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New solution to the problem of bipolar injection in insulator and semiconductor devices. Solids with an arbitrary distribution of defects

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Abstract

A problem of the bipolar space charge transport between the anode and the cathode is presented. The space charge is formed by free electrons and holes as well as by trapped electrons. It was found that interactions between carriers can be described by the $n(p)$ relationships. Also, it was found that there exist the conditions in which the metal–solid–metal system can act as an n–p or p–n blocking diode. Ohm, Fowler–Nordheim, Schottky and Child's law are obtained.

Keywords: Electrical conduction; Child's law; Mobility; Charge injection; Insulators; Charge carriers

1. Introduction

Different mathematical methods describing double injection in a solid have been considered. A problem of the carrier generation has been studied by Dumke [1, 2], Simmons [3–5] and by Hindley [6]. A small signal theory for semiconductors has been presented by Henisch and Manifacier [7–11]. The carrier flow through an insulator has been solved by a regional approximation method [12–16]. A problem of the $n(p)$ relationships has been considered by Schwob [17, 18]. In this concept, the transition region between the anode and cathode regions has been distinguished. Using quasineutrality assumption, the different relations $n(p)$ between positive and negative charge carrier concentrations have been obtained. In general, with this assumption a set of the solutions is very limited [19–27]. Moreover, in the case of strong asymmetric double injection, quasineutrality assumption cannot be made [28–30]. In this work we will present our theoretical analysis describing the space charge transport between the two electrodes. The purpose of this work is to find new

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relationships $n(p)$ and to define the current–voltage characteristics for space charge conditions.

2. The basic equations

In our theoretical analysis we make the following assumptions:

(I) The planar capacitor with the anode $x = 0$ and the cathode $x = L$ will be used (Fig. 1).

(II) The potential barrier width at the electrodes is small in comparison with the mean free path and there are no surface states at the metal–bulk interfaces.

(III) There exist many structural defects (Fig. 2) and the concentration of atoms is possibly maximal (Fig. 3).

(IV) There exist the recombination centers and the trapping levels (Fig. 4).

(V) The carrier diffusion is unimportant [31, 32].

(VI) The carrier mobilities are independent of the electric field intensity.

(VII) The bulk acts as an unlimited reservoir of traps.

With these assumptions the basic equations are the continuity equation, the Gauss equation, the generation-recombination equations and the integral condition. These equations are as follows:

$$\varepsilon_\infty \frac{\partial E(x,t)}{\partial x} = q\left\{(p(x,t) - p_0) - (n(x,t) - n_0) - \sum_{i=1}^{m} (n_{ti}(x,t) - n_{ti,0})\right\}, \qquad (1)$$

$$\frac{\partial}{\partial x}\{[\mu_p p(x,t) + \mu_n n(x,t)]E(x,t)\} + \frac{\partial p(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} - \sum_{i=1}^{m} \frac{\partial n_{ti}(x,t)}{\partial t} = 0, \qquad (2)$$

$$\frac{\partial n_{ti}(x,t)}{\partial t} = c_{ni} N_{ti} n(x,t) - v_{ni} n_{ti}(x,t), \quad i = 1, 2, \ldots, m, \qquad (3)$$

$$\frac{\partial n(x,t)}{\partial t} = -\beta(p(x,t)n(x,t) - n_0 p_0) - \sum_{i=1}^{m} \frac{\partial n_{ti}(x,t)}{\partial t} + \frac{\partial}{\partial x}[\mu_n n(x,t)E(x,t)], \qquad (4)$$

$$\int_0^L E(x,t)\,\mathrm{d}x = V, \quad V > 0, \qquad (5)$$

where q is the electron charge, ε_∞ is the high-frequency permittivity, μ_n and μ_p are the electron and hole mobilities, respectively, E the electric field intensity, x the distance from the electrode, t the time, p and n are the hole and electron concentrations, respectively, n_{ti} the trapped electron concentration in the ith trapping level, m the number of the trapping levels, $p_0 = n_0 + \sum_{i=1}^{m} n_{ti,0}$ are the equilibrium concentrations of carriers, v_{ni}, β and c_{ni} are the generation-recombination parameters, N_{ti} the concentration of traps in the ith trapping level, V the applied voltage and L the distance between the electrodes.

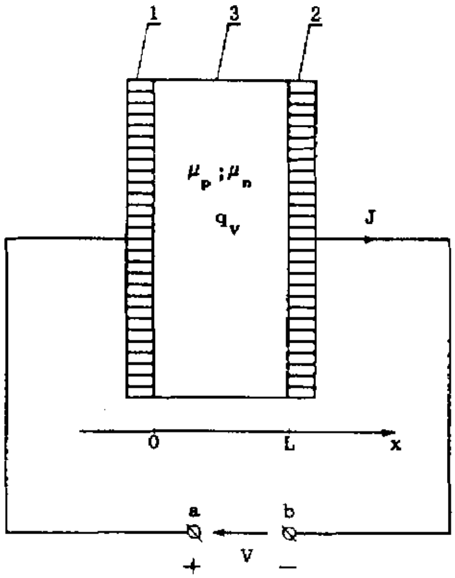

Fig. 1. The planar capacitor system. J is the total current density; q_v is the space charge density. 1, anode; 2, cathode, 3, solid (semiconductor, insulator).

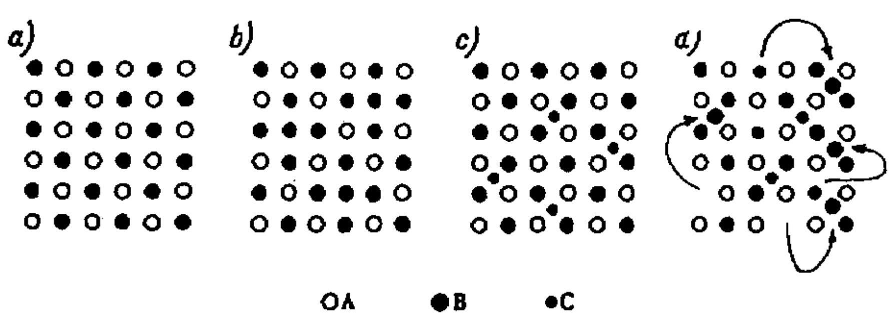

Fig. 2. The structural defects in the bulk. (a) The perfect crystalline lattice formed by the A and B atoms. (b) Some A atoms are replaced by the B atoms. (c) Pollutants and impurities are interpreted by the C atoms. (d) Some B atoms are replaced by the C atoms and some B atoms are displaced. The arrows indicate the Frenkl defects.

For such space charge problem we will find the relationships $n(p)$ and we will define new space charge density distribution for the analytical form of the current–voltage characteristics.

3. The solution of the problem

In this part we shall show that a function $n(p)$ is valid for the steady and transient state of the current flow. First, to this end, let us investigate the steady state.

