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Some further description for a current flow trough an amorphous solid. A case of an imperfect contact.

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Including a secondary electron emission for the cathode in a double injection problem, a new mathematical model is presented. In a planar capacitor system the carrier mobilities are discontinuous. The metal-solid-metal system can act as an n-n or p-n blocking diode. A new boundary condition for the "V" type current-voltage characteristic is determined.

Keywords: space charge transport, carrier injection, solid, trapping levels

1. INTRODUCTION

One of the purposes of space charge theory is to find the electrical field distribution in the metal-solid-metal system [1–3]. Using allowed electron transition concept, the current flow between the two electrodes can be determined. This property is expressed by a current-voltage characteristic. Generally, in order to find this function, it is necessary to give the boundary conditions describing carrier injection from the metal into a solid. In this paper, that concept will be continued.

The purpose of this work is to develop a space-charge problem and to find new internal and boundary conditions corresponding to the electric field distribution of a voltage or current stabiliser.

2. THE MATHEMATICAL MODEL

In this paper, we will consider a solid in which the trapping levels will be grouped into the four permissible energy levels [4–8]. With this assumption, the so-called effective parameters such as the frequency parameters c_{21} and c_{t12} as well as the recombination parameters c_{12} and c_{t21} will be used. For the trapped electrons, the concentrations of traps in the first and second trapping level will be represented by N_{t1} and N_{t2}, respectively. Analogously, for the trapped holes, the concentrations of traps in the first and second trapping level will be equal to P_{t1} and P_{t2}, respectively. The system of atoms will be treated as an unlimited reservoir of traps, that is $P_{t1} >> p_{t1}$; $P_{t2} >> p_{t2}$; $N_{t1} >> n_{t1}$ and $N_{t2} >> n_{t2}$. The metal-solid-metal system will be represented by a planar capacitor system with the anode $x = 0$ and the cathode $x = L$. Also, L denotes the distance between the electrodes. Moreover, we will assume that the diffusion current is negligible [9, 10]. For a solid, we will assume that the polarisation effect is characterised by the dielectric constant ε. Additionally, the mobilities μ_n and μ_p of free electrons and holes (respectively) are independent of the electric field intensity E. For the $x = L$ contact, we will consider a special case of electron emission from the cathode into a solid. Here, for the bulk plane and for the metal plane, we will assume that the contact acts as a system of points placed in a vacuum. For such a contact structure, a secondary electron emission occurs [11–13]. This property will be represented by a vacuum capacitor with the contact voltage V_2 and a distance parameter denoted by $L - d$. In our considerations, for the planar capacitor system, the basic equations such as the Gauss equation; the continuity equation; the generation-recombination equations and the field integral will be used. On this basis, the space charge transport through the bulk is described by

$$\frac{\varepsilon}{q}\frac{\partial E(x,t)}{\partial x} = p(x,t) + p_{t1}(x,t) + p_{t2}(x,t) - (n(x,t) + n_{t1}(x,t) + n_{t2}(x,t)) \tag{1}$$

$$\frac{\partial}{\partial x}\left\{\left[\mu_n n(x,t) + \mu_p p(x,t) + \mu_{t1} n_{t1}(x,t) + \mu_{t2} n_{t2}(x,t)\right] E(x,t)\right\} + \frac{\partial p(x,t)}{\partial t} + \frac{\partial p_{t1}(x,t)}{\partial t} + \frac{\partial p_{t2}(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0 \tag{2}$$

$$\frac{\partial n(x,t)}{\partial t} = v_{t1}P_{t1} + v_{n1}n_{t1}(x,t) - c_n N_{t1} n(x,t) - C_t n(x,t) p_{t1}(x,t) + \frac{\partial}{\partial x}\left[\mu_n n(x,t) E(x,t)\right] \tag{3}$$

$$\frac{\partial p_{t1}(x,t)}{\partial t} = v_{t1}P_{t1} + c_{t21}P_{t1}p_{t2}(x,t) - c_{t12}P_{t2}p_{t1}(x,t) - C_t n(x,t) p_{t1}(x,t) \tag{4}$$

$$\frac{\partial p_{t2}(x,t)}{\partial t} = c_{t12}P_{t2}p_{t1}(x,t) - c_{t21}P_{t1}p_{t2}(x,t) - v_{t2}p_{t2}(x,t) + c_{t2}P_{t2}p(x,t) \tag{5}$$

$$\frac{\partial n_{t1}(x,t)}{\partial t} = c_{21}N_{t1}n_{t2}(x,t) - v_{n1}n_{t1}(x,t) - c_{12}N_{t2}n_{t1}(x,t) + c_n N_{t1} n(x,t) + \frac{\partial}{\partial x}\left[\mu_{t1} n_{t1}(x,t) E(x,t)\right] \tag{6}$$

