BULLETIN OF THE POLISH
ACADEMY OF SCIENCES
TECHNICAL SCIENCES
VOL. 36, No. 1–2, 1988

VARIA

Relations $n(p)$ Between Positive and Negative Charge Carrier Concentrations in Conditions of Bimolecular Recombination

by

Bronisław ŚWISTACZ

Presented by L. BADIAN on March 4, 1987

Summary. A model of dielectric with positive and negative charge carriers is presented. The trapped charge is omitted. Electric conductivity is analysed in steady state at constant voltage. It is demonstrated that in conditions of external electric field the carrier concentrations may satisfy the law of mass action $np = \text{const}$.

1. Introduction. Two basic equations of electric charge transport are used in analyses of electric conductivity in a solid dielectric, namely

– the Gauss equation

$$\frac{\partial}{\partial x}(\varepsilon E) = e_0[(p-p_0)-(n-n_0)] \tag{1}$$

– the continuity equation

$$\frac{\partial}{\partial x}(\mu_p pE + \mu_n nE] + \frac{\partial p}{\partial t} - \frac{\partial n}{\partial t} = 0 \tag{2}$$

where $e_0 = 1.6 \cdot 10^{-19}$C, ε – high-frequency electric permeability; μ_p and μ_n – mobility of positive and negative charge carriers, respectively; E – electric field intensity, x – distance of the point from the electrode, t – time; p and n – nonequilibrium concentrations of positive and negative charge carriers, respectively; p_0 and n_0 – equilibrium concentrations of positive and negative charge carriers, respectively.

Equations (1) and (2) describe charge transport in a plane condenser system. A full description of electric conductivity requires an additional equation describing processes inside the dielectric. This may be written as [3]

$$F\left(\frac{\partial n}{\partial t}; \frac{\partial n}{\partial x}; \frac{\partial E}{\partial x}; E; n; p\right) = 0. \tag{3}$$

The form of the F function depends on processes occurring in the dielectric during electric charge transport. In equations (1)–(3) diffusion and trapping processes are omitted ($n_0 = p_0$).

In this work we consider the case of the F function describing the law of bimolecular carrier recombination with the trapping process omitted [1, 3]

$$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}(\mu_n nE) + \beta(np - n_0^2) = 0 \tag{4}$$

(where β is the recombination coefficient). We intend to determine the $n(p)$ relations in steady state at constant voltage

$$\int_0^L E dx = U; \qquad U = \text{const} \tag{5}$$

where L — distance between electrodes, U — voltage applied to the electrodes. We will also determine some current-voltage dependences in conditions in which $n(p)$ dependences between carrier concentrations exist.

2. Problem solution. The steady state equation is obtained from equations (1)–(4) assuming $\partial/\partial t = 0$. After omitting the trapping process ($n_0 = p_0$), the steady state of electric conductivity in the dielectric is described by the following equations

$$\frac{d}{dx}(\varepsilon E) = e_0(p-n) \tag{6}$$

$$j = e_0 \mu_n nE + e_0 \mu_p pE; \quad j = \text{const} \tag{7}$$

$$\frac{d(\mu_n nE)}{dx} - \beta(np - n_0^2) = 0 \tag{8}$$

where j is the electric field density.

The electric field intensity $E(x)$ satisfies the voltage condition (5). As announced in the Introduction, we shall determine the $n(p)$ relations together with some current-voltage dependences $j = j(U)$ or $U = u(j)$ corresponding to them. To do this we determine the value of concentration p from (6)

$$p = \frac{\varepsilon}{e_0}\frac{dE}{dx} + n \tag{9}$$

and substitute in (7)

$$j = e_0 \mu_n nE + e_0 \mu_p E\left(\frac{\varepsilon}{e_0}\frac{dE}{dx} + n\right). \tag{10}$$

From this we get the value of concentration n

$$n = \frac{j}{e_0(\mu_n+\mu_p)E} - \frac{\varepsilon\mu_p}{e_0(\mu_n+\mu_p)}\frac{dE}{dx}. \tag{11}$$

Taking into consideration relations (9) and (11) we get the value of concentration p

$$p = \frac{j}{e_0(\mu_n+\mu_p)E} + \frac{\varepsilon\mu_n}{e_0(\mu_n+\mu_p)}\frac{dE}{dx}. \tag{12}$$