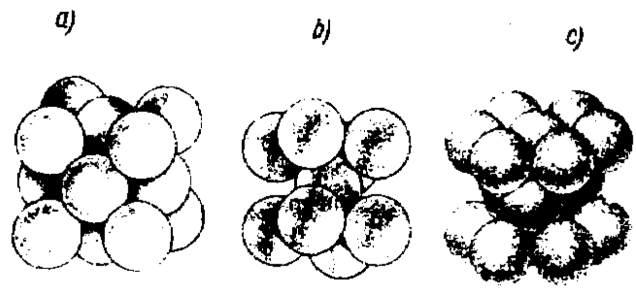

Fig. 3. The possible maximal concentration of atoms in space. The atoms are interpreted by the spheres with the same radius: (a) the regular system with coordinate number 12; (b) the regular system with coordinate number 8; (c) the hexagonal system with coordinate number 12.

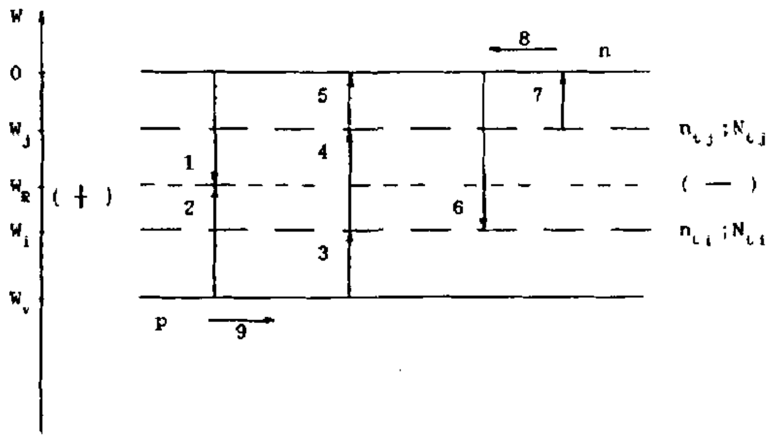

Fig. 4. The energy diagram and allowed electron transitions. W denotes the total energy of an electron; W_i, W_j and W_R are the trapping levels, W_v is the valence level, (+) and (—) denote the anode and cathode, respectively. 1 and 2, hole electron recombination with the rate βnp; 3, 4 and 5, hole electron pair generation with rate $\beta n_0 p_0$; 6, $c_{ni} n N_{ti}$; 7, $v_{nj} n_{tj}$; 8, $\mu_n nE$; 9, $\mu_p pE$.

3.1. The steady state

From (1)–(4) it follows that the equations describing the stationary state have the following form:

$$\varepsilon_\infty \frac{\mathrm{d}E(x)}{\mathrm{d}x} = q\left(p(x) - n(x) - \sum_{i=1}^{m} n_{ti}(x)\right), \tag{1a}$$

$$J = q\mu_n n(x)E(x) + q\mu_p p(x)E(x), \quad J = \text{constant}, \tag{2a}$$

$$n(x) = \theta_{0i} n_{ti}(x), \qquad \theta_{0i} = \frac{v_{ni}}{c_{ni} N_{ti}}, \quad i = 1, 2, \ldots, m, \tag{3a}$$

$$\frac{d}{dx}[\mu_n n(x)E(x)] - \beta(p(x)n(x) - n_0 p_0) = 0, \tag{4a}$$

$$\int_0^L E(x)\,dx = V, \tag{5a}$$

where J is the electric current density. Next, by combining (1a)–(4a), we obtain the following differential equation:

$$-\varepsilon_\infty \mu_n \mu_p \frac{d}{dx}\left(E\frac{dE}{dx}\right) = \frac{\beta J^2}{q(\mu_p + \mu_n/\theta)E^2}\left[\left(1 + \frac{\varepsilon_\infty \mu_n E}{\theta J}\frac{dE}{dx}\right)\left(1 - \frac{\varepsilon_\infty \mu_p E}{J}\frac{dE}{dx}\right) - \frac{q^2(\mu_n + \theta\mu_p)^2 n_0 p_0 E^2}{\theta J^2}\right], \quad \theta = 1 + \sum_{i=1}^{m}(1/\theta_{0i})^{-1}. \tag{6}$$

Eq. (6) is easily solved when the carrier mobilities satisfy the condition $\mu_n = \theta\mu_p$. With this assumption, we shall make use of the following substitutions:

$$y = E^2, \qquad \frac{dy}{dx} = w(y), \qquad \frac{d^2y}{dx^2} = w\frac{dw}{dy}. \tag{7}$$

By combining (6) and (7), we obtain the Bernoulli-type differential equation

$$-w\frac{dw}{dy} = \frac{\beta J^2}{q\varepsilon_\infty \mu_p^3 \theta y}\left[1 - \left(\frac{\varepsilon_\infty \mu_p}{2J}w\right)^2 - \frac{(2q\mu_p)^2\theta n_0 p_0 y}{J^2}\right]. \tag{8}$$

It is well-known that the Bernoulli differential equation always leads to the linear differential equation. In our case, using the substitution [33]

$$z = 1 - \left(\frac{\varepsilon_\infty \mu_p}{2J}w\right)^2, \tag{8a}$$

we get the following linear equation:

$$\frac{dz}{dy} - \frac{\varepsilon_\infty \beta z}{2q\mu_p\theta y} = -\frac{2\varepsilon_\infty \beta q \mu_p n_0 p_0}{J^2} \tag{9}$$

for which the general integral has the form

$$z = Cy^\chi - \frac{2\varepsilon_\infty \beta q\mu_p n_0 p_0}{J^2(1-\chi)}y, \qquad \chi = \frac{\varepsilon_\infty \beta}{2q\mu_p\theta}, \quad \chi \neq 1, \tag{10}$$

where C is a constant of integration. In the case when $\chi = 1$, the above solution must be replaced by

$$z = Cy - \left(\frac{\sigma_\Omega}{J}\right)^2 y\ln y, \qquad \sigma_\Omega = 2q\mu_p(\theta n_0 p_0)^{1/2}, \quad \chi = 1. \tag{11}$$

Now, we shall show that a function $n(p)$ exists. From (1a)–(2a) and (8a) it follows that the carrier concentration product can be expressed by

$$n(x)p(x) = \frac{J^2 z}{4q^2\mu_p^2 0y}. \tag{12}$$

Hence, on the basis of (10) and (11) we obtain the following two relationships:

$$n(x)p(x) = n(0)p(0) = \text{constant for } n_0 = p_0 = 0 \tag{13}$$

as well as

$$n(x)p(x) = \frac{\chi n_0 p_0}{\chi - 1} = \text{constant}, \quad \text{and } C = 0. \tag{14}$$

The relationship (13) is written for the perfect insulator. In this case the product np depends on the boundary conditions. The relationship (14) is independent of the boundary conditions. In the particular case when $\chi \gg 1$ the solution (14) leads to the law of mass action $n(x)p(x) = n_0 p_0$. It is worth noting that this law is also determined by the Fermi–Dirac (or Boltzmann) distribution function which is written at the thermodynamic equilibrium conditions. Since the function (14) is independent of carrier injection from the electrodes into the bulk, therefore Eqs. (1)–(3) and (14) can define the new model of electric conduction. We may suppose that this is possible when these equations determine the stable state. In what follows, we shall investigate the stability of (1a)–(3a) and (14).

