x)$ is given for arbitrary integration constants C_3 and C_4. Thus, formulas (21) and (22) as well as boundary functions $j = f_0[E(0)]$ and $j = f_L[E(L)]$ describing the mechanism of charge injection from electrode $x = 0$ and $x = L$ into the dielectric, together describe the current-voltage dependence in parametric form. It is worth noting that the density of space charge $q_v = \varepsilon \frac{dE}{dx}$

$$q_v = \frac{\varepsilon\alpha\left(x + \frac{C_3}{2\alpha}\right)}{E}; \quad E > 0. \tag{23}$$

From (22) it results that the integration constant C_3 may be negative, for example when boundary values $E(0)$ and $E(L)$ are identical. The condition $E(0) = E(L)$ may be satisfied when boundary functions f_0 and f_L are identical: $f_0[E(0)] \equiv f_L[E(L)]$; then we have $C_3 = -\alpha L$. Hence, it follows from (23) that in region $0 \leqslant x < L/2$ the values of $q_v < 0$, and in region $L/2 < x \leqslant L$ the values of $q_v > 0$. Thus, when carrier injection is symmetrical, $f_0 \equiv f_L$, the metal-dielectric-metal system behaves as a p-n junction. A similar result was obtained in [4]. It is worth noting that in conditions of carrier injection symmetry, then parameters z_1 and z_0 take the values $z_1 = -z_0 = \sqrt{\alpha}L/2$, and (23) may assume a simpler form when $E(L) \gg z_1$ or $z_1 \gg E(L)$. Assume that $E(L) \ll z_1$. After introducing $W = E(L)/z_1$, formula (22) takes the form

$$U = \frac{z_1^2}{2\sqrt{\alpha}}\left\{2W + (W^2-1)\,ln\left|\frac{W+1}{W-1}\right|\right\}. \tag{24}$$

Since parameter $|W| \ll 1$, we have

$$ln\left|\frac{W+1}{W-1}\right| = 2\left(W + \frac{W^3}{3} + \frac{W^5}{5}\ldots\right) \tag{25}$$

i.e.

$$U = \frac{z_1^2}{\sqrt{\alpha}}\left\{W + (W^2-1)\left(W + \frac{W^3}{3} + \frac{W^5}{5} + \ldots\right)\right\}; \qquad |W| \ll 1. \tag{26}$$

Limiting ourselves to third power terms, (24) takes the final form

$$U = \frac{2}{3}\cdot\frac{z_1^2}{\sqrt{\alpha}}\cdot W^3; \qquad W = \frac{E(L)}{z_1}; \qquad z_1 = \frac{\sqrt{\alpha}L}{2} \tag{27}$$

$$U = \frac{4E^3(L)}{3\alpha L}; \qquad E(L) \ll \frac{\sqrt{\alpha}L}{2}. \tag{28}$$

Proceeding analogously in the case when $E(L) \gg z_1 = \dfrac{\sqrt{\alpha}L}{2}$, the formula (21) becomes the linear function

$$U = LE(L); \qquad E(L) \gg \frac{\sqrt{\alpha}L}{2}. \tag{29}$$

If the boundary functions $j = f_0[E(0)] \equiv f_L[E(L)]$ are increasing, the current-voltage dependence $j = j(U)$ described by (28) is low-current and (29) is high-current.

Example: there are given boundary functions $j = a_0 E^2(0) \equiv a_L E^2(L)$ which means that $a_0 = a_L$ and $E(0) = E(L)$. It this case (28) takes the form of the slowly increasing power function $j \sim U^{2/3}$ and (29) becomes a rapidly increasing quadratic function $j \sim U^2$. This is known as the effect of current-voltage characteristic switching.

4. Conclusions. It was found, in agreement with [4], that generation processes are characterized by the distribution of electric field intensity $E^2(x)$

described by the trinomial square $E^2(x) = \alpha x^2 + bx + c$, with $\alpha > 0$ and parameter α depending not on current density j but on internal parameters of the dielectric. In this case only those mechanisms of carrier injection are allowed for which boundary values $E(0)$ and $E(L)$ are identical: $E(0) = E(L)$. This may happen when carrier injection is symmetrical, that is to say when the boundary functions $j = f_0[E(0)]$ and $j = f_L[E(L)]$ describing the mechanism of carrier injection from the electrodes to the dielectric are identical: $f_0 \equiv f_L$. In such conditons the metal-dielectric-metal system behaves like an n-p junction and the current-voltage characteristic has the singular point $E_{cr} = \sqrt{\alpha}L/2$ (critical value of electric field intensity). If the boundary function $j = f_0[E(0)] \equiv f_L[E(L)]$ is increasing, the function $j = j(U)$ is low current for $E(L) \ll E_{cr}$, and the function $j = f_L(U/L)$ is high current for $E(L) \gg E_{cr}$. This is the so called current-voltage characteristic switching.

In this work it was not univocally determined whether the current-voltage characteristic switching effect is characteristic only for generation processes or whether it is also a result of other possible processes such as generation-recombination of carriers. This problem is the subject of a separate paper.

INSTITUTE OF FUNDAMENTAL ELECTRONICS AND ELECTROTECHNOLOGY, TECHNICAL UNIVERSITY, PL. GRUNWALDZKI 13, 50-372 WROCŁAW
(INSTYTUT PODSTAW ELEKTROTECHNIKI I ELEKTROTECHNOLOGII, POLITECHNIKA WROCŁAWSKA)

REFERENCES

[1] M.A. Lampert, P. Mark, *Current injection in solids*, Academic Press, London, 1970.

[2] B. Świstacz, *Transient states of electric conduction in solid dielectric including carrier recombination, I. Ideal dielectric without traps, II. Ideal dielectric with deep traps*, Bull. Pol. Ac.: Tech., 33 (1985), 587–603.

[3] B. Świstacz, *Kinetic model of carrier recombination in solid dielectric*, Bull. Pol. Ac.: Tech. 35 (1987), 101–113.

[4] B. Świstacz, *The effect of generation-recombination processes and of injection mechanism on current-voltage dependences in the metal-dielectric-metal system. I. Carrier generation*, Bull. Pol. Ac.: Tech., 35 (1987), 319–328.