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Electron-Ion Conductance in the Metal-Dielectric-Metal System with Two Given Boundary Conditions II. Generation-Recombination with Two Mobilities

by
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Summary. Transport of free positive charge carriers and of trapped negative charge carriers in a model dielectric is considered. It was found that when recombination processes predominate there is no switching of the current-voltage characteristic.

1. Introduction. In Part I of this work we presented a dielectric model in which transport of stationary state electric charge in the flat capacitor system is described by the following equations

$$\frac{\varepsilon}{e_0}\frac{dE}{dx} = p - n - n_t; \quad n_t = \sum_{i=1}^{m} n_{t_i} \tag{1}$$

$$j = e_0 \cdot \mu_p \cdot pE + e_0 \cdot \mu_n \cdot nE + e_0 \cdot \mu_t \cdot n_t E; \quad j = \text{const} \tag{2}$$

$$\sum_{i=1}^{m} \{C_{n_i} nN_{t_i} + v_{p_i} N_{t_i} - v_{n_i} n_{t_i} - C_{p_i} pn_{t_i}\} + \frac{d}{dx}(\mu_t n_t E) = 0 \tag{3}$$

$$\sum_{i=1}^{m} \{v_{n_i} n_{t_i} - C_{n_i} nN_{t_i}\} + \frac{d}{dx}(\mu_n nE) = 0 \tag{4}$$

with the given integration condition

$$\int_0^L E\,dx = U; \quad U = \text{const} \tag{5}$$

were $e_0 = 1.6\cdot10^{-19}C$; ε — high-frequency electric permeability; x — distance between point and electrode; E — electric field intensity; p, n — concentration of free carriers of positive and negative charge, respectively; μ_p, μ_n — mobility of free carriers of positive and negative charge, respectively; m — number of discretely distibuted trapping levels; $C_{n_i}, C_{p_i}, v_{n_i}, v_{p_i}$ — recombination-generation coefficients of the i-th group of trapping states; N_{t_i} — concentration of negative charge carriers trapped on the "i-th" trapping level; μ_t — mobility of trapped carriers; L — distance between electrodes; U — voltage applied to the electrodes; j — density of electric current.

Equations (2) and (3) are written with the assumption that the dielectric is an infinite reservoir of trapping states ($N_{t_i} \gg n_{t_i}$) and that the mobilities of carrier trapped on the various trapping levels are the same. Similarly as in Part I, the subsequent considerations will concern electric conductance in amorphic bodies or strongly doped (high concentration of traps) crystalline structures. In such materials, marked by large numbers of structure defects and impurities (traps), only a very small mean free path of negative charge carriers is possible; in the case of deep traps there is a very small concentration n of these carriers, and then we can assume that $\mu_n = 0$, meaning that there occurs transport of positive ions and mobility of carriers in trapping states (negative ions). In such conditions, in order to describe conductivity fully, we must introduce the boundary functions $j = f_0(E(x=0)]$ and $j = f_L[E(x=L)]$ describing the mechanism of charge injection into the dielectric from electrode $x = 0$ and $x = L$, respectively [1, 2].

The aim of this paper is to determine the current-voltage dependences $j = j(U)$ or $U = U(j)$ in the case of ion conductance (i.e. when $\mu_n = 0$) with given boundary functions f_0 and f_L.

2. Solution of the problem. In determining the current-voltage dependence, we make an additional assumption concerning the value of parameters θ_{0_i} described by the formula

$$\theta_{0_i} = \frac{v_{n_i}}{C_{n_i} N_{t_i}}; \quad i = 1,2,\ldots,m \tag{6}$$

and the value of the recombination coefficients C_{p_i}. In what follows we assume that $\theta_{0_1} \approx \theta_{0_2} \approx \ldots \theta_{0_m} = \theta_0$ (this is possible when $\theta_{0_i} \ll 1$ — deep traps), and that $C_{p_1} \approx C_{p_2} \approx \ldots C_{p_m} = C_p$. This is also possible because coefficients C_{p_i} depend on the cariers' heat velocity and on capture cross-section.

Given these assumptions, we will analyse ion conductance, i.e. $\mu_n = 0$ in conditions of carrier generation and recombination.