3.2. The transient state

In order to define the problem of the stability we must take into account the equations describing the transient state of the current flow. To this end, we shall use the dimensionless variables system

$$Q_1 = \frac{p}{K^{1/2}}, \qquad Q_2 = -\frac{n}{K^{1/2}}, \qquad Q_{2ti} = -\frac{n_{ti}}{K^{1/2}},$$

$$Q_1^0 = \frac{p_0}{K^{1/2}}, \qquad Q_2^0 = -\frac{n_0}{K^{1/2}}, \qquad Q_{2ti}^0 = -\frac{n_{ti,0}}{K^{1/2}},$$

$$K = \frac{\chi n_0 p_0}{\chi - 1}, \qquad x^* = \frac{x}{L}, \qquad E^* = \frac{\varepsilon_\infty E}{qLK^{1/2}}, \qquad V^* = \frac{\varepsilon_\infty V}{qL^2 K^{1/2}},$$

$$t^* = \frac{\mu_p q K^{1/2} t}{\varepsilon_\infty}, \qquad \tau_{ni} = \frac{\mu_p q K^{1/2}}{\varepsilon_\infty c_{ni} N_{ti}}, \qquad \tau_{gi} = \frac{\mu_p q K^{1/2}}{\varepsilon_\infty v_{ni}}. \tag{15}$$

Therefore, the transient state is described by the following equations:

$$\frac{\partial}{\partial x^*}\{(Q_1-\theta Q_2)E^*\}+\frac{\partial Q_1}{\partial t^*}+\frac{\partial Q_2}{\partial t^*}+\sum_{i=1}^{m}\frac{\partial Q_{2ti}}{\partial t^*}=0, \tag{16}$$

$$\frac{\partial E^*}{\partial x^*}-Q_1-Q_2-\sum_{i=1}^{m}Q_{2ti}=0, \tag{17}$$

$$\frac{\partial Q_{2ti}}{\partial t^*}=\frac{Q_2}{\tau_{ni}}-\frac{Q_{2ti}}{\tau_{gi}},\quad i=1,2,\ldots,m, \tag{18}$$

$$Q_1Q_2+1=0, \tag{19}$$

with the voltage condition

$$\int_0^1 E^*\,\mathrm{d}x^*=V^*, \tag{20}$$

where E^*, Q_1, Q_2 and Q_{2ti} depend on the distance x^* and the time t^*. Next, let us define the equations of the steady state for the model (16)–(19). These equations have the form

$$\frac{\mathrm{d}}{\mathrm{d}x^*}\{[Q_1^*(x^*)-\theta Q_2^*(x^*)]E_s^*(x^*)\}=0, \tag{21}$$

$$\frac{\mathrm{d}E_s^*(x^*)}{\mathrm{d}x^*}-Q_1^*(x^*)-\theta Q_2^*(x^*)=0,\qquad Q_1^*(x^*)Q_2^*(x^*)+1=0,$$

$$Q_{2ti}^*(x^*)=Q_2^*(x^*)/\theta_{0i},\quad i=1,2,\ldots,m, \tag{22}$$

with the integral condition

$$\int_0^1 E_s^*(x^*)\,\mathrm{d}x^*=V^*. \tag{23}$$

Eqs. (21) and (22) correspond to (1a)–(3a) and (14). From (21)–(23) we obtain three solutions. These solutions are as follows:

(a) The first solution is the uniform electric field E_s^*,

$$E_s^*(x^*)=V^*,\qquad Q_1^*(x^*)=Q_1^0,\qquad Q_2^*(x^*)=Q_2^0,\qquad Q_{2ti}^*(x^*)=Q_{2ti}^0 \tag{24}$$

for $0\leqslant x^*\leqslant 1$.

(b) The second and third solutions are the monotonic functions E_s^* and Q_2^*,

$$E_s^*\frac{\mathrm{d}E_s^*}{\mathrm{d}x^*}=\pm[J_s^{*2}-4\theta E_s^{*2}]^{1/2},\qquad Q_2^*=-\frac{J^*\pm[J^{*2}-4\theta E_s^{*2}]^{1/2}}{2\theta E_s^*} \tag{25}$$

for $0\leqslant x^*\leqslant 1$ where J^* is the constant which depends on the current density J in the form

$$J^*=\varepsilon_\infty J/(q^2\mu_p LK). \tag{25a}$$

Let us now introduce the small parameter $\varepsilon = \varepsilon(x^*, t^*)$ into the function $Q_2^*(x^*) = -f(x^*)$ in order to obtain the functions $Q_2 = Q_2(x^*, t^*), Q_1 = Q_1(x^*, t^*)$ and $Q_{2ti} = Q_{2ti}(x^*, t^*)$ in the form

$$Q_2 = -f + \varepsilon, \qquad Q_1 = -\frac{1}{Q_2} = \frac{1}{f} + \frac{\varepsilon}{f^2} + \frac{\varepsilon^2}{f^3} + \cdots, \qquad Q_{2ti} = (-f + \varepsilon)/\theta_{0i},$$

$$f = f(x^*). \tag{26}$$

Next, we must introduce the assumptions defining the ε-function space. These assumptions are as follows:

(i) all the functions $\varepsilon = \varepsilon(x^*, t^*)$ are infinitesimal in the region $0 \leqslant x^* \leqslant 1$ and $t^* \geqslant 0$,

(ii) all the derivatives of the functions $\varepsilon = \varepsilon(x^*, t^*)$ are continuous and limited in the region $0 \leqslant x^* \leqslant 1$ and $t^* \geqslant 0$,

(iii) in the ε-function space there exist the uniform convergent sequences $\varepsilon_\nu (\nu = 1, 2, 3, \ldots)$,

(iv) the boundary values of $\varepsilon(0, t^*)$ are equal to zero for $t^* \geqslant t_0^*$, where t_0^* is a finite number. Thus, in this space the distance $\rho(\varepsilon_1, \varepsilon_2)$ between two arbitrary points ε_1 and ε_2 can be determined by

$$\rho(\varepsilon_1, \varepsilon_2) = \max_{x^*, t^*} \left|\frac{\partial \Delta\varepsilon}{\partial t^*}\right| + \max_{x^*, t^*} \left|\frac{\partial \Delta\varepsilon}{\partial x^*}\right| + \max_{x^*, t^*} |\Delta\varepsilon|, \qquad \Delta\varepsilon = \varepsilon_1 - \varepsilon_2.$$

