J. Phys.: Condens. Matter 7 (1995) 2563–2585. Printed in the UK

Bipolar space-charge problem for semiconductors and insulators

Bronisław Swistacz

The Technical University of Wrocław, Institute of Electrical Engineering and Technology, Wybrzeża St, Wyspiańskiego 27, 50-370 Wrocław, Poland

Received 13 June 1994, in final form 1 December 1994

Abstract. In this work we make use of classical analysis for electrical conduction with one or two boundary conditions. The space charge is defined by free holes and electrons as well as by trapped electrons. Different interactions between carriers such as carrier recombination and generation are considered. We have determined the conditions in which a metal–solid–metal system can act as an n–p junction. Also, we have determined the conditions in which the current–voltage dependence can be strongly non-linear and discontinuous.

1. Introduction

Numerous analytical methods describing double injection in insulators have been proposed. Fundamental concepts for double injection are a regional approximation method [1–4] and small-signal theory [5, 6]. Those concepts contain fundamental physical processes, but the mathematical methods are not mathematically clear. Usually, in these methods the divergence of the electric field has been equal to zero. With this assumption, boundary conditions are very limited. In the case of strong asymmetric double injection, this assumption is not possible. This assumption ought to be determined by boundary functions describing the mechanisms of carrier injection from the electrode into a solid.

The purpose of this work is to present our theoretical analysis of this problem and to find new current–voltage characteristics for space-charge conditions.

2. The model system and the basic equations

The nature of the space-charge phenomenon in solids can be explained by the behaviour of the electrons surrounding the positive atomic nucleus in terms of the total energy of the electrons. The total energy is the sum of the potential and kinetic energies. This energy of every electron in the normal atom is a negative quantity. A zero reference level for an electron is at an infinite distance from the nucleus (in the case of an isolated atom). It is very well known that there exist only discrete energy states that are permissible in a given atom. The electrons occupying these states can absorb or emit discrete amounts of energy. Under these conditions, the electron must pass from one state to another. The additional kinetic energy can be given by a photon, a phonon or an external electric field. In solids,

0953-8984/95/132563+23$19.50

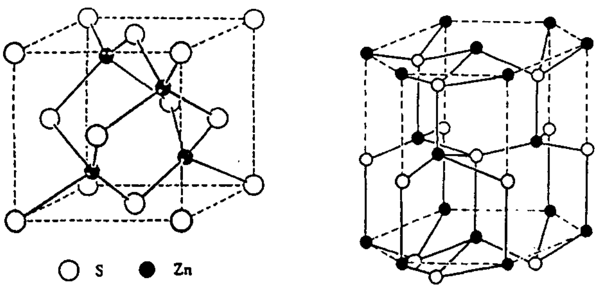

Figure 1. The α-ZnS (left) and β-ZnS (for α-CdS and β-CdS) crystal lattices. The α-ZnS (the regular system) structure is typical for diamond, Ge, Si and β-SiC. Diamond, ZnS and SiC are typical insulators (table 1). For diamond, ZnS and CdS, a strong effect of light on electrical conduction is observed. In the case of CdS, a strong effect of γ-radiation on electrical conduction occurs.

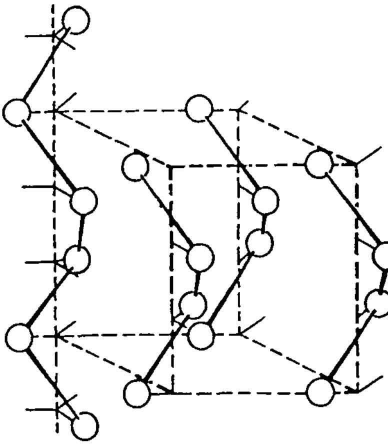

Figure 2. The Se long-chain structure (amorphous solid). Selenium is a typical photoconductor. In the absence of light the Se structure is a good insulator. The As:Se mixture has the same electrical properties.

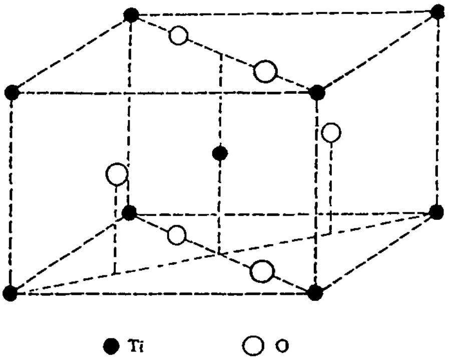

Figure 3. The TiO_2 crystal lattice. This material is a typical insulator (table 1) with large relative dielectric constant $\varepsilon_r = 114$.

atoms are spaced closely together (as an example, the structures of ZnS, Se, TiO_2 and SiO_2 are shown in figures 1–4).

Thus, many more permissible energy states are available because of the interaction forces between adjacent atoms (figure 5). However, in the case of pure insulators (some insulators are shown in table 1) the electron with the greatest negative energy (the valence electron) cannot absorb additional energy in small amounts. In general, in impure crystalline or amorphous structures there exist many crystal defects, pollutants and impurities (figure 6). Thus, the concentration of atoms can be sufficiently great (figure 7). Under these conditions the valence electron can absorb additional (phonon and photon) kinetic energy in small

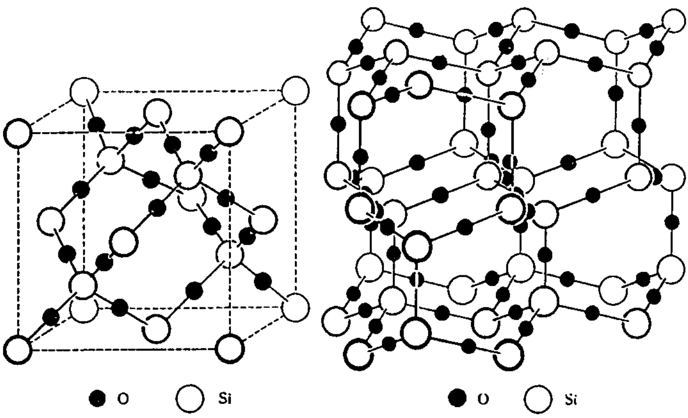

Figure 4. The SiO_2 regular (left) and hexagonal crystal lattices. This material is a typical insulator (table 1).

amounts and occupy a higher energy state (the trapping level). In figure 8, this is illustrated by arrow 4. If the additional (phonon and photon) kinetic energy is sufficiently great, then this electron can occupy higher trapping levels (figure 8, arrow 5). In particular, the trapped electron occupying the highest trapping level can pass to the zero reference level (figure 8, arrow 6). The vacancy (hole) left by the trapped electron represents an unfilled energy state. Such electron transition defines electron–hole pair generation.

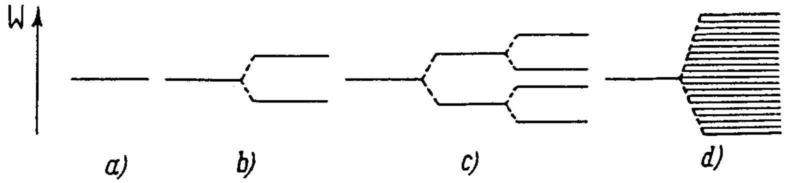

Figure 5. The splitting of an energy level in an atom. (*a*) The given energy level of an isolated atom. (*b*) The splitting of this level in the case of two atoms. (*c*) The splitting of this level in the case of four atoms. (*d*) The same situation in the case of many atoms. The total energy of a valence electron in a given atom is denoted by W.

The inverse process occurs under low-temperature conditions and in the absence of light and radiation. Under these conditions, the free and trapped electrons lose a portion of the kinetic energy because of the Coulomb force between the given electron and the positive nucleus of an adjacent atom. Thus, the free electron can pass to the empty energy state of the adjacent vacancy (figure 8, arrows 1, 2 and 3). Such electron transition defines electron–hole recombination.

If additional kinetic energy is given by an external electric field to the free electron, then electron flow with mobility μ_n occurs. Also, the external electric field can pull away the valence electron from a normal atom when the total energy of this electron is sufficiently great. In this case, the valence electron can pass to the unfilled energy state of the adjacent vacancy. This valence electron flow is equivalent to vacancy (hole) flow with mobility μ_p.

In general, the number of generation–recombination parameters is very great. In order to find analytical or numerical solutions, it is necessary to know the relations between them. In

Table 1. Some insulator and semiconductor materials.