From (11) and (12) we determine the various terms in equation (8)

$$\frac{d}{dx}(\mu_n nE) = -\frac{\varepsilon\mu_n\mu_p}{e_0(\mu_n+\mu_p)}\frac{d}{dx}\left(E\frac{dE}{dx}\right) \tag{13}$$

$$np = \left(\frac{j}{e_0(\mu_n+\mu_p)E}\right)^2\left(1+\frac{\varepsilon\mu_n E}{j}\frac{dE}{dx}\right)\left(1-\frac{\varepsilon\mu_p E}{j}\frac{dE}{dx}\right). \tag{14}$$

Hence, basing on (8), we obtain the following differential equation describing the distribution of electric field intensity $E(x)$

$$-\frac{\varepsilon\mu_n\mu_p}{e_0(\mu_n+\mu_p)}\frac{d}{dx}\left(E\frac{dE}{dx}\right) = \frac{\beta j^2}{e_0^2(\mu_n+\mu_p)^2E^2}\left[\left(1+\frac{\varepsilon\mu_n E}{j}\frac{dE}{dx}\right)\cdot\left(1-\frac{\varepsilon\mu_p E}{j}\frac{dE}{dx}\right) - \frac{e_0^2(\mu_n+\mu_p)^2 n_0^2 E^2}{j^2}\right]. \tag{15}$$

Equation (15) is easily solved in the case of identical carrier mobilities. In the further considerations we assume identical carrier mobilities $\mu_n = \mu_0 = \mu$. In this case we make the following substitutions

$$y = E^2; \quad \frac{dy}{dx} = w(y); \quad \frac{d^2y}{dx^2} = w\frac{dw}{dy} \tag{16}$$

and thus equation (15) takes the form of the Bernoulli equation

$$-w\frac{dw}{dy} = \frac{\beta j^2}{e_0\varepsilon\mu^3 y}\left[1-\left(\frac{\varepsilon\mu}{2j}w\right)^2 - \frac{(2e_0\mu n_0)^2}{j^2}y\right]. \tag{17}$$

After substituting

$$z = 1-\left(\frac{\varepsilon\mu}{2j}w\right)^2 \tag{18}$$

we get the linear equation

$$\frac{dz}{dy} - \frac{\varepsilon\beta z}{2e_0\mu y} = -\frac{2\varepsilon\beta\mu e_0 n_0^2}{j^2} \tag{19}$$

with the general integral of the form

$$Z = C_1 y^{\chi} - \frac{2\varepsilon\beta\mu e_0 n_0^2}{j^2(1-\chi)}y; \quad \chi = \frac{\varepsilon\beta}{2e_0\mu} \tag{20}$$

(where C_1 is the integration constant). In the case when parameter $\chi = 1$, i.e. when $\beta = \frac{2e_0\mu}{\varepsilon}$, the solution (20) must be replaced by

$$Z = C_1 y - \left(\frac{2e_0\mu n_0}{j}\right)^2 y \ln y; \quad \chi = 1. \tag{21}$$

It is worth noting that the carrier concentration product may be expressed with variables z and y

$$np = \frac{j^2}{4e_0^2\mu^2 y}\left[1-\left(\frac{\varepsilon\mu w}{2j}\right)^2\right] = \frac{j^2 z}{4e_0^2\mu^2 y}. \tag{22}$$

Hence, basing on (20), we get the equation

$$np\frac{4e_0^2\mu^2 y}{j^2} - \left(\frac{2e_0\mu n_0}{j}\right)^2 \frac{\chi}{(\chi-1)} y = C_1 y^\chi \tag{23}$$

or its equivalent form

$$np - \frac{\chi n_0^2}{\chi-1} = C_1' y^{\chi-1} = C_1' E^{2\chi-2}; \quad C_1' = \frac{C_1 j^2}{(2e_0\mu)^2} \tag{24}$$

where C_1' is a new, suitably selected integration constant. Equation (24) is the sought dependence between n, p and E which may be written in the implicit form

$$f(n, p, E) = 0 \tag{25}$$

with function f characterizing both the interior and the edges of the dielectric. A particular case of relations (21) (24) and (25) are $n(p)$ dependences. In the case of the ideal dielectric $n_0 = 0$ when parameter $\chi = 1$, we get

$$n(x)p(x) = n(0)p(0) = \text{const} \tag{26}$$

whereas when the integration constant $C_1 = 0$, we get the second $n(p)$ dependence

$$n(x)p(x) = \frac{\chi n_0^2}{\chi-1} = \text{const} \tag{27}$$

which for $\chi \gg 1$ becomes the law of mass action

$$n(x)p(x) = n_0^2. \tag{28}$$

Dependence (28) was the object of resorption state analysis in [2].