In such ε-function space we can define the so-called orbital stability. Generally, the stability problem can be solved by the variational method (the first approximation method). The main idea of the first approximation method is to find the difference between the left-hand side of (16)–(19) and (21), (22) in the form

$$\Delta F = F(\varepsilon) - F(\varepsilon = 0) = \delta F(\varepsilon) + \delta^2 F(\varepsilon) = 0, \qquad \lim_{\rho \to 0} \delta^2 F(\varepsilon)/\rho(\varepsilon, 0) = 0,$$

$$\int_0^1 (E^*(\varepsilon) - E_s^*)\, dx^* = \int_0^1 (\delta E^*(\varepsilon) + \delta^2 E^*(\varepsilon))\, dx^* = 0,$$

$$\lim_{\rho \to 0} \delta^2 E^*(\varepsilon)/\rho(\varepsilon, 0) = 0,$$

where $F(\varepsilon)$ is the left-hand side of (16)–(19), $F(\varepsilon = 0)$ the left-hand side of (21), (22), $\delta F(\varepsilon)$ the main linear part of ΔF, $\delta^2 F(\varepsilon)$ the nonlinear part of ΔF. Now, let us define the variation δE^*. This functional is of the form

$$\delta E^* = \int_0^{x^*} \left(\theta + \frac{1}{f^2}\right) \varepsilon\, dx^* - \int_0^1 dx^* \int_0^{x^*} \left(\theta + \frac{1}{f^2}\right) \varepsilon\, dx^* = \int_{x_1^*}^{x^*} \left(\theta + \frac{1}{f^2}\right) \varepsilon\, dx^*,$$

$$x_1^* \in \langle 0, 1 \rangle.$$

We shall assume that the variation δE^* is arbitrary small, that is $\delta E^* = 0$. This condition denotes that the perturbation of the electric field intensity is infinitesimal. The stability of the system can be determined by the following implication:

$$\text{if } \delta F(\varepsilon) = 0 \text{ and } \delta E^* = 0 \quad \text{then } \lim_{t^* \to \infty} \varepsilon(x^*, t^*) = 0 \text{ and } \lim_{\varepsilon \to 0} \rho(\varepsilon) = 0$$

for $0 \leqslant x^* \leqslant 1$, where $\rho(\varepsilon) = \rho(\varepsilon, 0)$. Let us illustrate this problem for (24) and (25). First, we shall investigate the stability of (24). In this case we must assume that $f \equiv -Q_2^0$. Hence, on the basis of (24) and (16)–(19) we ascertain that the condition $\delta F(\varepsilon) = 0$ has the form

$$\theta \frac{\partial \varepsilon}{\partial t^*} + Q_1^0 \frac{\partial E^*}{\partial x^*} = 0 \quad \text{and} \quad \frac{\partial E^*}{\partial x^*} = 2\theta\varepsilon;$$

therefore,

$$\frac{\partial \varepsilon}{\partial t^*} + 2Q_1^0\varepsilon = 0. \tag{27}$$

Hence, the small parameter ε is of the form

$$\varepsilon(x^*, t^*) = \varepsilon(x^*, 0)\exp(-2Q_1^0 t^*), \quad 0 \leqslant x^* \leqslant 1. \tag{27a}$$

Now, we can define the functions $\varepsilon(x^*, 0)$ for which the condition $\delta E^* = 0$ is satisfied. As an example, let us take into account the following function:

$$\varepsilon(x^*, 0) = \begin{cases} \varepsilon_1(x^*) & \text{for } x^* \in \langle 0; x_0^* \rangle \\ 0 & \text{for } x^* \in \langle x_0^*; 1 \rangle \end{cases} \quad (x_0^* \to 0+),$$

where ε_1 is the arbitrary differentiable function. In particular case when ε_1 is of the form $\varepsilon_1 = \phi(x^*, \nu)\sin(\nu x^*)$ or $\varepsilon_1 = \phi(x^*, \nu)\cos(\nu x^*)$, where ϕ is the arbitrary differentiable and infinitesimal function as $\nu \to \infty$, then we have $\delta E^* = 0$ for all values of $x_0^* \in \langle 0; 1 \rangle$. This property follows from one of the Dirichlet theorems. We can easily verify that the variations $\delta^2 E^*(\varepsilon)/\rho(\varepsilon)$ and $\rho(\varepsilon)$ are arbitrary small as $\varepsilon \to 0$. Thus, the solution (24) is stable. Now, let us investigate the stability of the system for (25). We *must notice that the solution (25) defines the positive or negative space charge density.* As an example, let us consider the function $f(x^*)$ describing the negative space charge density, that is $f(x^*) > 1$ and $\mathrm{d}f/\mathrm{d}x^* > 0$ for $x^* \in \langle 0; 1 \rangle$. To this end, we shall introduce the new symbols

$$a = a(x^*) = \theta + \frac{1}{f^2(x^*)}, \qquad b = b(x^*) = \frac{1}{f^2(x^*)} - \theta,$$

$$g = g(x^*) = \theta f(x^*) + \frac{1}{f(x^*)}.$$

With these symbols, the operator $F(\varepsilon)$ is defined by

$$\frac{\partial(Q_1 + Q_2)}{\partial t^*} + \sum_{i=1}^{m} \frac{\partial Q_{2i}}{\partial t^*} = a\frac{\partial \varepsilon}{\partial t^*} + \cdots,$$

$$Q_1 - \theta Q_2 = g + \varepsilon b + \cdots,$$

$$\frac{\partial}{\partial x^*}\{(Q_1 - \theta Q_2)E^*\} = \left(\frac{\mathrm{d}g}{\mathrm{d}x^*} + \frac{\partial(\varepsilon b)}{\partial x^*}\right)(E_s^* + \delta E^*) + (g + \varepsilon b)\left(\frac{\mathrm{d}E_s^*}{\mathrm{d}x^*} + a\varepsilon\right) + \cdots.$$

Thus, taking into account the condition $\delta E^* = 0$, we can write the condition $\delta F(\varepsilon) = 0$ in the following form:

$$a\frac{\partial \varepsilon}{\partial t^*} + \frac{\partial(bE_s^*\varepsilon)}{\partial x^*} + ag\varepsilon = 0. \tag{28}$$

Next, using the substitution $\vartheta = \ln|\varepsilon b E_s^*|$ and the theory of characteristics, we can write the following ordinary equations:

$$\frac{\mathrm{d}x^*}{\mathrm{d}t^*} = \frac{bE_s^*}{a} \quad \text{and} \quad \frac{\mathrm{d}\vartheta}{\mathrm{d}t^*} = -g.$$