Material	Band gap at $T = 300$ K (eV)	Property
SiO_2	8.0	insulator
Al_2O_3	5.8	insulator
C (diamond)	5.4	insulator
SnO_2	3.71	insulator
ZnS	3.58–3.67	insulator
TiO_2	3.67	insulator
ZnO	3.2	insulator
SiC	$\simeq 3$	insulator
AlP	$\simeq 2.5$	insulator
CdS	2.4	insulator
β-SiC	2.3	insulator
ZnTe	2.3	insulator
GaP	2.24	insulator
AlSb	1.60	semiconductor
GaAs	1.45	semiconductor
Si	1.10	semiconductor
GaSb	0.67	semiconductor
Ge	0.66	semiconductor
Se	—	insulator semiconductor
As:Se	—	insulator semiconductor

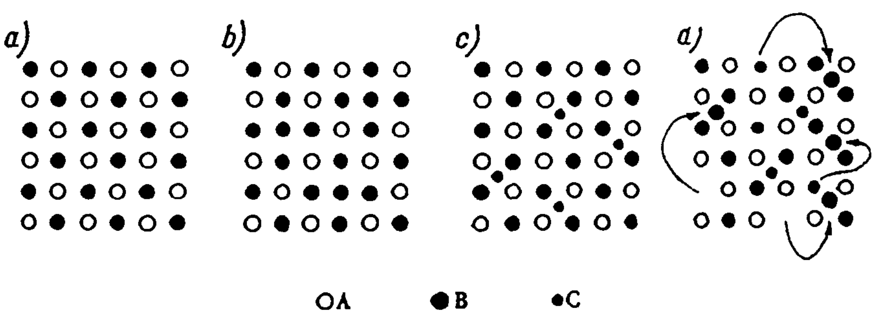

Figure 6. The possible defects in a crystalline structure. (*a*) The perfect crystalline structure formed by the A and B atoms. (*b*) Some A atoms are replaced by B atoms. (*c*) The solid interstitial solution of the C atoms. (*d*) The solid interstitial solution of C and B atoms (the arrows indicate Frenkl defects), and B atoms replaced by C atoms.

order to avoid this difficulty, we have grouped the trapping levels into two levels (figure 8). With this assumption, electron passage from lower to higher energy level is characterized by the generation parameters ν_p, c_{21} and ν_n. Electron passage from higher to lower energy level is characterized by the recombination parameters c_n, c_{12} and C_p.

In the theoretical analysis we make the following assumptions:

(i) A planar capacitor with anode $x = 0$ for injecting holes and cathode $x = L$ for injecting electrons will be used (figure 9).

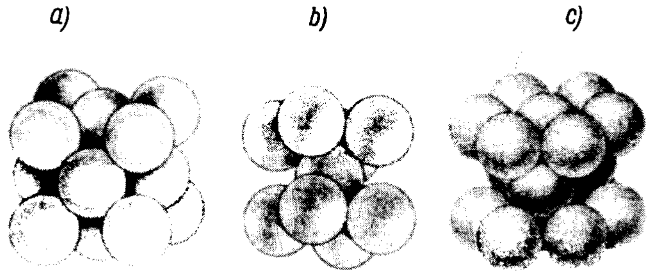

Figure 7. The possible maximal concentrations of atoms in space. The atoms are interpreted by spheres with the same radius. (*a*) The regular system with coordination number = 12. (*b*) The regular system with coordination number = 8. (*c*) The hexagonal system with coordination number = 12.

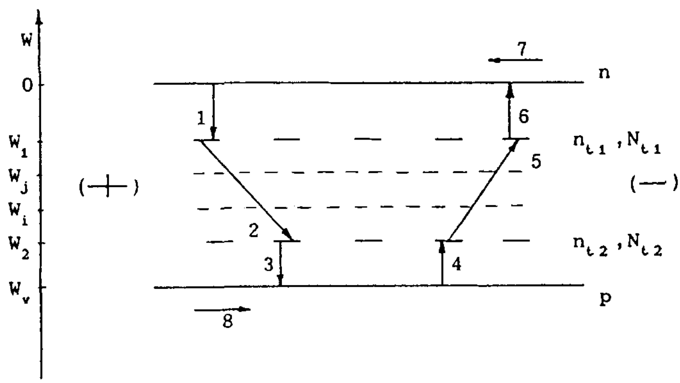

Figure 8. The energy diagram and allowed electron transitions between trapping levels. W is the total energy of an electron, (+) and (−) denote the anode and the cathode, respectively. The transition energies are: 1, $c_n N_{t1} n$; 2, $c_{12} N_{t2} n_{t1}$; 3, $C_p p n_{t2}$; 4, $\nu_p N_{t2}$ (and the hole activation energy is $W_p = W_2 - W_v$); 5, $c_{21} N_{t1} n_{t2}$; 6, $\nu_n n_{t1}$ (and the electron activation energy is $W_n = -W_1$); 7, $\mu_n n E$; and 8, $\mu_p p E$.

(ii) The potential barrier width at the anode and the cathode is much smaller than the mean free path.

(iii) The mobilities of the free carriers are independent of the electric field intensity and carrier diffusion is unimportant.

(iv) There are no surface states at the metal–bulk contact.

For such internal processes we shall define the space-charge transport equations. These equations are the Gauss equation, the continuity equation, the generation–recombination equations [7–9] and the field integral, which are written for the planar capacitor system as follows:

$$\varepsilon \frac{\partial E(x,t)}{\partial x} = q[p(x,t) - n(x,t) - n_{t1}(x,t) - n_{t2}(x,t)] \tag{1}$$

$$\frac{\partial}{\partial x}\{[\mu_n n(x,t) + \mu_p p(x,t)]E(x,t)\} + \frac{\partial p(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0 \tag{2}$$

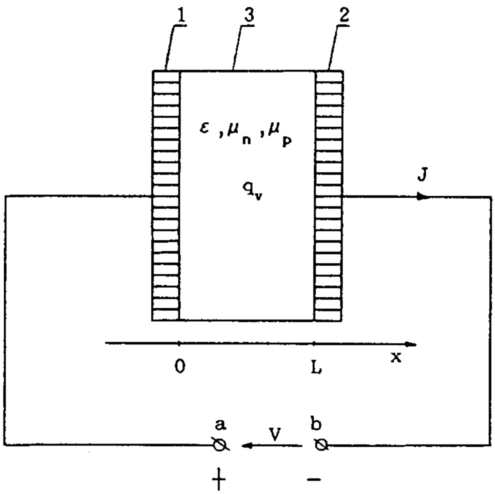

Figure 9. The planar capacitor system: 1, the anode; 2, the cathode; 3, solid (insulator or semiconductor); a and b, the voltage (or current) terminals; q_v, the space-charge density; and J, the total current density.

$$\frac{\partial n_{t1}(x,t)}{\partial t} = c_n N_{t1} n(x,t) + c_{21} N_{t1} n_{t2}(x,t) - \nu_n n_{t1}(x,t) - c_{12} N_{t2} n_{t1}(x,t) \qquad N_{t1} \gg n_{t1} \tag{3}$$

$$\frac{\partial n_{t2}(x,t)}{\partial t} = \nu_p N_{t2} + c_{12} N_{t2} n_{t1}(x,t) - c_{21} N_{t1} n_{t2}(x,t) - C_p p(x,t) n_{t2}(x,t) \qquad N_{t2} \gg n_{t2} \tag{4}$$

$$\frac{\partial n(x,t)}{\partial t} = \nu_n n_{t1}(x,t) - c_n N_{t1} n(x,t) + \frac{\partial}{\partial x}[\mu_n n(x,t) E(x,t)] \tag{5}$$

$$\int_0^L E(x,t)\,\mathrm{d}x = V = \text{constant} \qquad V > 0. \tag{6}$$

Here q is the electric charge, ε the dielectric constant, x the distance from the electrode, t the time, E the electric field intensity, p the free-hole concentration, n the free-electron concentration, n_{t1} and n_{t2} the trapped electron concentrations in the first and second trapping level, N_{t1} and N_{t2} the concentrations of traps, L the distance between the electrodes, and V the applied voltage. In equations (1)–(5) we have assumed that $N_t \gg n_t$, that is, the bulk acts as an unlimited reservoir of carriers. For such a space-charge transport model we shall find the current–voltage characteristics.

3. The solution of the problem

From (1)–(6) it follows that the steady state of electrical conduction is described by

$$\frac{\varepsilon}{q}\frac{\mathrm{d}E(x)}{\mathrm{d}x} = p(x) - n(x) - n_{t1}(x) - n_{t2}(x) \tag{1a}$$

$$J = qE(x)[\mu_n n(x) + \mu_p p(x)] \qquad J = \text{constant} \tag{2a}$$

$$C_n n(x) - \nu_n n_{t1}(x) = C_{12} n_{t1}(x) - C_{21} n_{t2}(x) \tag{3a}$$

$$C_{12} n_{t1}(x) - C_{21} n_{t2}(x) = C_p p(x) n_{t2}(x) - \nu_p N_{t2} \tag{4a}$$

$$\frac{d}{dx}[\mu_n n(x) E(x)] = C_n n(x) - \nu_n n_{t1}(x) \tag{5a}$$

$$\int_0^L E(x)\,dx = V = \text{constant} \qquad V > 0 \tag{6a}$$

where $C_n = c_n N_{t1}$, $C_{12} = c_{12} N_{t2}$, $C_{21} = c_{21} N_{t1}$. We shall consider two particular forms of the electric field distributions $E(x)$ determining the space-charge density. These distributions are as follows: the space-charge distribution $\varepsilon dE/dx$ is determined by two mechanisms of carrier injection, that is $E(x) = E(x, J, C_1, C_2)$; the space-charge distribution $\varepsilon dE/dx$ is determined by one mechanism of carrier injection, that is $E(x) = E(x, J, C_1)$. Here C_1 and C_2 are constants of integration.