We begin our analysis of the problem by introducing the auxiliary parameters

$$\lambda = \frac{e_0 \mu_t p E}{j}; \quad r = \frac{\mu_p}{\mu_t} \tag{7}$$

Taking (1)–(4) into consideration, ion conductance is described by the system of two equations

$$r\cdot\frac{d\lambda}{dx} = \frac{e_0}{j}\cdot\sum_{i=1}^{m} v_{p_i} N_{t_i} - \frac{C_p j\lambda(1-r\lambda)}{e_0(\mu_t E)^2} \tag{8}$$

$$\frac{\varepsilon\mu_t E}{j}\frac{dE}{dx} = \lambda(1+r\theta) - \theta; \ \theta = 1 + m\theta_0 = 1 + \frac{m v_{n_1}}{C_{n_1} N_{t_1}}. \tag{9}$$

We first consider the case when parameters $v_{p_i} = 0$, i.e. when recombination processes predominate. After dividing (9) and (8) by sides and taking into account the values of parameters $v_{p_i} = 0$, we get

$$\chi\frac{dE}{d\lambda} = \frac{[\lambda(1+r\theta)-\theta]E}{\lambda\left(\lambda-\frac{1}{r}\right)}; \quad \chi = \frac{\varepsilon C_p}{e_0\mu_t} \tag{10}$$

$$\frac{d\lambda}{dx} = -\frac{C_p j\lambda(1-r\lambda)}{e_0 r(\mu_t E)^2}; \quad \frac{d\lambda}{dx} < 0. \tag{11}$$

The solution of (10) is the function $E(\lambda)$

$$E = K\cdot\lambda^{a_1}\cdot\left|\lambda-\frac{1}{r}\right|^{a_2}; \quad a_1 = \frac{r\theta}{\chi}; \quad a_2 = \frac{1}{\chi}; \quad K = \text{const} \tag{12}$$

where K is the integration constant. The value of the integration constant K may be detemined on the basis of boundary values $E(0)$ and $\lambda(0)$

$$K = \frac{E_{(0)}}{\lambda_{(0)}^{a_1}\left|\lambda_{(0)}-\frac{1}{r}\right|^{a_2}}. \tag{13}$$

We then write the integration condition (5) in the form

$$U = \int_{\lambda_0}^{\lambda_1} E\left(\frac{dx}{d\lambda}\right)d\lambda; \quad \lambda_0 = \lambda_{(0)}; \quad \lambda_1 = \lambda_{(L)}. \tag{14}$$

Hence, on the basis of (10) and (11) we get

$$U = -\frac{e_0\mu_t^2 K^3}{C_p j}\int_{\lambda_0}^{\lambda_1} \lambda^{3a_1-1}\left|\lambda-\frac{1}{r}\right|^{3a_2-1} d\lambda; \quad \lambda_1 < \lambda_0 \tag{15}$$

$$-\int_{\lambda_0}^{\lambda_1} \lambda^{2a_1-1}\left|\lambda-\frac{1}{r}\right|^{2a_2-1} d\lambda = \frac{C_p j L}{e_0\mu_t^2 K^2}; \quad \lambda_1 < \lambda_0. \tag{16}$$

Thus, formulas (12), (13), (15) and (16) together with the boundary functions $j=f_0[E(0)]$ and $j=f_L[E(L)]$ describe the current-voltage dependences in parametric form.

Example: $r \gg 1$, i.e. $\mu_p \gg \mu_t$, and so (15) and (16) take the form

$$U=\frac{e_0\mu_t^2K^3}{C_pj(3a_{12}-1)}\{\lambda_0^{3a_{12}-1}-\lambda_1^{3a_{12}-1}\} \tag{15a}$$

$$\lambda_0^{2a_{12}-1}-\lambda_1^{2a_{12}-1}=\frac{C_pjL(2a_{12}-1)}{e_0\mu_t^2K^2} \tag{16a}$$

where

$$a_{12}=a_1+a_2=\frac{r\theta+1}{\chi} \tag{17}$$

whereas (12) and (13) become

$$\frac{E(L)}{\lambda_1^{a_{12}}}=K; \quad \frac{\lambda_0}{\lambda_1}=\left\{\frac{E(0)}{E(L)}\right\}^{1/a_{12}}. \tag{13a}$$

Thus, after transformations, we get the final form of dependence $j=j(U)$ in parametric form

$$U=\frac{(2a_{12}-1)L\left\{E(0)^{3-\frac{1}{a_{12}}}-E(L)^{3-\frac{1}{a_{12}}}\right\}}{(3a_{12}-1)\left\{E(0)^{2-\frac{1}{a_{12}}}-E(L)^{2-\frac{1}{a_{12}}}\right\}} \tag{18}$$

$$j=f_0[E(0)] \quad \text{and} \quad j=f_L[E(L)]$$

where f_0 and f_L are functions describing the mechanism of carrier injection into the dielectric from electrodes $x=0$ and $x=L$.