Hence,

$$t^* = \varphi(x^*) - \varphi(x^*(t^*=0)) \quad \text{and} \quad \vartheta = -\int_0^{t^*} g[x^*(s)]\,\mathrm{d}s - G_1(x^*(t^*=0));$$

therefore,

$$\varepsilon(x^*, t^*) = \pm\frac{1}{b(x^*)E_s^*(x^*)}\exp\left\{G[\varphi(x^*) - t^*] - \int_0^{t^*} g[x^*(s)]\,\mathrm{d}s\right\}$$

or

$$\varepsilon(x^*, t^*) = \varepsilon(0, t^* - \varphi(x^*))\frac{b(0)E_s^*(0)}{b(x^*)E_s^*(x^*)}\exp\left\{\int_{t^*}^{t^*-\varphi(x^*)} g[x^*(s)]\,\mathrm{d}s\right\}, \tag{28a}$$

where

$$\varphi(x^*) = \int_0^{x^*}\frac{a(s)\,\mathrm{d}s}{b(s)E_s^*(s)}, \qquad G = -G_1[\varphi^{-1}],$$

and G_1 is an arbitrary differentiable function defining the initial values of the function ϑ, and φ^{-1} is the inverse function. If the condition $\delta E^* = 0$ is to be satisfied, we must assume that the values of G_1 are infinite large. Let us notice that the functions $E_s^*(x^*)$, $\varphi(x^*)$ and $g(x^*)$ are limited. Since

$$\int_{t^*}^{t^*-\varphi(x^*)} g[x^*(s)]\,\mathrm{d}s = -g[x^*(c)]\varphi(x^*), \quad t^* < c < t^* - \varphi(x^*)$$

and $\varepsilon(0, t^*) = 0$ for $t^* \geqslant t_0^*$, therefore the function $\varepsilon(x^*, t^*)$ is equal to zero for $t^* \geqslant t_0^* + \max_{x^*}|\varphi(x^*)|$ and for $x^* \in \langle 0; 1\rangle$. Analogously, proceeding for the function $f(x^*) < 1$, we ascertain that the steady state of (16)–(19) is stable.

Using the boundary condition $\varepsilon(1, t^*)$, we also ascertain that the system is stable. Now, let us define the physical aspect of (27a) and (28a). First, let us notice that the solution (28a) describes the propagation of the perturbation between the electrodes. In general, the boundary functions $\varepsilon(0, t^*)$ and $\varepsilon(1, t^*)$ describe the mechanisms of carrier injection from the electrodes into the bulk. As an example, let us take into account the

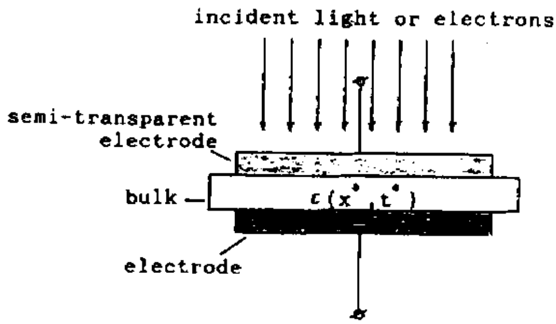

Fig. 5. The optic-field technique realizing the propagation of the space charge perturbation $\varepsilon(x^*, t^*)$ between the two electrodes.

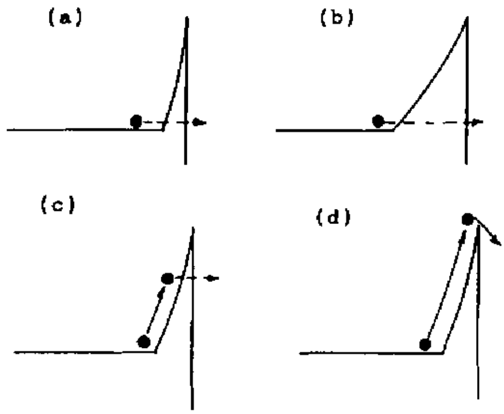

Fig. 6. Electron transition through a potential barrier: (a) ohmic-field emission; (b) field emission; (c) thermionic-field emission; (d) thermionic emission.

boundary function $\varepsilon(0, t^*)$ describing the mechanism of hole injection from the anode into the bulk (electron emission from the bulk into the anode). Using the optic-field technique [34–38] (Fig. 5), it was found that the space charge can be induced in the bulk at the bulk–metal contact. This phenomenon can be explained by the mechanisms describing the carrier flow through a potential barrier [39–40]. These mechanisms are as follows:

(a) quantum-mechanical tunnelling through the barrier from the edge of the depletion region (the field-emission current (Figs. (6a) and (b)),

(b) quantum-mechanical tunnelling through a part of the barrier (the thermionic-field emission current (Fig. 6(c)),

(c) the electron emission from the bulk over the top of the barrier (the thermionic-emission current (Fig. 6(d)),

(d) recombination in the depletion region (Fig. 7).

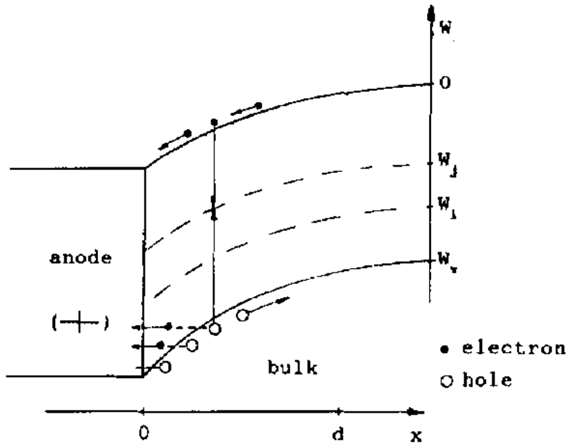

Fig. 7. The recombination current flow through a potential barrier. The barrier width is denoted by d.

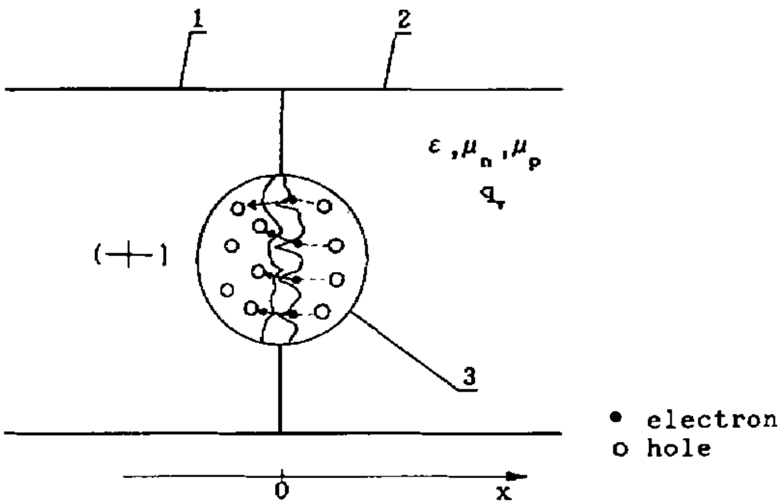

Fig. 8. Ionization of atoms in the bulk at the anode: 1, the anode; 2, the bulk; 3, the microscopic structure.