It is very well known that electron and hole emission from the metal into the bulk can be described by the following mechanisms [10–14]:

(i) quantum-mechanical tunnelling through the barrier (the field emission current, figures 10(*a*) and (*b*));

(ii) quantum-mechanical tunnelling through a part of the barrier (the thermionic field emission current, figure 10(*c*));

(iii) electron emission over the top of the barrier (the thermionic emission current, figure 10(*d*)); and

(iv) recombination in the depletion region (the recombination current, figure 11).

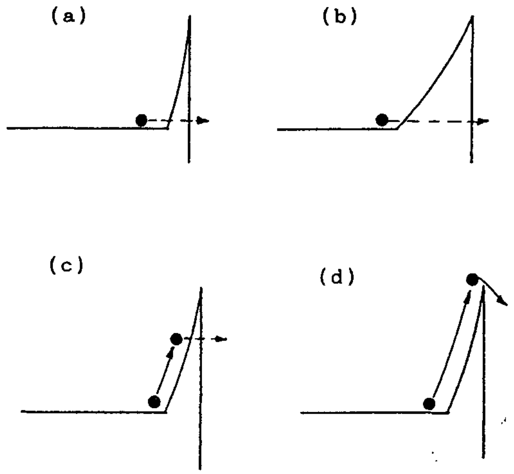

Figure 10. The possible mechanisms of electron transition through the potential barrier: (*a*) ohmic field emission; (*b*) field emission; (*c*) thermionic field emission; and (*d*) thermionic emission.

With assumption (ii) from section 2, the emission current density J depends on the barrier height and on the electric field intensity E_0 at the injecting contact. This boundary function can be written as $J = f_0(E_0)$, where f_0 is the function describing the mechanism of carrier injection from the electrode into the bulk.

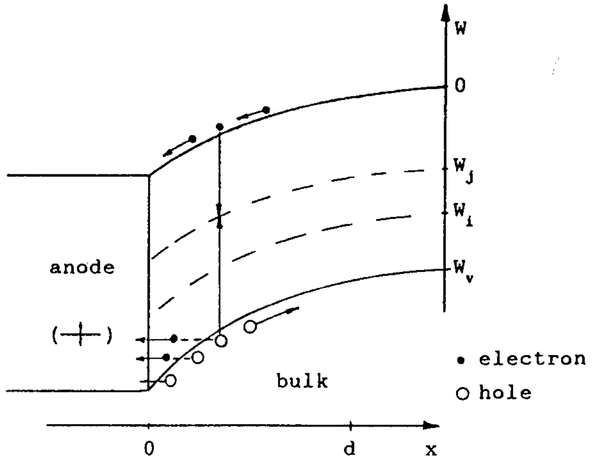

Figure 11. Recombination in the depletion region at the anode. The barrier width is d.

For the function $E = E(x, J, C_1, C_2)$ the voltage condition (6a) is of the form

$$V = \int_0^L E(x, J, C_1, C_2)\,\mathrm{d}x = V(J, C_1, C_2) \tag{6b}$$

and

$$E(0) = E(x = 0, J, C_1, C_2) \qquad \text{and} \qquad E(L) = E(x = L, J, C_1, C_2). \tag{6c}$$

If the current–voltage characteristics are to be evaluated, it is necessary to give two boundary functions $J = f_0[E(0)]$ and $J = f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes $x = 0$ and $x = L$ into the bulk, respectively. Thus, the current–voltage dependence can have the parametric form

$$V = V[E(0), E(L)] \qquad J = f_0[E(0)] \qquad J = f_L[E(L)] \tag{6d}$$

for double injection. In the case when $E = E(x, J, C_1)$, then (6d) results in

$$V = V[E(0)] \qquad J = f_0[E(0)] \tag{6e}$$

for injected holes or

$$V = V[E(L)] \qquad J = f_L[E(L)] \tag{6f}$$

for injected electrons. In what follows, we shall find the functions $J = J(V)$ or $V = V(J)$, which can be evaluated by the use of (6d)–(6f).

In this section we shall consider a few of the cases of internal interactions between carriers for which the analytical form of $E(x)$ and of the current–voltage characteristics can be found.

3.1. The carrier recombination conditions

In this section we shall consider the case of asymmetric double injection for carrier recombination. In this case we assume that $\nu_n = \nu_p = c_{21} = 0$ in (3*a*)–(5*a*). Next, introducing the new variables A_1, A_2, A_3, A_4 in the form

$$A_1 = q\mu_n pE/J \qquad A_2 = q\mu_n nE/J$$
$$A_3 = q\mu_n n_{t1}E/J \qquad A_4 = q\mu_n n_{t2}E/J \tag{7}$$

we obtain the following equations

$$\rho A_1 + A_2 = 1 \qquad \rho = \mu_p/\mu_n \tag{8}$$

$$\mu_n E\, dA_2/dx = C_n A_2 \tag{9}$$

which result in

$$-\frac{\varepsilon C_n}{\rho J}\frac{dE}{dA_1} = \frac{A_1}{A_2} - 1 - \frac{C_n}{C_{12}} - \frac{q\mu_n C_n E}{C_p J A_1}. \tag{10}$$

The analytical form of $E(x)$ will be found when (i) $C_n \leqslant C_{12}$ and $c_n = C_p$, (ii) $N_{t1} \gg n$ and $N_{t1} \gg p$, and (iii) $A_1 \ll 1/(1+\rho)$ (a case of strong asymmetric double injection). With these assumptions, we obtain

$$dE/dA_1 = q\mu_p E/\varepsilon C_p A_1 \tag{10a}$$

and therefore

$$E = K_1 A_1^{\chi} \qquad \chi = q\mu_p/\varepsilon C_p \tag{10b}$$

where K_1 is a constant of integration. Next, taking into account (10*b*), (9) and (8), we have

$$\frac{\mu_n K_1 (1-A_2)^{\chi}}{\rho^{\chi} A_2}\, dA_2 = C_n\, dx. \tag{11}$$

The solution of equation (11) has the form

$$\ln A_2 + \sum_{i=1}^{\chi} \frac{1}{i}\binom{\chi}{i}(-A_2)^i = \frac{\rho^{\chi} C_n x}{\mu_n K_1} + K' \qquad K' = \text{constant} \tag{11a}$$

for natural values of χ. Using the power-series representation to the logarithm and limiting the power series to the power $\chi + 1$, the result (11*a*) takes the form

$$-\sum_{i=1}^{\chi}\left[\frac{(1-A_2)^i}{i} - \frac{1}{i}\binom{\chi}{i}(-A_2)^i\right] - \frac{(\rho A_1)^{\chi+1}}{\chi+1} = \frac{\rho^{\chi} C_n x}{\mu_n K_1} + K'. \tag{11b}$$

Let us notice that the sum of terms containing the power $(-A_2)^k$ in the expression $\sum_{i=1}^{\chi}(1-A_2)^i/i$, where $k = 1, 2, \ldots, \chi$, takes the value

$$(-A_2)^k \frac{1}{k}\binom{\chi}{k}.$$

Thus, equation (11*b*) leads to

$$A_1 = \frac{1}{\rho}\left(K_2 - \frac{(\chi+1)\rho^\chi C_n x}{\mu_n K_1}\right)^{1/(\chi+1)} \tag{12}$$

where K_2 is a constant of integration. Hence on the basis (10*b*), we obtain the electric field distribution in the following form

$$E(x) = K_3\left(K_2 - \frac{(\chi+1)C_n x}{\mu_n K_3}\right)^{\chi/(\chi+1)} \qquad K_3 = K_1/\rho^\chi. \tag{13}$$

By calculating the integral (6*a*), we get the current–voltage dependence in the parametric form

$$V = \frac{(\chi+1)L}{2\chi+1}\frac{E_{(0)}^{i+1} - E_{(L)}^{i+1}}{E_{(0)}^{i} - E_{(L)}^{i}} \qquad i = (\chi+1)/\chi \tag{14}$$

and $J = f_0[E(0)]$, $J = f_L[E(L)]$.