Example: there are given boundary functions $j \sim E^2(0)$ and $j \sim E^2(L)$, and this means that $j \sim U^2$. Equations (15) and (16) may be used since $r \neq 1$, i.e. $\mu_p \neq \mu_t$.

If $r=1$, i.e. $\mu_t=\mu_p=\mu$, and parameter $\theta \approx 1$ ($m\theta_0 \ll 1$ — deep traps), then from (9) and (12) we get the differential equation

$$\varepsilon\mu E\frac{dE}{dx}=\pm\sqrt{j^2-K_1E^{\chi}}; \quad \mu=\mu_t=\mu_p; \quad K_1=\text{const} \tag{19}$$

where K_1 is the new, suitably selected, integration constant. If we limit our considerations to the (+) sign, from (19) and the integration condition (5) there result the relationships

$$U=\varepsilon\mu\int_{E(0)}^{E(L)}\frac{E^2\,dE}{\sqrt{j^2-K_1E^{\chi}}} \tag{20}$$

$$\int_{E(0)}^{E(L)}\frac{E\,dE}{\sqrt{j^2-K_1E^{\chi}}}=\frac{L}{\varepsilon\mu}. \tag{21}$$

Equation (19) was analysed in [3] when parameter $\chi=2$. As an example, we will consider the case when $\chi=1$, i.e. $C_p=\frac{e_0\mu_p}{\varepsilon}$. Then (20) and (21) take the form

$$U=\frac{2\mu\varepsilon}{5K_1}\{E^2(0)\sqrt{j^2-K_1E(0)}-E^2(L)\sqrt{j^2-K_1E(L)}\}+\frac{4j^2L}{5K_1} \tag{22}$$

$$3j^2(E^2(L)-E^2(0))+K_1(E^3(L)-E^3(0))=\left(\frac{3K_1L}{2\mu\varepsilon}\right)^2+\frac{3L}{\varepsilon\mu}\cdot(2j^2+ \tag{23}$$

$$+K_1\cdot E(L))\sqrt{j^2-K_1E(L)}.$$

Since $j^2-K_1E^{\chi}=(e_0\mu E)^2\,(p-n_t)^2>0$ and $p \neq n_t$. Thus, assuming the strong inequality $p \gg n_t$, it would be a good approximation to assume in (23) the condition $j^2-K_1\cdot E(L)\approx j^2$ from which there results the following formula defining the value of the integration constant K_1

$$K_1=\frac{2(\mu\varepsilon)^2(E^3(L)-E^3(0))}{(3L)^2}\left\{1+\sqrt{1-\frac{(3L)^2[6Lj^3-3\mu\varepsilon j^2(E^2(L)-E^2(0))]}{(\mu\varepsilon)^3(E^3(L)-E^3(0))^2}}\right\}. \tag{24}$$

Formulas (22) and (24) and boundary functions $j_0=f_0[E(0)]$ and $j=f_L[E(L)]$ describe the current-voltage dependence in parametric form.

Example: when there are given the boundary functions $j=a_0E^k(0)$ and $j=a_LE^k(L)$ $(a_0>a_L)$, the dependence $U=U(j)$ may take the boundary form

$$U=\alpha_1j^{\frac{k-1}{k}}+\alpha_2j^{\frac{2k-3}{k}}; \quad j \gg j_g; \quad k \geq 1; \quad \alpha_2>0; \tag{25}$$

where α_1 and α_2 are constants depending on internal parameters μ, ε, C_p, L, and on boundary parameters a_0 *and* a_L; j_g — a boundary value such that when $j \gg j_g$ the expression under the square root in (24) is close to 1. From (25) it results that for $k=1$, $\frac{dU}{dj}<0$ and for $k=2$, $j \sim U^2$. Summing up this part of the work, it is worth noting that when parameter $\chi \approx 0$, i.e. when $\varepsilon C_p \ll e_0\mu_t$, there results from (20) and (21) the relationship $U=U[E(0),E(L)]$