In general, the bulk-contact surface is not homogeneous. On this basis, we can suppose that the bulk-contact surface acts as the system of the points. In the bulk at the contact $x = 0$ the valence electrons can be pulled away from the normal atoms and accumulated in the points (Fig. 8). Next, these electrons can pass from the points into the anode. Similarly, proceeding for the contact $x = L$, we ascertain that the boundary functions $\varepsilon(0, t^*)$ and $\varepsilon(1, t^*)$ exist and the propagation of the perturbation $\varepsilon(x^*, t^*)$ in the form (27a) and (28a) is practically made.

Now, we can define the importance of (13) and (14) for the current–voltage characteristics.

3.3. The current–voltage characteristics

It is worth noting that the solution (24) defines Ohm's law $J^* = (Q_1^0 - \theta Q_2^0)V^*$. The similar result can follow from (25). In this case, the function $E_s^*(x^*)$ has the form

$$2\theta^{1/2}E_s^*(x^*) = \{J^{*2} - 16\theta^2(x^* + C)^2\}^{1/2}, \tag{29}$$

where C is a constant of integration. Next, using the boundary condition $J^* = 2Q_1^0 E_s^*(x^* = 0)$ for (29), we obtain the following current–voltage dependence:

$$16\theta^{3/2}V^* = J^{*2}\arcsin\left(\frac{4\theta}{J^*}\right) + 4\theta[J^{*2} - 16\theta^2]^{1/2}. \tag{29a}$$

In the particular case when $J^* \gg 4\theta$, then $J^* = 2\theta^{1/2}V^*$, that is $J = \sigma V/L$, where the conductivity parameter σ is

$$\sigma = \left(\frac{\chi}{\chi - 1}\right)^{1/2}\sigma_\Omega. \tag{29b}$$

In the special case when $\chi \gg 1$, that is $\varepsilon_\infty\beta \gg 2q\mu_p\theta$ the conductivity parameter σ becomes the ohmic conductivity $\sigma_\Omega = 2q(\mu_p\mu_n n_0 p_0)^{1/2}$. The other interesting result follows from (1a)–(4a) and (13). In this case, the electric field intensity distribution satisfies the equation

$$E\frac{dE}{dx} = \frac{1}{\varepsilon_\infty\mu}(J^2 - A_1E^2)^{1/2}, \qquad \mu = \mu_p = \mu_n/\theta, \tag{30}$$

therefore

$$E = A\left\{J^2 - \left(\frac{A_1}{\varepsilon_\infty\mu}\right)^{1/2}(x + B)^2\right\}^{1/2}, \qquad A = A_1^{-1/2}, \tag{30a}$$

where A and B are constants of integration. Making use of the boundary condition $dE/dx|_{x=0} = 0$, we obtain the current–voltage dependence $J = J(V)$ in the following parametric form:

$$V = \frac{\varepsilon_\infty\mu E^3(0)}{2J}\left\{\arcsin\left(\frac{JL}{\varepsilon_\infty\mu E^2(0)}\right) + \frac{L}{\varepsilon_\infty\mu E^2(0)}\left[J^2 - \left(\frac{J^2L}{\varepsilon_\infty\mu E^2(0)}\right)^2\right]^{1/2}\right\} \tag{31}$$

and $J = f_0[E(0)]$, where $f_0[E(0)]$ is the boundary function describing the mechanism of carrier injection from the electrode $x = 0$ into the bulk. Moreover, with the boundary condition $dE/dx|_{x=0} = 0$, we have

$$E(L) = E(0)\left\{1 - \left(\frac{JL}{\varepsilon_\infty\mu E^2(0)}\right)^2\right\}^{1/2}. \tag{31a}$$

Let us consider some special cases of f_0. The boundary function is quadratic $J = a_0E^2(0)$, where a_0 is the boundary parameter. In this case we have $J \propto V^2$. In the particular case when $a_0 = \varepsilon_\infty\mu/L$ we get Child's law in the form

$$J = \frac{16}{\pi^2}\varepsilon_\infty\mu\frac{V^2}{L^3} \tag{31b}$$

and $E(L) = 0$. From (31) we can obtain the other functions $J = J(V)$, namely

$$J = f_0[V/L] \quad \text{for } \frac{\varepsilon_\infty\mu V^2}{L^3} \gg f_0[V/L] \quad \text{or} \quad f_0^{-1}(J) \gg \left(\frac{JL}{\varepsilon_\infty\mu}\right)^{1/2}, \tag{32}$$

where f_0^{-1} is the inverse function. For instance, if f_0 is the Fowler–Nordheim function in the form $f_0 = a_0E_0^2\exp(-b_0/E_0)$, where a_0 and b_0 are the boundary parameters and $E_0 = E(0)$, then we have

$$J = \frac{a_0V^2}{L^2}\exp(-b_0L/V) \quad \text{for } a_0 \ll \frac{\varepsilon_\infty\mu}{L}. \tag{32a}$$

If f_0 is linear, that is $f_0 = \sigma_0E(0)$, where σ_0 is the boundary parameter, then we obtain Ohm's law $J = \sigma_0V/L$ for $V \gg \sigma_0L^2/\varepsilon_\infty\mu$ or $J \gg \sigma_0^2L/\varepsilon_\infty\mu$. Analogously, proceeding for the Schottky function $f_0 = J_0\exp(b_1E_0^{1/2})$, where J_0 and b_1 are the boundary parameters, we ascertain that there exist the finite numbers V_1 and V_2, such that

$$J = J_0\exp[b_1(V/L)^{1/2}] \quad \text{for } V \in \langle V_1, V_2\rangle \tag{32b}$$

and $\varepsilon_\infty\mu L^{-3} \gg J_0$ (comparison of values).

4. Discussion

A problem of the $n(p)$ relationships has been presented by Schwob [17, 18]. The fundamental assumption of this method is the condition $\varepsilon_\infty\mathrm{d}E/\mathrm{d}x \approx 0$ for $x \in (0, L)$. Using the boundary conditions $E(0) = E(L) = 0$ and $\mu_p = \mu_n = \mu$, Schwob determined the different functions $n(p)$ and interpreted them in the n–p plane. In this plane, he distinguished four regions which correspond to the configurations p–i–n, n–i–p, n–i–n, p–i–p. In general, let us notice that the fundamental assumption of this method with the boundary conditions $E(0) = E(L) = 0$ are not mathematically clear when the potential barrier width at the electrodes is much smaller than the distance L. In this case, the voltage condition (5a) and quasineutrality assumption must lead to the function $E(x) \equiv V/L$. Also, from (1)–(5) it follows that the fundamental problem of the current flow between the anode and cathode is to determine the space charge density distribution. In order to compare our methodology with the regional approximation method presented by [12–18], let us return to (7), (8a) and (10). From these equations it follows that there exists a function $E(x)$ described by

$$\frac{\varepsilon_\infty\mu_p}{2J}\frac{\mathrm{d}E^2}{\mathrm{d}x} = \left\{1 - CE^{2\chi} + \frac{\gamma E^2}{1-\chi}\right\}^{1/2}, \qquad \gamma = \frac{2q\beta\varepsilon_\infty\mu_pn_0p_0}{J^2}. \tag{33}$$