For example, when the boundary functions f_0 and f_L describe the tunnel effect in the form $J = a_0E^2(0)$ and $J = a_LE^2(L)$ ($a_L > a_0$) or these functions are linear, then we have $J \sim V^2$ or $J \sim V$, respectively. Different experiments, for low-temperature conditions, show that the linear and quadratic functions $J(V)$ occur at low- and high-voltage conditions, respectively. For another example, with Schottky injection at the boundaries $x = 0$ and $x = L$ so that $J = J_0\exp(b_0E_{(0)}^{1/2})$ and $J = J_0\exp(b_LE_{(L)}^{1/2})$ ($b_L > b_0$), then the function $J(V)$ is $J \sim \exp(\text{constant} \times V^{1/2})$. Similarly, for the Pool boundary function, we get $J \sim \exp(\text{constant} \times V)$. These functions, at the same temperature conditions, are acceptable for the mean values of V. In the case when χ is a rational number $\chi = m/n$, we can make use of the following substitution

$$z^n = 1 - A_2 = \rho A_1 \qquad \mathrm{d}A_2 = (-n)z^{n-1}\,\mathrm{d}z \tag{15}$$

for which equation (11) takes the form

$$\frac{(-n)z^m z^{n-1}}{1-z^n}\,\mathrm{d}z = \frac{\rho^{m/n}C_n}{\mu_n K_1}\,\mathrm{d}x. \tag{16}$$

Hence, we get

$$z^m\ln(1-z^n) - m\int_0^z z^{m-1}\ln(1-z^n)\,\mathrm{d}z = \frac{\rho^{m/n}C_n x}{\mu_n K_1} + K' \tag{17}$$

where $K' = $ constant. Since $z \ll 1$, the function $\ln(1-z^n)$ may be expressed by a power series. By combining (17*a*) and limiting the power series to the first approximation, we obtain

$$-\frac{nz^{m+n}}{m+n} = \frac{\rho^{m/n}C_n x}{\mu_n K_1} + K'. \tag{17a}$$

Next, taking into consideration the substitution (15) and $\chi + 1 = (m+n)/n$, we notice that equation (17*b*) results in (12) with the constant $K_2 = -K'(\chi+1)$. On this basis we ascertain that the current–voltage dependence (15) is satisfied for rational values of χ.

3.2. The carrier generation–recombination conditions with immobile positive charge $\mu_p = 0$

Now we shall take into account the problem of space charge when there is no electron transition from a normal atom to the adjacent vacancy. With additional assumption of coefficient equality $C_{12} = C_{21}$, we have

$$A_2 = 1 \qquad A_3 = A_4 = \frac{C_n}{\nu_n} = \frac{1}{\theta_0} \qquad A_1 = \frac{(q\mu_n)^2 \nu_p N_{t2} \theta_0 E^2}{C_p J^2}. \tag{18}$$

Hence, on the basis (3*a*)–(5*a*), we obtain the differential equation

$$\frac{\varepsilon\mu_n E}{J}\frac{dE}{dx} = \frac{(q\mu_n)^2 \nu_p N_{t2} \theta_0 E^2}{C_p J^2} - \left(1 + \frac{2}{\theta_0}\right) \tag{19}$$

for which the general integral has the form

$$E = \frac{J}{\beta^{1/2}}\left[\theta + C\exp\left(\frac{2\beta x}{\varepsilon\mu_n J}\right)\right]^{1/2}$$
$$\theta = 1 + 2/\theta_0 \qquad \beta = (q\mu_n)^2 \nu_p N_{t2}\theta_0 / C_p \tag{20}$$

where C is a constant of integration, which can be expressed by the boundary value for $x = L$

$$C = \left(\frac{\beta E^2(L)}{J^2} - \theta\right)\exp\left(\frac{-2\beta L}{\varepsilon\mu_n J}\right). \tag{20a}$$

From (20), (20*a*) and (6*a*) it follows that the voltage function $V = V(E(L), J)$ has the form

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}}\left(\frac{\beta^{1/2}}{J}[E(L) - E(0)] + \frac{\theta^{1/2}}{2}\ln\left|\frac{[\beta^{1/2}E(L) - J\theta^{1/2}][\beta^{1/2}E(0) + J\theta^{1/2}]}{[\beta^{1/2}E(L) + J\theta^{1/2}][\beta^{1/2}E(0) - J\theta^{1/2}]}\right|\right) \tag{21}$$

where

$$E(0) = \frac{J}{\beta^{1/2}}\left\{\theta\left[1 - \exp\left(\frac{-2\beta L}{\varepsilon\mu_n J}\right)\right] + \frac{\beta E^2(L)}{J^2}\exp\left(\frac{-2\beta L}{\varepsilon\mu_n J}\right)\right\}^{1/2}. \tag{21a}$$

Therefore, the current–voltage characteristic is described parametrically by (21) and (21*a*) and the boundary function $J = f_L[E(L)]$. In the most simple case when the boundary function is linear $J = (\beta/\theta)^{1/2}E(L)$, we obtain Ohm's law $J = \sigma V/L$ with the conductivity parameter

$$\sigma = \frac{q\mu_n(\nu_p N_{t2}\theta_0)^{1/2}}{C_p^{1/2}(1 + 2/\theta_0)^{1/2}}. \tag{22}$$

An interesting case occurs when the electrode $x = L$ injects an infinite quantity of electrons, that is $E(L) \to 0$. Then, (21) and (21*a*) yield

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}}\left\{\frac{\theta^{1/2}}{2}\ln\left|\frac{1 + [1 - \exp(-2\beta L/\varepsilon\mu_n J)]^{1/2}}{1 - [1 - \exp(-2\beta L/\varepsilon\mu_n J)]^{1/2}}\right| - \theta^{1/2}\left[1 - \exp\left(\frac{-2\beta L}{\varepsilon\mu_n J}\right)\right]^{1/2}\right\} \tag{23}$$

and additionally, for values of the current density $J \gg 2\beta L/\varepsilon\mu_n$, we have

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}}\left[\frac{\theta^{1/2}}{2}\ln\left(\frac{1+W}{1-W}\right) - \theta^{1/2}W\right] \qquad W = \left(\frac{2\beta L}{\varepsilon\mu_n J}\right)^{1/2}. \tag{23a}$$

Expanding the logarithm as far as terms in W^3, we obtain

$$V = \frac{\varepsilon\mu_n J^2}{\beta^{3/2}}\left[\theta^{1/2}\left(W + \frac{W^3}{3}\right) - \theta^{1/2}W\right] = \frac{\varepsilon\mu_n\theta^{1/2}J^2}{3\beta^{3/2}}W^3 \tag{23b}$$

or the equivalent form

$$J = \tfrac{9}{8}\varepsilon\mu_n\theta^{-1}V^2/L^3 \tag{23c}$$

which is Child's law.

3.3. The hole–electron pair generation

In this section we shall consider the problem of space charge when carrier generation is dominant. In this case we assume that $C_n = C_{12} = C_p = 0$ in (3*a*)–(5*a*). Under these conditions, we obtain the following differential equation

$$\mathrm{d}E/\mathrm{d}x = \alpha_1(x - x_0)/E - \alpha_2 \tag{24}$$

where

$$\alpha_1 = \frac{(\mu_p + \mu_n)q\nu_p N_{t2}}{\varepsilon\mu_n\mu_p}$$

$$\alpha_2 = \frac{q\nu_p N_{t2}}{\varepsilon}\left(\frac{1}{C_{21}} + \frac{1}{\nu_n}\right) \tag{24a}$$

$$x_0 = L + \frac{[(\rho+1)A_2(L) - 1]J}{(\rho+1)q\nu_p N_{t2}}.$$

The boundary parameter x_0 is the constant of integration.

From (24) we get two singular solutions

$$E_1(x) = z_1(x - x_0) \qquad E_2(x) = z_2(x - x_0) \tag{24b}$$

where

$$z_{1,2} = -\tfrac{1}{2}\alpha_2 \pm \tfrac{1}{2}(\alpha_2^2 + 4\alpha_1)^{1/2} \qquad z_1 > 0 \qquad z_2 < 0 \tag{24c}$$

for $x_0 \leqslant 0$ or $x_0 \geqslant L$, respectively. Using the integral condition (6*a*), we can find the relation between the voltage V and the boundary parameter x_0 in the form

$$V = \tfrac{1}{2}[z_2(L - x_0)^2 - z_2x_0^2] = \tfrac{1}{2}L[2E(L) - z_2L] \tag{24d}$$

for $x_0 \geqslant L$, or

$$V = \tfrac{1}{2}[z_1(L - x_0)^2 - z_1x_0^2] = \tfrac{1}{2}L[2E(0) + z_1L] \tag{24e}$$

for $x_0 \leqslant 0$. Hence, it follows that the current–voltage dependence $J(V)$ is determined by the boundary function $J = f_L[E(L)]$ or $J = f_0[E(0)]$ describing the mechanism of carrier injection from the electrode $x = L$ or $x = 0$ into the bulk. Since $E(x) \geqslant 0$ for $x \in \langle 0; L \rangle$, this dependence is as follows:

$$J = f_L[(V - V_{02})/L] \qquad \text{for } V \geqslant V_{02} = |z_2|L^2/2 \tag{24f}$$

or

$$J = f_0[(V - V_{01})/L] \qquad \text{for } V \geqslant V_{01} = z_1 L^2/2. \tag{24g}$$