We now proceed to some of the current-voltage dependences corresponding to relations (26)–(28). In the case of relation (27), the integration constant C_1 in equation (20) is zero. Taking into consideration substitutions (18) and (16) we get the differential equation describing the distribution of electric field intensity $E(x)$

$$\frac{\varepsilon\mu E}{j}\frac{dE}{dx} = \left[1-\left(\frac{2e_0\mu n_0}{j}\right)^2 \frac{\chi E^2}{\chi-1}\right]^{1/2}; \quad \chi > 1 \tag{29}$$

having the general integral of the form

$$E = \frac{j}{\sigma}\left(\frac{\chi}{\chi-1}\right)^{-1/2}\left\{1-\left[\frac{\chi\sigma^2(x+C_2)}{(\chi-1)\varepsilon\mu j}\right]^2\right\}^{1/2}; \quad \sigma = 2e_0\mu n_0 \tag{30}$$

where C_2 is the integration constant. The integration constant C_2 is determined on the basis of the characteristic of charge injection from the electrode into the dielectric described by the function of current density j and electric field intensity $E(0)$ next to the electrode $x = 0$ of the form $j = f_0[E(0)]$. In this research we considered the case when the edge $x = 0$ is described by the linear function

$$j = \left(\frac{\chi-1}{\chi}\right)^{-1/2} \sigma E(0).$$

The integration constant C_2 is then equal zero. Taking into consideration the voltage condition (5) we get the current-voltage dependence

$$U = \frac{j}{2\sigma}\left(\frac{\chi}{\chi-1}\right)^{-1/2}\left\{\frac{(\chi-1)\varepsilon\mu j}{\chi\sigma^2}\arcsin\left(\frac{\chi\sigma^2 L}{(\chi-1)\varepsilon\mu j}\right) + L\left[1-\left(\frac{\chi\sigma^2 L}{(\chi-1)\varepsilon\mu j}\right)^2\right]^{1/2}\right\}. \tag{31}$$

If the value of current density satisfies the condition

$$j \gg \frac{\chi\sigma^2 L}{(\chi-1)\varepsilon\mu}$$

the dependence (31) takes the simple form

$$j = \left(\frac{\chi-1}{\chi}\right)^{-1/2} \sigma\frac{U}{L}. \tag{32}$$

When parameter $\chi \gg 1$, the linear dependence (32) takes the form of the Ohm law $j = \sigma U/L$.

When relation (26) is satisfied, the electric field intensity satisfies the equation

$$E\frac{dE}{dx} = \frac{1}{\varepsilon\mu}(j^2 - C_3 E^2)^{1/2}; \quad C_3 = j^2 C_1 \tag{33}$$

which has the general solution of the form

$$E = \frac{1}{\sqrt{C_3}}\sqrt{j^2 - \left(\frac{C_3}{\varepsilon\mu}\right)^2 (x+C_4)^2} \tag{34}$$

where C_3 and C_4 are integration constants.

The integration constants are determined on the basic of characteristics of charge injection from the electrode $x = 0$ into the dielectric, $j = f_0[E(0)]$, and the electrode $x = L$, $j = f_L[E(L)]$. An interesting case is when the boundary values $n(0)$ and $p(0)$ are equal, $n(0) = p(0)$. It is easy to demonstrate that the equality $n(0) = p(0)$ holds when the boundary value $n(L)$ is described by the relations [3]

(35) $$n(L) = \frac{f_L[E(L)] - e_0 \mu p(L) E(L)}{e_0 \mu E(L)}$$

(36) $$E(L) = \frac{E(0)}{\varepsilon\mu}\left\{1 - \left(\frac{jL}{\varepsilon\mu E^2_{(0)}}\right)^2\right\}^{1/2}.$$

In such a case, the integration constants C_3 and C_4 take the values

(37) $$C_3 = \left(\frac{j}{E(0)}\right)^2, \quad C_4 = 0.$$

Hence, basing on (34) and (5), we get the current-voltage dependence in parametric form

(38) $$U = \frac{\varepsilon\mu E^3_{(0)}}{2j}\left\{\arcsin\left(\frac{jL}{\varepsilon\mu E^2_{(0)}}\right) + \frac{L}{\varepsilon\mu E^2_{(0)}}\sqrt{j^2 - \left(\frac{j^2 L}{\varepsilon\mu E^2_{(0)}}\right)^2}\right\} \quad j = f_0[E_{(0)}].$$