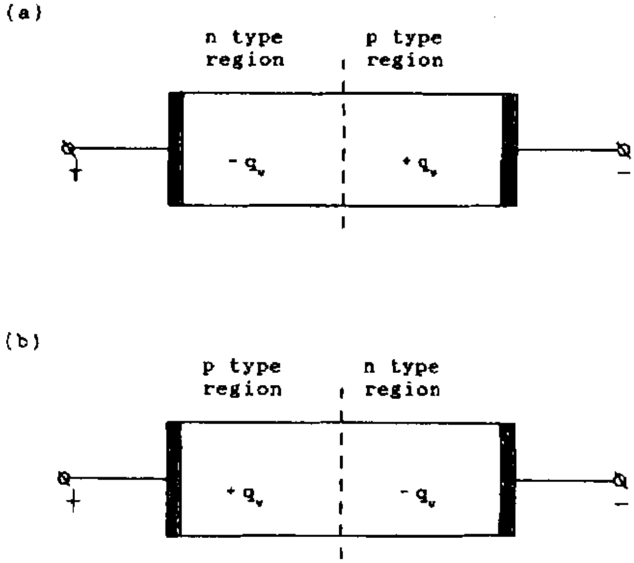

Fig. 9. The possible space charge distribution between the anode and cathode. (a) The planar capacitor system acts as an n–p blocking diode with the current–voltage dependence (33ac). (b) The planar capacitor system acts as a p–n blocking diode with the current–voltage characteristic (33ba).

In the case when $\chi \approx 0$, that is, the generation processes are dominant, we have

$$E^2(x) = \alpha x^2 + C_1 x + C_2, \qquad \alpha = \frac{2q\beta n_0 p_0}{\varepsilon_\infty \mu_p}, \tag{33a}$$

where C_1 and C_2 are constants of integration. Thus, the current–voltage dependence is determined by (5a) and by the boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes $x = 0$ and $x = L$ into the bulk [41]. In this case, it is possible for the boundary functions to be identical: $f_0[E(0)] \equiv f_L[E(L)]$, that is $E(0) = E(L)$ and $C_1 = -\alpha L$. Under these conditions, the space charge density $q_v = \varepsilon_\infty \mathrm{d}E/\mathrm{d}x$ is of the form

$$q_v = \varepsilon_\infty \alpha(x - L/2)/E, \quad E > 0. \tag{33ab}$$

Thus, a negative space charge is distributed in the region $x \in \langle 0; L/2)$ and a positive space charge occurs in the region $x \in (L/2; L\rangle$. Therefore, the metal–solid (insulator or semiconductor)–metal system acts as an n–p junction (Fig. 9(a)). In this case the current–voltage characteristic has the following parametric form:

$$V = \frac{1}{2\alpha^{1/2}}\left(E^2(L) - \frac{\alpha L^2}{4}\right)\ln\left|\frac{E(L) + \frac{1}{2}\alpha^{1/2}L}{E(L) - \frac{1}{2}\alpha^{1/2}L}\right| + \tfrac{1}{2}LE(L) \tag{33ac}$$

and $J = f_L[E(L)]$.

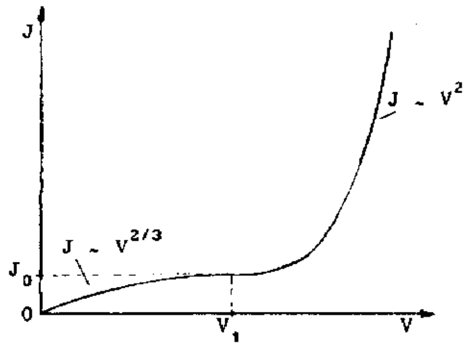

Fig. 10. The shape of (33ac) in the case of the quadratic boundary function $J \sim E^2(L)$. V_1 is $V_1 = \frac{1}{4}\alpha^{1/2}L^2$ and $J_0 = f_L(2V_1/L)$. This characteristic is typical for the SiC structure.

The shape of the $J(V)$ curve (33ac) with the quadratic boundary function $J = a_L E^2(L)$ where a_L is the boundary parameter, is shown in Fig. 10. Another interesting result follows from (33) for $\chi = 2$.

Assuming that $E(0) = E(L) = 0$ and the recombination processes are dominant, we can show that (33) leads to

$$E(x) = \left[\frac{2JL}{\pi\varepsilon_\infty\mu_p}\cos\left(\frac{\pi}{2L}(2x - L)\right)\right]^{1/2} \tag{33b}$$

for which the current–voltage characteristic is also Child's law

$$J = \frac{\pi^3\varepsilon_\infty\mu_p}{8a^2}\frac{V^2}{L^3}, \qquad a = \int_0^{\pi/2}\sqrt{\cos t}\,dt. \tag{33ba}$$

From (33b) it follows that the system acts as a p–n junction (Fig. 9(b)). Thus, from our considerations it follows that the space charge regions are determined by the internal processes and by the mechanisms describing carrier injection from the electrodes into the bulk.

In the regional approximation method the space charge regions have been assumed. This is the fundamental difference between our methodology and that theory.

5. Conclusions

From our considerations it follows Eqs. (1a)–(4a) can result in the relationships between n, p and E in the form $f_{np}(n, p, E, n_0, p_0, C) = 0$, that is

$$np - \frac{\chi n_0 p_0}{\chi - 1} - CE^{2\chi - 2} = 0, \tag{34}$$

where C is a constant of integration. An interesting case of the generation-recombination processes occurs when the (1)–(5) electron passages are very quick (Fig. 4).

For such allowed electron transitions we can suppose that the interactions between carriers are independent of the external electric field. Formally, this property can be characterized by the particular integral of (10), that is, C is equal to zero. For example, this is acceptable at the room temperature $T = 300$ K when the energy separations in the band gap are $\leqslant 0.1$ eV [41]. In this case the function f_{np} defines the relationship $n(p)$ in the form $n(x)p(x) = \chi n_0 p_0/(\chi - 1)$. This relationship corresponds to the three stable states of the current flow through the bulk. These states are described by the uniform electric field and by the increasing or descreasing electric field intensity distribution $E(x)$. Generally, for the monotonic function $E(x)$ the current–voltage characteristic $J = J(V)$ is nonlinear. In particular, this characteristic becomes linear $J = \sigma V/L$, where σ depends on the ohmic conductivity σ_Ω in the form (29b). For the perfect insulator, that is $n_0 = p_0 = 0$, the function f_{np} is equivalent to (13). With the boundary condition $p(0) = n(0)$, the current–voltage dependence becomes Child, Flower–Nordheim and Schottky's law in the form (32a), (32b). In general, under conditions of the generation-recombination processes (Fig. 4) the electric field intensity distribution is described by (33). From this equation it follows that the system can act as an n–p blocking diode when the generation processes are dominant. Also, from (33) it follows that the system can act as a p–n blocking diode when the recombination processes are dominant. The above results can explain the experiment characteristics $J(V)$ for the insulator and semiconductor materials such as Al_2O_3, anthracene, ZnS, SiC, CdS, TiO_2, GaAs, Ge and Si.