From (24a) it follows that the constant of integration $A_2(L)$ depends on the boundary values $E(L)$ in the form

$$A_2(L) = \frac{q\nu_p N_{t2} E(L)}{|z_2|J} + \frac{1}{\rho + 1} \qquad E(L) = z_2(L - x_0).$$

Since $A_2(L) < 1$, the boundary function f_L and the boundary parameter $E(L)$ must satisfy the following condition

$$E(L) < \frac{\rho |z_2| f_L[E(L)]}{q(\rho + 1)\nu_p N_{t2}}. \tag{24h}$$

Analogously, proceeding for the condition $A_2(0) < 1$, we can find the condition for the boundary function f_0 and the boundary parameter $E(0)$. It is worth noting that the Schottky function and the Fowler–Nordheim function as well as the power boundary function fulfil the condition (24h). Returning to (24), we can find the general integral in the form

$$\left|\frac{E}{x - x_0} - z_1\right|^{\gamma_1} \left|\frac{E}{x - x_0} - z_2\right|^{\gamma_2} = C|x - x_0| \tag{25}$$

where

$$\gamma_1 = -z_1/(z_1 - z_2) \qquad \gamma_2 = z_2/(z_1 - z_2)$$

and C is a constant of integration. With the additional assumptions that $\nu_n = \nu_p = C_{21}$ and $4\alpha_1 \gg \alpha_2^2$ (carriers are not too mobile), equation (25) can be written as

$$\left|\frac{E}{x - x_0} - \alpha_1^{1/2}\right|^{-1/2} \left|\frac{E}{x - x_0} + \alpha_1^{1/2}\right|^{-1/2} = C|x - x_0|. \tag{25a}$$

Hence, we obtain the electric field distribution

$$E(x) = [K + \alpha_1(x - x_0)^2]^{1/2} \tag{26}$$

where K is a constant of integration. Substituting (26) into (6a), we get

$$V = \frac{1}{2}\left(\frac{K}{\alpha_1^{1/2}} \ln\left|\frac{E(L) + \alpha_1^{1/2}(L - x_0)}{E(0) - \alpha_1^{1/2} x_0}\right| + (L - x_0)E(L) + x_0 E(0)\right). \tag{27}$$

The constants of integration K and x_0 can be expressed by

$$K = E^2(0) - \frac{[E^2(0) - E^2(L) + \alpha_1 L^2]^2}{4\alpha_1 L^2} \qquad x_0 = \frac{E^2(0) - E^2(L) + \alpha_1 L^2}{2\alpha_1 L}. \tag{28}$$

Thus, referring to (6*d*), we remark that two mechanisms of carrier injection from the electrodes $x = 0$ and $x = L$ into the bulk, which are described by the boundary functions $J = f_0[(E(0)]$ and $J = f_L[E(L)]$, define the current–voltage characteristics in parametric form. It is possible for the boundary functions f_0 and f_L to be identical, $f_0[E(0)] \equiv f_L[E(L)]$, that is $E(0) = E(L)$ and $x_0 = L/2$. Then, the functions (27)–(28) lead to the current–voltage dependence $V = V(J)$ in the following parametric form:

$$V = \frac{\alpha_1^{-1/2}}{2}\left(E^2(L) - \frac{\alpha_1 L^2}{4}\right)\ln\left|\frac{E(L) + \frac{1}{2}\alpha_1^{1/2}L}{E(L) - \frac{1}{2}\alpha_1^{1/2}L}\right| + \tfrac{1}{2}LE(L) \qquad J = f_L[E(L)]. \tag{29}$$

From (29) it follows that $V \to V_1 = \alpha_1^{1/2}L^2/4$ and $\mathrm{d}V/\mathrm{d}E(L) \to \infty$ as $E(L) \to \alpha_1^{1/2}L/2$. If $J = f_L[E(L)]$ is monotonic, then $\mathrm{d}J/\mathrm{d}V \to 0$ and $J \to J_0 = f_L(2V_1/L)$ as $V \to V_1$. Thus, the function $J(V)$ is defined and differentiable for all values of $V \geqslant 0$. If $E(L) \ll \alpha_1^{1/2}L/2$, then (29) results in

$$E(L) = (\tfrac{3}{4}\alpha_1 LV)^{1/3} \qquad \text{and} \qquad J = f_L[E(L)]. \tag{29a}$$

If $E(L) \gg \alpha_1^{1/2}L/2$, that is, the electric field becomes uniform, then $J = f_L(V/L)$. Finally, we ascertain that the inverse function to (29) is typical for a blocking diode. For the characteristic (29), the space-charge density $q_v(x)$ has the form

$$q_v(x) = \varepsilon\alpha_1(x - L/2)/E(x) \qquad E(x) > 0. \tag{29b}$$

Hence, it follows that a negative charge is distributed in the region $0 \leqslant x < L/2$ and a positive charge in the region $L/2 \leqslant x < L$. Therefore, the system acts as an n–p junction.

3.4. Absence of electron transition between trapping levels

A space charge can be formed when electron transition between trapping levels does not occur (the case of insulators). In this case we assume that $C_{12} = C_{21} = 0$ in (3*a*)–(5*a*), obtaining

$$A_2(x) = A_2(L) \qquad A_3 = A_2/\theta_0 \qquad \theta_0 = \nu_n/C_n$$
$$A_4 = \theta_p(q\mu_n E)^2/A_1 J^2 \qquad A_1 = (1 - A_2)\mu_n/\mu_p \qquad \theta_p = \nu_p N_{t2}/C_p \tag{30}$$

and

$$\frac{\varepsilon\mu_n E}{J}\frac{\mathrm{d}E}{\mathrm{d}x} = \alpha - \alpha_1 E^2 \qquad \alpha_1 = \frac{\theta_p}{A_1}\left(\frac{q\mu_n}{J}\right)^2 \qquad \alpha = A_1 - A_2 - A_3 = \text{constant} \tag{31}$$

where $A_2(L)$ is a constant of integration. From (31) we get the function $E(x)$ in the form

$$E(x) = \left[\alpha_0 J^2 + C\exp\left(-\frac{2\beta_0 x}{J}\right)\right]^{1/2} \qquad \alpha_0 = \frac{\alpha A_1}{\theta_p(q\mu_n)^2} \qquad \beta_0 = \frac{\theta_p q^2\mu_n}{A_1\varepsilon} \tag{32}$$

where C is a new constant of integration. Next, expressing the constant of integration by the boundary values $E(0)$ and $E(L)$, we get

$$E^2(0) = \alpha_0 J^2 + C \qquad E^2(L) = \alpha_0 J^2 + C\exp(-2\beta_0 L/J). \tag{32a}$$

Let us note that equations (32*a*) result in a transcendental equation for the constant α or A_1. For (31) the voltage conditions (6*a*) can be written as

$$V = \int_0^L E(x)\,\mathrm{d}x = \int_{E(0)}^{E(L)} E\frac{\mathrm{d}x}{\mathrm{d}E}\,\mathrm{d}E = \frac{\varepsilon\mu_n}{J}\int_{E(0)}^{E(L)} \frac{E^2}{\alpha - \alpha_1 E^2}\,\mathrm{d}E. \tag{33}$$

In what follows, we shall discuss the importance of the parameter α for (33). In the case when $\alpha > 0$, then (33) becomes

$$V = \frac{\varepsilon\mu_n a}{2Ja_1^3}\ln\left|\frac{[a_1E(L)+a][a_1E(0)-a]}{[a_1E(L)-a][a_1E(0)+a]}\right| + \frac{\varepsilon\mu_n}{Ja_1^2}[E(0)-E(L)]$$

$$a_1 = \alpha_1^{1/2} \qquad a = |\alpha|^{1/2}. \tag{33a}$$

If the electrode $x = 0$ injects an infinite quantity of holes, that is $E(0) \to 0$, then (32*a*) and (33*a*) result in

$$V = \frac{\varepsilon\mu_n a}{2Ja_1^3}\ln\left|\frac{a_1E(L)+a}{a_1E(L)-a}\right| - \frac{\varepsilon\mu_n}{Ja_1^2}E(L) \tag{33b}$$

and

$$E(L) = C[\exp(-2\beta_0 L/J) - 1] \qquad C = -\alpha_0 J^2. \tag{33c}$$

For values $a_1E(L) \ll a$, we can expand the logarithm to obtain

$$V = \frac{\varepsilon\mu_n a}{Ja_1^3}\left(\frac{a_1}{a}E(L) + \frac{1}{3}\frac{a_1^3}{a^3}E^3(L) + \cdots\right) - \frac{\varepsilon\mu_n}{Ja_1^2}E(L).$$

Next, taking into account only terms up to the power 3, we get

$$V = (\varepsilon\mu_n/3J\alpha)E^3(L). \tag{33d}$$

In the particular case when $J \gg 2\beta_0 L$, we have

$$E^2(L) \simeq -2C\beta_0 L/J = 2\alpha_0\beta_0 LJ = (2\alpha L/\varepsilon\mu_n)J.$$

Therefore, the current–voltage characteristic has the form

$$J = f_L(3V/2L) \tag{33e}$$

where f_L is the boundary function describing the mechanism of electron injection from the electrode $x = L$ into the bulk.