$$U=\frac{2L(E^3(L)-E^3(0))}{3(E^2(L)-E^2(0))}; \quad \chi \approx 0. \tag{26}$$

Thus, in the case when the internal parameters satisfy the relation $\varepsilon C_p \ll e_0\mu_t$, the formula (26) and boundary functions $j = f_0[E(0)]$ and $j = f_L[E(L)]$ describe the current-voltage characteristic in parametric form. Formula (26) is a particular case of (18), since from the boundary transition $\chi \to 0$ there results the boundary transition of parameter $a_{12} \to \infty$. We will now derive a differential equation describing the distribution of electric field intensity $E(x)$ for simultaneously existing generation and recombination processes, i.e. $v_{p_i} \neq 0$ and $C_p \neq 0$. We obtain this equation by transforming (8) and (9)

$$(27)\quad \frac{\varepsilon\mu_p\theta_1}{2j}\frac{d}{dx}\left(\frac{dE^2}{dx}\right) = \frac{e_0}{j}\sum_{i=1}^{m} v_{p_i}\cdot N_{t_i} - \frac{C_p j}{e_0(\mu_t E)^2}\left\{\theta_1\left(\frac{\varepsilon\mu_t}{2j}\frac{dE^2}{dx} + \theta\right)\cdot\left[1 - r\theta_1\left(\frac{\varepsilon\mu_t}{2j}\frac{dE^2}{dx} + \theta\right)\right]\right\}; \quad \theta_1 = (1 + r\theta)^{-1}$$

which, when $r = 1$ ($\mu_p = \mu_t$) and $\theta = 1$ ($m\theta_0 \ll 1$ — deep traps) takes the simpler form

$$(28)\quad \frac{\varepsilon\mu}{4j}\frac{d}{dx}\left(\frac{dE^2}{dx}\right) = \frac{e_0}{j}\sum_{i=1}^{m} v_{p_i}\cdot N_{t_i} - \frac{C_p j}{4e_0(\mu E)^2}\left\{1 - \left(\frac{\varepsilon\mu}{2j}\frac{dE^2}{dx}\right)^2\right\} \quad \mu = \mu_t = \mu_p.$$

It is easy to demonstrate that (28) may be reduced to a first-order Bernoulli-type equation [3] the solution of which is of the form

$$(29)\quad \frac{\varepsilon\mu}{2j}\frac{dE^2}{dx} = \sqrt{1 - CE^{2\chi_1} + \frac{\gamma E^2}{1-\chi_1}}; \quad \chi_1 = \frac{\varepsilon C_p}{2e_0\mu}; \quad \gamma = \frac{2e_0\varepsilon\mu}{j^2}\sum_{i=1}^{m} v_{p_i}N_{t_i}; \quad \chi_1 \neq 1$$

where C is the integration constant. Equation (29) is esily integrated for $\chi_1 \approx 0$, $\chi_1 = 1/2$ and $\chi_1 = 2$. For example, when $\chi_1 \approx 0$, i.e. for $C_p \ll 2e_0\mu/\varepsilon$, the solution of (29) is the $E(x)$ function

$$(30)\quad E^2(x) = \alpha x^2 + ax + b; \quad \alpha = \frac{2e_0}{\varepsilon\mu}\sum_{i=1}^{m} v_{p_i}N_{t_i}; \quad a = \text{const},\ b = \text{const}$$

where a and b are integration constants. As we can see, the current-voltage characteristic switching is possible in this case. When $\chi_1 = 1/2$, equation (29) can be integrated in the elementary manner but the integration constants satisfy transcendental equations and the determination of function $j = j(U)$ with given boundary functions becomes a fairly complex problem. If $\chi_1 = 2$, one of the solutons of (29) is the $E(x)$ function

$$(31)\quad E^2(x) = \frac{\gamma}{2C_1} + 2C_2^2\exp\left(\frac{2\sqrt{C_1}jx}{\varepsilon\mu}\right) \pm \sqrt{\left(E^2(x) - \frac{\gamma}{2C_1}\right)^2 + K_2};$$

$$K_2 = \frac{4C_1 - \gamma^2}{4C_1^2}; \quad C_1 > 0$$

where C_1 and C_2 are suitably selected integration constants.