Acknowledgements

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References

[1] A.G. Milnes, Deep Impurities in Semiconductors, Wiley Intersciences, New York, 1973.

[2] K.C. Kao and W. Hwang, Electrical Transport in Solids, Pergamon, Oxford, 1981.

[3] J.G. Simmons, Theory of steady state photoconductivity in amorphous semiconductors, J. Non-Cryst. Solids, 8–10 (1972) 947–953.

[4] J.G. Simmons and M.C. Tam, Theory of isothermal current and the direct determination of trap parameters in semiconductors and insulators containing arbitrary trap distributions, Phys. Rev. B, 7 (1973) 3706–3713.

[5] J.G. Simmons and G.W. Taylor, Nonequilibrium steady-state statistics and associated for insulators and semiconductors containing an arbitrary distribution of traps, Phys. Rev. B, 4 (1971) 502–511.

[6] H.K. Hindley, Avalanche ionization in amorphous semiconductors, J. Non-Cryst. Solids, 8–10 (1972) 557–562.

[7] J.C. Manifacier and H.K. Henisch, Minority-carrier injection into semiconductors, Phys. Rev. B, 17 (1978) 2640–2647.

[8] J.C. Manifacier and H.K. Henisch, Minority-carrier injection into semiconductor containing traps, Phys. Rev. B, 17 (1978) 2648–2654.

[9] J.C. Manifacier and H.K. Henisch, The interpretation of ohmic behaviour in semi-insulating gallium–arsenide system, J. Appl. Phys., 52 (1981) 5195–5201.

[10] J.C. Manifacier, Y. Moreau and H.K. Henisch, A new static method for measuring-carrier lifetime, J. Appl. Phys., 54 (1983) 5158–5160.

[11] H.K. Henisch, Threshold switching form of super-conductivity, Mater. Res. Bull., 24 (1989) 749–752.

[12] M.A. Lampert and P. Mark, Current Injection in Solids, Academic Press, New York, 1970.

[13] L.M. Rosenberg and M.A. Lampert, Double injection in the perfect insulators: further analytic results, J. Appl. Phys., 41 (1970) 508–521.

[14] A. Waxman and M.A. Lampert, Double injection in insulators. II. Further analytic results with negative resistance, Phys. Rev. B, 1 (1970) 2735–2745.

[15] K.C. Kao, Double injection in solids with non-ohmic contacts. 1. Solids without defects, J. Phys. D, 17 (1984) 1433–1448.

[16] K.C. Kao, Double injection in solids with non-ohmic contacts. 2. Solids with defects, J. Phys. D, 17 (1984) 1449–1467.

[17] H.P. Schwob and I. Zschokke-Granacher, A new approach to double injection, Solid State Electron., 15 (1972) 271–276.

[18] H.K. Schwob and D.F. Williams, Double injection in insulators: Theoretical descriptions and analysis of some double-injection experiments in anthracene crystals, J. Appl. Phys., 45 (1974) 2638–2649.

[19] B. Swistacz, Some conditions for strong asymmetric double injection in insulators and semiconductors, Arch. Elektrotechnik, 78 (1995) 111–116.

[20] T. Bhattacharya, A note on the space-charge problem, IMA J. Appl. Math., 48 (1992) 117–124.

[21] C.J. Budd, A. Friedman, B. McLeod and A.A. Wheeler, The space charge problem, SIAM J. Appl. Math., 50 (1990) 181–198.

[22] G. Cimatti, Existence of weak solution for the space-charge problem, IMA J. Appl. Math., 44 (1990) 185–195.

[23] A. Skopec, Determination of the resorption current due to space-charge in a dielectric using a step-by-step method, J. Electrostatics, 8 (1979) 13–18.

[24] A. Skopec and E. Smycz, Transformation of initial conditions into boundary conditions for the calculation of the resorption current, J. Electrostatics, 8 (1979) 29–34.

[25] E. Smycz, Boundary problem in the SCM theory – characteristics of contact surfaces, J. Electrostatics, 15 (1984) 173–181.

[26] R. Gerhard-Multhaupt, Electretes-dielectrics with quasi-permanent charge or polarization, IEEE Trans. Electr. Insul., EI-22 (1987) 531–554.

[27] H.J. Wintle, Discharge current following space-charge-limited-current consolidations in insulators, J. Appl. Phys., 62 (1987) 2933–2936.

[28] K.C. Kao, New theory of electrical discharge and breakdown in low-mobility condensed insulators, J. Appl. Phys., 55 (1984) 752–755.

[29] J. Simon and J.-J. Andre, Molecular Semiconductors, Photo-Electrical Preperties and Solar Cells, Springer, Berlin and New York, 1985.

[30] N. Sankarraman, P.H. Niedermann, R.J. Noer and O. Fischer, Characterization of enhanced field emission sites on niobium surface due to heat treatment, J. Phys., 47 C-7 Suppl. (1986) 133–138.

[31] R.B. Schilling, Transient space-charge-limited currents including diffusion, Solid State Electron., 10 (1967) 689–699.

[32] R.B. Schilling and H. Schachter, Neglecting diffusion in space-charge-limited currents, J. Appl. Phys., 38 (1967) 841–844.

[33] B. Swistacz, Relations $n(p)$ between positive and negative charge carrier concentrations in conditions of bimolecular recombination, Bull. Pol. Acd. Sci. Tech. Sci., 36 (1987) 133–139.

[34] A. Many and G. Rakavy, Theory of transient space-charge-limited-current in solids in the presence of trapping, Phys. Rev., 126 (1962) 1980–1988.

[35] R.O. Loutfy, J.H. Sharp, C.K. Hsiao and R. Ho, Phtalocyanine organic solar cells. Indium/x-metal free phtalocyanine Schottky barriers, J. Appl. Phys., 52 (1982) 5218–5230.

[36] H.K. Henisch and J.C. Manifacier, Space-charge conduction and realaxation in dielectic films, J. Appl. Phys., 61 (1987) 5379–5385.

[37] M. Zahn, Keer electro-optic mapping measurement in electron beam irradiated polymethylmethacrylate, IEEE Trans. Electr. Insul., EI-22 (1987) 181–185.

[38] J.D. Cross, Influence of electrode processes on transient conduction phenomena in nonpolar liquid, J. Electrostatics, 29 (1992) 55–72.

[39] I.P. Batra, E. Tekman and S. Ciraci, Theory of Schottky-barrier and metallization, Prog. Surf. Sci., 36 (1992) 289–361.

[40] H. Matsuura and H. Okushi, Schottky barrier junctions of hydrogenated amorphous silicon–germanium alloys, J. Appl. Phys., 62 (1987) 2871–2879.

[41] B. Swistacz, Bipolar space-charge problem for semiconductors and insulators, J. Phys.: Condens. Mater., 7 (1995) 2563–2585.