When $\alpha < 0$, then (33) leads to

$$V = \frac{\varepsilon\mu_n}{a_1^2 J}\left(E(0) - E(L) + \frac{1}{b}\{\tan^{-1}[bE(L)] - \tan^{-1}[bE(0)]\}\right) \qquad b = a_1/a. \tag{33f}$$

Analogously, proceeding for $E(L) \to 0$, we obtain the $J(V)$ curve in the form

$$J = f_0(3V/2L) \tag{33g}$$

where f_0 is the boundary function describing the mechanism of hole injection from the electrode $x = 0$ into the bulk.

When $a_0 = \alpha = 0$, that is $A_2 = 1/(1+\rho\theta)$ and $A_1 = \theta/(1+\rho\theta)$ where $\theta = 1 + 1/\theta_0$, then the function (32) becomes

$$E(x) = E(0)\exp(-\beta_0 x/J) \qquad E(0) = E(L)\exp(\beta_0 L/J). \tag{34}$$

Hence, on the basis (6*a*) we obtain the current–voltage characteristic in the following parametric form

$$V = \frac{JE(L)}{\beta_0}[\exp(\beta_0 L/J) - 1] \qquad \text{and} \qquad J = f_L[E(L)]. \tag{34a}$$

For $J \gg \beta_0 L$ the characteristic (34) leads to $J = f_L(V/L)$.

4. Discussion

In this section we compare our analysis with two analytical methods that have solved the problem of double injection in solids. A regional approximation method for double injection in insulators and semiconductors has been used by Lampert and Schwob [1–4]. In this method, positive and negative as well as quasi-neutral charge regions have been distinguished in the region $x \in \langle 0, L\rangle$. These regions are as follows:

(i) the first region is $x \in \langle 0, x_1\rangle$ (the anode region) in which $\varepsilon \mathrm{d}E/\mathrm{d}x = f_1(n, p) > 0$;
(ii) the second region is $x \in \langle x_1, x_2\rangle$ in which $\varepsilon \mathrm{d}E/\mathrm{d}x = f_2(n, p) \simeq 0$;
(iii) the third region is $x \in \langle x_2, L\rangle$ (the cathode region) in which $\varepsilon \mathrm{d}E/\mathrm{d}x = f_3(n, p) < 0$.

Here f_1, f_2 and f_3 are the given functions. With the boundary conductions such as (*a*) $E(0) = E(L) = 0$ and (*b*) continuity of the electric field at the junction planes $x = x_1$ and $x = x_2$, the current–voltage characteristics have been obtained. These functions can be $J \sim V$, $J \sim V^2/L^3$ (Child's law), $J \sim V^3/L^5$ or $J \sim V^{l+1}/(V_0 - V)^l$ where V_0 and l are constant parameters characterizing the material.

A small-signal theory for the diffusion problem has been presented by Manifacier and Henisch [5, 6]. In this method, the space-charge regions (i)–(iii) have also been distinguished and $x_1 \to 0$ and $x_2 \to L$ when $E(0) = E(L) = 0$ or $E(0) = E(L) = V/L$. With the condition $\varepsilon \mathrm{d}E/\mathrm{d}x = f_2(n, p) \simeq 0$, the diffusion problem equations have been written as linearized equations. The fundamental problem of this method is to find the functions $p(x)$ and $n(x)$ as well as $V(\mu_n, \mu_p)$ for the various boundary parameters $\mathrm{d}n/\mathrm{d}x$ and $\mathrm{d}p/\mathrm{d}x$ at the planes $x = 0$ and $x = L$. Usually, in this method the current–voltage characteristic is linear $J \sim V$.

According to our considerations, we notice that the space-charge regions are determined by the transport equations (1*a*)–(6*a*) and the boundary functions $f_0[E(0)]$ and $f_L[E(L)]$ describing the mechanisms of carrier injection from the electrodes into the bulk. This is the fundamental difference between our methodology and those theories.

In order to discuss the stationary state we ought to know the transient state describing the space-charge transport. This problem has not been discussed by the regional approximation method or by the small-signal theory. On this basis we ascertain that the assumptions (i)–(iii) of these theories are not mathematically clear. Also, with these assumptions the set of solutions is very limited [15–20].

In order to show this problem let us return to our analysis. From (29*b*) it follows that the functions $f_1(n, p)$ and $f_2(n, p)$ are determined by the boundary functions $f_0[E(0)]$ and $f_L[E(L)]$. As another example, from the singular solutions (24*b*) it follows that the function $E(x, C_1)$ characterizing the transport of one carrier can be determined by two mobilities μ_p and μ_n. In order to explain this mathematical detail we must take into consideration (1)–(6). Next, using theory of characteristics for (1)–(5), we obtain the following ordinary equations:

$$\begin{aligned} \mathrm{d}x_n/\mathrm{d}t &= -\mu_n E(x_n(t), t) \\ \mathrm{d}n(x_n(t), t)/\mathrm{d}t &= f_n(n, p, n_{t1}, n_{t2}) \end{aligned} \tag{35}$$

as well as

$$\begin{aligned} \mathrm{d}x_p/\mathrm{d}t &= -\mu_p E(x_p(t), t) \\ \mathrm{d}p(x_p(t), t)/\mathrm{d}t &= f_p(n, p, n_{t1}, n_{t2}) \end{aligned} \tag{36}$$

where f_n and f_p are given functions. Taking into account the boundary parameters $x_n(\lambda) = L$ and $x_p(\lambda) = 0$, where λ is the initial time and $\lambda \geqslant 0$, we ascertain that double injection can occur when $E(0, t) > 0$ and $E(L, t) > 0$.

With this assumption, the initial conditions $p(x_p(0), 0)$, $n(x_n(0), 0)$, $n_{t1}(x, 0)$, $n_{t2}(x, 0)$ and the boundary conditions $p(0, t)$ and $n(L, t)$ determine the transient state for double injection. The fundamental problem of our considerations is to find the physical aspect for the boundary values of $p(0, t)$ and $n(L, t)$. In this paper we assumed that the convection current $J(x, t)$ and the electric field intensity are continuous at the plane $x = 0$ and $x = L$. From the field theory it follows that this assumption is equivalent to $q_s(t)|_{x=L}^{x=0} \equiv 0$ where q_s is the surface charge at $x = 0$ and $x = L$. With this assumption we can write the following boundary conditions:

$$\begin{aligned} J(0, t) &= q[\mu_n n(0, t) + \mu_p p(0, t)]E(0, t) = f_0[E(0, t)] \\ J(L, t) &= q[\mu_n n(L, t) + \mu_p p(L, t)]E(L, t) = f_L[E(L, t)] \end{aligned} \tag{37}$$

as well as

$$\begin{aligned} &\varepsilon E(x, t) = \int_0^x q_v(x, t)\,\mathrm{d}x + \varepsilon E(0, t) \\ &\varepsilon L E(0, t) = \varepsilon V - \int_0^L (L - x) q_v(x, t)\,\mathrm{d}x \qquad q_v = q(p - n - n_{t1} - n_{t2}). \end{aligned} \tag{38}$$

The physical importance is $E(x, t) > 0$ for $x \in \langle 0, L\rangle$ and $t \geqslant 0$. Thus, this condition defines a set of $\{V, f_0, f_L\}$. For such a set our considerations are valid. From (37) it follows that the singular solutions (24*b*) can be determined by

$$\begin{aligned} p(0, t) &= \frac{\mu_n}{q(\mu_n + \mu_p)}\left(\frac{f_L[E(L, t)]}{\mu_n E(0, t)} + \varepsilon(\alpha_2 + z_2)\right) \\ n(L, t) &= \frac{\mu_n}{q(\mu_n + \mu_p)}\left(\frac{f_L[E(L, t)]}{\mu_n E(L, t)} - \frac{\varepsilon\mu_p}{\mu_n}(\alpha_2 + z_2)\right) \end{aligned} \tag{39}$$

or by

$$p(0,t) = \frac{\mu_n}{q(\mu_n+\mu_p)}\left(\frac{f_0[E(0,t)]}{\mu_n E(0,t)} + \varepsilon(\alpha_2+z_1)\right)$$
$$n(L,t) = \frac{\mu_n}{q(\mu_n+\mu_p)}\left(\frac{f_0[E(0,t)]}{\mu_n E(L,t)} - \frac{\varepsilon\mu_p}{\mu_n}(\alpha_2+z_1)\right). \tag{40}$$

Therefore, we see that singular solutions and current–voltage characteristics (24*f*) and (24*g*) can exist. Analogously, from (37) it follows that the solution (34) is determined by the condition

$$n(L,t) = f_L[E(L,t)]/[q\mu_n(1+\rho\theta)E(L,t)].$$

In general, the main idea of the concept of space charge is to identify the internal and boundary processes that occur in the metal–solid–metal system. Usually, this problem is solved by the following electric field conditions [21–38]:

(i) the transient state of the discharging capacitor, characterized by

$$\int_0^L E(x,t)\,dx = 0 \qquad \text{for } t \geqslant 0 \tag{41}$$

(ii) the transient state or the steady state of the charging capacitor, described by

$$\int_0^L E(x,t)\,dx = V(t) \not\equiv 0 \qquad \text{for } t \geqslant 0 \tag{42}$$

where the voltage function is usually of the form $V(t)$ = constant or $V(t) = V_0\sin(\omega t)$, and the parameters V_0 and ω are given; and

(iii) the open system, in which the total current density $J_t(t)$ is

$$J_t(t) = \varepsilon\partial E(x,t)/\partial t + J_c(x,t) \equiv 0 \tag{43}$$

where J_c is the convection current.