If $E^2(x) \gg \frac{\gamma}{2C_1}$, equation (31) is reduced to the form

$$(32)\quad E(x) = C_2\exp\left(\frac{\sqrt{C_1}jx}{\varepsilon\mu}\right)$$

i.e.

$$(33)\quad E(x) = E(0)\left\{\frac{E(L)}{E(0)}\right\}^{x/L}$$

and hence, in the light of (5) and boundary condition, we have the dependence $j = j(U)$ in parametric form

$$(34)\quad U\cdot ln\frac{E(L)}{E(0)} = L(E(L) - E(0)); \quad j = f_0[E(0)]; \quad j = f_L[E(L)].$$

It is easy to demonstrate that condition $E^2 \gg \frac{\gamma}{2C_1}$ can be satisfied for the assumed boundary functions. Another solution of (29) for $\chi_1 = 2$ is the function

$$(35)\quad E^2(x) + \frac{\gamma}{2C} = \sqrt{\frac{K_3}{C}}\cos\left(\frac{2j\sqrt{C}x}{\varepsilon\mu} - C_1\right); \quad K_3 = \frac{\gamma^2 + 4C}{4C}$$

where C_1 is a suitably selected integration constant. Since $\cos\delta = \cos(-\delta)$, it results from (35) that there is possible transport for which $E(0) = E(L)$. This condition is possible only when

$$(36)\quad \frac{2j\sqrt{C}L}{\varepsilon\mu} - C_1 = C_1; \quad C_1 = \frac{j\sqrt{C}L}{\varepsilon\mu}.$$

Hence

$$(37)\quad E^2(x) + \frac{\gamma}{2C} = \sqrt{\frac{K_3}{C}}\cos\left[\frac{j\sqrt{C}}{\varepsilon\mu}(2x - L)\right].$$

In what follows we will be considering the carrier injection condition for which $E(0) \to 0$ and $E(L) \to 0$. We have

$$(38)\quad \cos C_1 = \frac{1}{\sqrt{1 + \frac{4C}{\gamma^2}}}.$$

If $\frac{2\sqrt{C}}{\gamma} \to \infty$, then $C_1 \to \frac{\pi}{2}$. In this case the value of the integration constant is

(39)
$$C = \left(\frac{\pi\varepsilon\mu}{2jL}\right)^2.$$

It results from this that condition $\frac{2\sqrt{C}}{\gamma} \to \infty$ is satisfied when

$$j \gg \frac{2e_0 L}{\pi} \sum_{i=1}^{m} \nu_{p_i} N_{t_i}.$$

It is further required that $\frac{2C}{\gamma} \to \infty$, this being equivalent to the inequality

$$\sum_{i=1}^{m} \nu_{p_i} N_{t_i} \ll \frac{\pi^2 \varepsilon\mu}{4e_0 L^2} \text{ (comparison of values)}$$

which means that recombination processes dominate. In this case (37) takes the form

(40)
$$E(x) = \sqrt{\frac{1}{\sqrt{C}} \cos\left[\frac{j\sqrt{C}}{\varepsilon\mu}(2x - L)\right]}.$$

Hence, on the basis of (5) and (39) we obtain Child's law $j \sim U^2/L^3$ in the form

(41)
$$j = \frac{\pi^3 \varepsilon\mu}{8a^2} \frac{U^2}{L^3}; \quad a = \int_0^{\pi/2} \sqrt{\cos t}\, dt.$$

It is worth noting that in this case the metal-dielectric-metal system behaves like a *p-n* junction.

3. Conclusions. Referring to Part I of this work, we find that when recombination processes dominate, the $E(x)$ function may display variable monotonicity in the entire region $0 \leqslant x \leqslant L$ and that symmetry of carrier injection into the dielectric from the electrodes, expressed as the identity of boundary functions $j = f_0[E(0)] \equiv f_L[E(L)]$, is admissible. In conditions of carrier injection symmetry and recombination, the current-voltage dependence may have the form of Child's law $j \sim U^2/L^3$. When parameter $\chi_1 = \frac{\varepsilon C_p}{2e_0\mu}$ ($\mu = \mu_t = \mu_p$) takes the value $\chi_1 = 1/2$, the dependence $j = j(U)$ may be decreasing (equation (25) when $k = 1$ and $\chi = 1$).

In this work we did not consider monopolar charge transport with two mobilities. This problem is the subject of a separate publication.

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