In the case when the boundary function f_0 is of the form

$$j = \frac{\varepsilon u}{L} E^2_{(0)}$$

the dependence (38) takes the form of Child's law

(39) $$j = \frac{16}{\pi^2}\varepsilon\mu\frac{U^2}{L^3}.$$

It results from equations (35) and (36) that the boundary value $n(L)$ is infinitely large, $n(L) = \infty$, when the function f_L is of the form

$$f_L[E(L)] = \sum_{d=0}^{n} a_\alpha E^\alpha(L) \qquad a_\alpha = \text{const},\ a_0 \neq 0.$$

3. Conclusions. The problem of the np product is a long-standing object of many and often controversial considerations and conclusions. Some authors questioned the constancy of this product, while others, admitting the possibility of its variability, do not give values of this product in conditions of space charge existence.

The objective of this research, summed up in the following conclusions resulting therefrom, was on the one hand to demonstrate that the np product may equal n_0^2 and on the other – to formulate conditions for and to give values of this product in other admissible conditions of carrier transport. An example of similar research is the monograph by Aderovitch [1] who allowed a number of possible recombination mechanisms.

According to our analysis, the carrier concentrations n and p may satisfy the law of mass action $np = \text{const}$,

– In the case of the ideal dielectric ($n_0 = p_0 = 0$) the value of the carrier concentration product depends on mechanisms of carrier injection from the electrodes into the dielectric. When such injection conditions are satisfied that the boundary conditions $n(0)$ and $p(0)$ are equal, $n(0) = p(0)$, and the boundary function f_0 is of the form $f_0 = aE^2_{(0)}$ ($a = \text{const}$), the current-voltage dependence is a quadratic function $j \sim U^2$; in particular, when parameter $a = \varepsilon\mu/L$, the quadratic dependence takes the form of Child's law, $j \sim U^2/L^3$.

– In the case of a nonideal dielectric, the value of the np product depends only on the internal parameters of the dielectric such as β, ε, μ, n_0. If the electrode $x = 0$ is described by the boundary function

$$f_0 = \left(\frac{\chi - 1}{\chi}\right)^{-1/2} \sigma E(0)$$

the current-voltage dependence $U = U(j)$ is a nonlinear functions. Function $U = U(j)$ becomes linear when current density satisfies the condition

$$j \gg \frac{\chi\sigma^2 L}{(\chi - 1)\varepsilon\mu}.$$

Acknowledgement. The present research was commissioned by the Department of Fundamental Electrotechnological Research of the Ministry of Metallurgy and Machine Industry and the Polish Academy of Sciences, and carried out at the Institute of Electrotechnology in Warsaw under the guidance of Professor L. Badian. The author expresses his warm thanks to Professor Badian for his numerous and valuable suggestions during the preparation of the manuscript.

INSTITUTE OF FUNDAMENTAL ELECTRONICS AND ELECTROTECHNOLOGY, TECHNICAL UNIVERSITY, PL. GRUNWALDZKI 13, 50 372 WROCŁAW
(INSTYTUT PODSTAW ELEKTROTECHNIKI I ELEKTROTECHNOLOGII, POLITECHNIKA WROCŁAWSKA

REFERENCES

[1] E. I. Aderovitch, *Toki dvoynoy inyekcyi v poluprovodnikach*, Moskwa, 1976.

[2] B. Świstacz, *Bipolar problem of space-charge in dielectric, I. System without trapping, II. Systems with trapping*, Bull. Pol. Ac.: Tech., **32** (1984) 427–455.

[3] B. Świstacz, *Transient states of electric conduction in solid dielectric including carrier recombination, I. Ideal dielectric without traps, II. Ideal dielectric with deep traps*, Bull. Pol. Ac.: Tech., **33** (1985) 587–603.

Б. Сьвистач, **Соединения $n(p)$ между концентрациями носителей положительного и отрицательного зарядов в условиях бимолекулярной рекомбинации**

В настоящей работе приводится модель диэлектрика с положительными и отрицательными носителями заряда. Не учитивался локализированный заряд. Проводился анализ электропроводности в состоянии, устоновленном при постоянном напряжении. В статье доказывается, что в условиях внешнего электрического поля концентрации поситeлей могут соответствовать закону действующих масс.