In the above, by making use of the current–voltage characteristic $J(V)$, we have identified the interior and the boundaries together.

The interior is described by electron transitions between the lower and higher energy levels (figure 8). Electron passage between the ith (lower) and jth (higher) trapping levels is characterized by the coefficients c_{ji} and c_{ij}, which define the rate of change of concentration by the equation

$$\partial n_{ti}/\partial t = c_{ji}n_{tj}N_{ti} - c_{ij}n_{ti}N_{tj}$$

where $i, j = 1, 2, \ldots, m$, and m is the number of trapping levels. The coefficients $C_{ij} = c_{ij}N_{tj}$ and c_{ji} have the form

$$c_{ji} = vs_i = v\pi r_{i0}^2 \qquad C_{ij} = \nu_{i0}\exp(-W_{ij}/kT) \tag{44}$$

where ν_{i0} is $\simeq 10^{12}$ s^{-1}, k is the Boltzmann constant, T the temperature, v the microscopic electron velocity, s_i the recombination cross section, r_{i0} the radius and $W_{ij} = W_j - W_i$ (figure 8). The radius r_{i0} is determined by

$$Z_i q^2/(4\pi\varepsilon r_{i0}) = kT = \tfrac{1}{2}m_e v^2 \tag{45}$$

where Z_i is the atomic number and m_e is the electron mass. The condition (45) defines the zero value of the total energy of an electron. In the case when the radius $r_i \leqslant r_{i0}$ the total energy is negative. The space-charge density determined by electron passage between the energy states is of the form

$$\varepsilon\frac{\partial E}{\partial x} = q\left((p - p_0) - (n - n_0) - \sum_{i=1}^{m}(n_{ti} - n_{ti,0})\right)$$

and

$$p_0 = n_0 + \sum_{i=1}^{m} n_{ti,0}$$

where p_0, n_0 and $n_{ti,0}$ are the equilibrium concentrations characterizing a solid at the temperature $T > 0$ K (the case of neutral traps). The frequency parameters ν_n and ν_p are determined by the Boltzmann factor in the form

$$\nu_n = \nu_{n0}\exp(-W_n/kT) \qquad \nu_p = \nu_{p0}(-W_p/kT) \tag{46}$$

where ν_{n0} and ν_{p0} are $\simeq 10^{12}$ s^{-1} and W_n and W_p are the electron and hole activation energies (figure 8), respectively. The parameter C_{21} is defined by the mean value of $\langle C_{ij}\rangle$.

The recombination parameters are of the form

$$C_p = vs_p = v\pi r_p^2 \qquad c_n = vs_n = v\pi r_n^2 \qquad c_{12} = \langle c_{ji}\rangle. \tag{47}$$

The radii r_n and r_p are determined by the condition (45). From (45) it follows that the product vs depends on the temperature in the form

$$vs = \frac{(Zq^2)^2}{4(2kT)^{3/2}\pi\varepsilon^2 m_e^{1/2}}. \tag{48}$$

Thus, under conditions of low temperature and the absence of photons, the frequency parameters ν_p, c_{21} and ν_n are very small, that is ν_p, ν_n, $C_{21} \to 0$ (this is also possible when the energies W_p, W_n and W_{ij} are sufficiently great). From (48) it follows that time constants τ_n and τ_{12} defined by

$$\tau_n = C_n^{-1} \qquad \tau_{12} = C_{12}^{-1} \tag{49}$$

are sufficiently small. For such internal processes, the space-charge transport is described by equation (10). With the assumptions $c_n = C_p$ and $C_n \leqslant C_{12}$, that is

$$Z_1 = Z_v \qquad \text{and} \qquad \tau_n \geqslant \tau_{12}$$

where Z_1 and Z_v are the atomic number in the first trapping level and in the valence level, respectively, we have analysed equation (10). We can show that the assumption (iii) of equation (10) is satisfied by (12)–(14). The shape of the $J(V)$ curve is shown in figure 12.

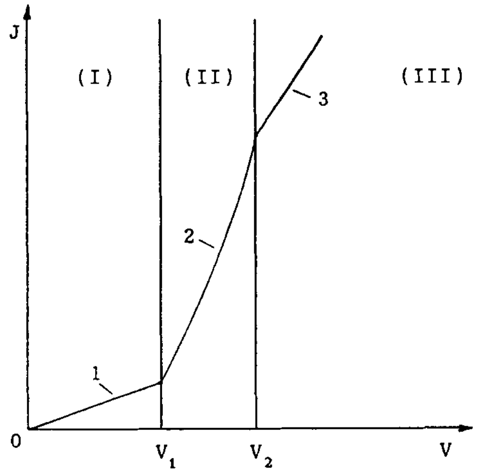

Figure 12. The shape of the current–voltage characteristic (14) and three voltage regions: (I) the low-voltage region with linear function 1; (II) the mean-voltage region with Schottky or Pool function 2; (III) the high-voltage region with quadratic function 3. The voltage values of V_1 and V_2 are determined by the continuity condition of the $J(V)$ curve. The scale is arbitrary for clarity.

This curve is obtained by experiment for insulator materials such as TiO_2, ZnS, CdS, Al_2O_3 and SiC at $T = 77$ K.

The inverse case occurs at high temperature and high-energy photon conditions. Under these conditions the recombination parameters c_n, c_{12} and C_p are sufficiently small, that is τ_n, τ_{12}, $C_p^{-1} \to \infty$, and the time constants τ_{gn}, τ_{21} and τ_p defined by

$$\tau_{gn} = \nu_n^{-1} \qquad \tau_{21} = C_{21}^{-1} \qquad \tau_p = \nu_p^{-1} \tag{50}$$

are sufficiently small (table 2). In this case the space-charge transport is described by equation (24). This equation leads to the displaced functions (24*f*) and (24*g*) (in figure 13, the shapes of these functions are shown). The displaced functions are obtained by experiment for insulator materials such as Se, Se:As, SiO_2, ZnS and CdS and for semiconductor materials such as Ge and Si. Also, equation (24) leads to the current–voltage characteristic (29). An example of the function (29) with the quadratic boundary function $J \sim E^2(L)$ is shown in figure 14. This function is typical for SiC.

In the case when $W_{i,i+1} > 2$ eV and W_n and W_p are sufficiently small (the case of pure insulators, which are placed in table 1), the space-charge transport is described by equation (31). This electric field intensity distribution occurs in typical insulators such as anthracene, TiO_2, Al_2O_3, CdS, ZnS, ZnO, SnO_2 and ZnTe.

A particular case occurs when the potential barrier between a normal atom and the adjacent vacancy (hole) is sufficiently high. In this case when additional kinetic energy is given by an external electric field to the valence electron in a small portion, electron transition between a normal atom and the adjacent vacancy cannot occur (the case of heavy holes, that is $\mu_p = 0$). This is acceptable for insulators. The space-charge transport under conditions of heavy holes with the time constants $\tau_{12} = \tau_{21}$ is described by (19). This equation leads to Child's law (23*c*), which is obtained by experiment for insulator materials such as anthracene, TiO_2, ZnS, CdS, ZnTe and Al_2O_3.

Table 2. The generation time constant of valence electrons at room temperature, $\tau_p = 10^{-12}\exp(W_p/kT)$, $kT = \frac{1}{40}$ eV.

$W_p = W_2 - W_v$ (eV)	τ_p
1.61	0.3×10^9 years
1.09	$\simeq$ 110 days
0.98	1 day
0.92	2.4 hours
0.81	1.5 min
0.69	1 s
0.58	10^{-2} s
0.52	10^{-3} s
0.40	10^{-5} s
0.29	10^{-7} s
0.18	10^{-9} s
0.12	10^{-10} s

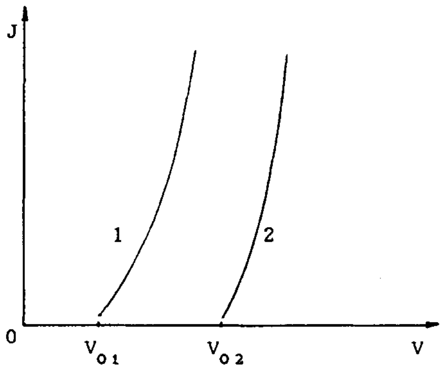

Figure 13. The displaced current–voltage characteristics (24*f*) and (24*g*): 1, the function $J = J(V - V_{01})$ determined by the positive space-charge density distribution $^+q_v(x) \equiv 2\varepsilon V_{01}/L^2$; 2, the function $J = J(V - V_{02})$ determined by the negative space-charge density distribution $^-q_v(x) \equiv -2\varepsilon V_{02}/L^2$. The scale is arbitrary for clarity.

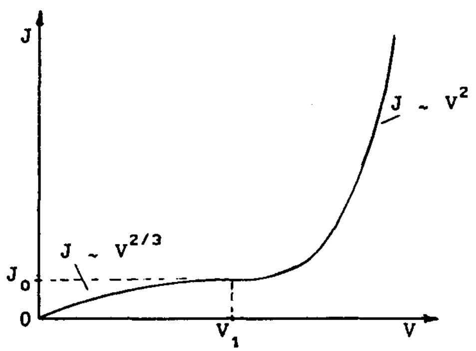

Figure 14. The shape of the curve (29) determined by the quadratic boundary function.

The linear function $J = \sigma V/L$ with the conductivity parameter (22) defines the ohmic conduction $\sigma_\Omega = q\mu_n n_0$, where the equilibrium concentration n_0 is $n_0 = p_0 - n_{t1,0} - n_{t2,0}$.

Now let us discuss the mechanism of carrier injection from the electrode into the bulk. As an example, let us take into account the mechanism of hole injection from the anode into the bulk.

Investigating the point/plane system with the $(+)/(-)$ and $(-)/(+)$ polarity electrodes, Kao [10] showed that the mechanism of hole injection from the anode into the bulk corresponds to the mechanism of electron injection from the cathode into the bulk. On this basis we can assume that the contact surfaces act as a system of points (figure 15). These surface structures have been verified by many thermal methods [29]. Thus, with this assumption, at the contact $x = 0$ the valence electrons are pulled away by the external field from the normal atoms. Next, on the points these electrons are accumulated. Under these conditions, electron emission from the points of the bulk into the anode occurs. On this basis, we can ascertain that the boundary function $J = f_0[E(0)]$ exists.

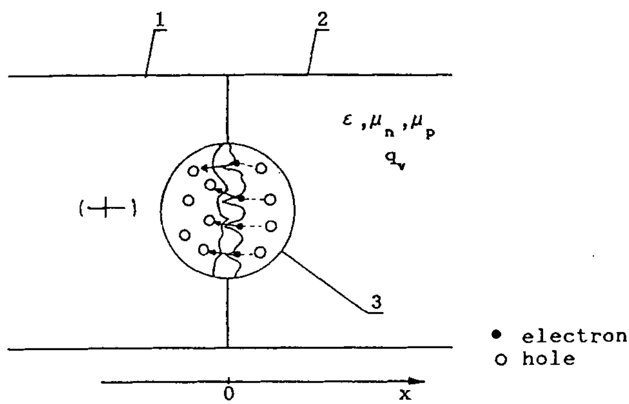

Figure 15. The ionization of atoms in the bulk at the anode: 1, the anode; 2, the bulk; 3, the microscopic structure of the metal–bulk interface. Electron emission from the bulk surface is indicated by the arrows.

5. Conclusions

In our work we have presented some results of the analysis of bipolar conduction in a metal–solid (semiconductor, insulator)–metal system in the steady state. We have characterized the generation and recombination processes. From these considerations it follows that the generation processes determine the electric field distribution in the form $E^2(x) = \alpha x^2 + ax + b$ where α is the material constant, a and b are constants of integration. In this case the symmetric injection of carriers is possible (the boundary functions f_L and f_0 are identical). Under these conditions, the function $E(x)$ is decreasing in the region $x \in \langle 0; L/2)$ and increasing in $x \in (L/2; L\rangle$, that is a negative and positive space charge occurs in the bulk. Also, the function $E(x)$ can be linear (the singular solutions (24*b*)). The carrier generation determines the $J(V)$ curve described by (27)–(28) as well as by (24*f*) or (24*g*). From (24*a*) and (35)–(38) it follows that there can be $x_0 \in (0; L)$. On this basis, we ascertain that the $J(V)$ curve can be discontinuous. The generation–recombination processes can determine the function $E(x)$ in the form (13), (20) and (32). From (20) and (20*a*) it follows that the function $E(x)$ can become uniform when the boundary function f_L is linear. For (13), (20) and (32) the $J(V)$ curve can be written as (14), (21), (23*c*), (33) or (34*a*). From (21) and (33*a*) it follows that the $J(V)$ curve can also be discontinuous.

Acknowledgments

This problem was sponsored and realized by Grant of K.B.N of The Polish Government.

References

[1] Lampert M A and Mark P 1970 *Current Injection in Solids* (New York: Academic)
[2] Rosenberg L M and Lampert M A 1970 *J. Appl. Phys.* **41** 508
[3] Waxman A and Lampert M A 1970 *Phys. Rev.* B **1** 2735
[4] Schwob H P and Williams D F 1974 *J. Appl. Phys.* **45** 2638
[5] Manifacier J C and Henish H K 1978 *Phys. Rev.* B **17** 2640
[6] Manifacier J C and Henish H K 1978 *Phys. Rev.* B **17** 2648
[7] Simmons J G and Taylor G W 1971 *Phys. Rev.* B **4** 502
[8] Simmons J G and Tam M C 1973 *Phys. Rev.* B **7** 3706
[9] Simmons J G 1972 *J. Non-Cryst. Solids* **8–10** 947
[10] Kao K C 1984 *J. Appl. Phys.* **55** 752
[11] Kao K C 1984 *J. Phys. D: Appl. Phys.* **17** 1433
[12] Kao K C 1984 *J. Phys. D: Appl. Phys.* **17** 1449
[13] Batra I P, Tekman E and Ciraci S 1991 *Prog. Surf. Sci.* **36** 289
[14] Matsuura H and Okushi H 1987 *J. Appl. Phys.* **62** 2871
[15] Stec Cz 1979 *J. Electrostat.* **8** 103
[16] Skopec A 1979 *J. Electrostat.* **8** 13
[17] Skopec A and Smycz E 1979 *J. Electrostat.* **8** 29
[18] Bhattacharya T 1992 *IMA J. Appl. Math.* **48** 117
[19] Budd C J, Friedman A, McLeod B and Wheeler A A 1990 *SIAM J. Appl. Math.* **50** 181
[20] Cimatti G 1990 *IMA J. Appl. Math.* **44** 185
[21] Many A and Rakavy G 1962 *Phys. Rev.* **126** 1980
[22] Batra I P, Kanazawa K K and Seki H 1970 *J. Appl. Phys.* **41** 3416
[23] Shatzkez M, Av-Ron M and Anderson R M 1974 *J. Appl. Phys.* **45** 2065
[24] Andress B, Fischer P and Rohl P 1977 *Prog. Colloid. Polym. Sci.* **62** 141
[25] Kao K C and Hwang W 1981 *Electrical Transport in Solids* (Oxford: Pergamon)
[26] Gerhard-Multhaupt R 1983 *Phys. Rev.* B **27** 2494
[27] Smycz E 1984 *J. Electrostat.* **15** 173
[28] Simon J and Andre J-J 1985 *Molecular Semiconductors. Photoelectrical Properties and Solar Cells* (Berlin: Springer)
[29] Sankaraman N, Niederman P H, Noer R J and Fischer O 1986 *J. Physique* **47** C7 Suppl. 133
[30] Leviner J 1986 *IEEE Trans. Electr. Insul.* **EI-21** 351
[31] Lang S B and Das-Gupta D K 1986 *J. Appl. Phys.* **59** 2151
[32] Lang S B and Das-Gupta D K 1986 *IEEE Trans. Electr. Insul.* **EI-21** 399
[33] von Seggern H 1986 *IEEE Trans. Electr. Insul.* **EI-21** 281
[34] Giacometti J A, Malmonge J A and Neto J M G 1987 *IEEE Trans. Electr. Insul.* **EI-21** 386
[35] Wintle H J 1987 *J. Appl. Phys.* **62** 2933
[36] Zahn M 1987 *IEEE Trans. Electr. Insul.* **EI-22** 181
[37] Mazurek B and Cross J D 1988 *J. Appl. Phys.* **63** 4899
[38] Conference, Technical University of Wrocław, Poland, 1990 *Mater. Sci.* **16** No 1–3