$$\frac{\partial n_{t2}(x,t)}{\partial t} = v_{p2}N_{t2} + c_{12}N_{t2}n_{t1}(x,t) - c_{21}N_{t1}n_{t2}(x,t) - C_p p(x,t) n_{t2}(x,t) + \frac{\partial}{\partial x}\left[\mu_{t2} n_{t2}(x,t) E(x,t)\right] \tag{7}$$

with the voltage condition for a solid

$$\int_0^d E(x,t)dx = V_1 \tag{8}$$

And also, the electron transport between the electrode and a solid is as follows

$$\varepsilon_0 \frac{\partial E_0(x,t)}{\partial x} = -q n_0(x,t) \tag{9}$$

$$\frac{\partial}{\partial x}\left[n_0(x,t)\vartheta(x,t)\right] = \frac{\partial n_0(x,t)}{\partial t} \tag{10}$$

$$-2q\int_L^x E_0(x,t)dx = m\vartheta^2(x,t) - m\vartheta^2(L,t) \tag{11}$$

$$\int_d^L E_0(x,t)dx = V_2 \tag{12}$$

Here, $q = 1.602 \cdot 10^{-19} C$, $\varepsilon_0 = 8.85 \cdot 10^{-12} F/m$; E_0 is the electric field intensity in a vacuum; m is the electron mass; x is the distance from the electrode, t is the time, ϑ is the velocity of an electron, n; n_0 and p are the free hole and electron concentrations, respectively, n_{t1}; n_{t2}; p_{t1}; p_{t2} are the concentrations of the trapped holes and electrons, respectively, μ_{t1} and μ_{t2} are the mobilities of trapped electrons, v_{p2}; v_{n1}; v_{t1}; v_{t2} denote the frequency parameters, c_n; C_p; C_t; c_{t2} denote the recombination parameters. The applied voltage V between the electrodes of a planar capacitor is

$$V = V_1 + V_2 \quad \text{and} \quad V = const. > 0 \tag{13}$$

With the above equations (1)–(13) we shall define the stationary state and we shall find different current-voltage characteristics.

3. THE STATIONARY STATE

Using condition $(\partial/\partial t) = 0$ in (1)–(12), the space charge transport through the planar capacitor system can be characterised by the electric field distributions $E(x)$ and $E_0(x)$. From (1)–(12) it follows that the stationary state is determined by the following equations

$$\frac{\varepsilon}{q}\frac{dE(x)}{dx} = p(x) + p_{t1}(x) + p_{t2}(x) - (n(x) + n_{t1}(x) + n_{t2}(x)) \tag{1a}$$

$$J = qE(x)\left[\mu_n n(x) + \mu_p p(x) + \mu_{t1} n_{t1}(x) + \mu_{t2} n_{t2}(x)\right]; \quad J = const. \tag{2a}$$

$$v_{t1}P_{t1} + v_{n1}n_{t1}(x) - c_n N_{t1} n(x) - C_t n(x) p_{t1}(x) + \frac{d}{dx}[\mu_n n(x)E(x)] = 0 \tag{3a}$$

$$v_{t1}P_{t1} + c_{t21}P_{t1}p_{t2}(x) - c_{t12}P_{t2}p_{t1}(x) - C_t n(x) p_{t1}(x) = 0 \tag{4a}$$

$$c_{t12}P_{t2}p_{t1}(x) - c_{t21}P_{t1}p_{t2}(x) - v_{t2}p_{t2}(x) + c_{t2}P_{t2}p(x) = 0 \tag{5a}$$

$$c_{21}N_{t1}n_{t2}(x) - v_{n1}n_{t1}(x) - c_{12}N_{t2}n_{t1}(x) + c_n N_{t1} n(x) + \frac{d}{dx}[\mu_{t1}n_{t1}(x)E(x)] = 0 \tag{6a}$$

$$v_{p2}N_{t2} + c_{12}N_{t2}n_{t1}(x) - c_{21}N_{t1}n_{t2}(x) - C_p p(x) n_{t2}(x) + \frac{d}{dx}[\mu_{t2}n_{t2}(x)E(x)] = 0 \tag{7a}$$

$$\int_0^d E(x)dx = V_1 \tag{8a}$$

$$\varepsilon_0 \frac{dE_0(x)}{dx} = -qn_0(x) \tag{9a}$$

$$J = qn_0(x)\vartheta(x) \tag{10a}$$

$$-2q\int_L^x E_0(x)dx = m\vartheta^2(x) - m\vartheta^2(L) \tag{11a}$$

$$\int_d^L E_0(x)dx = V_2 \tag{12a}$$

Here, J is the current density. The space charge transport through the system will be characterised by a current-voltage function in the form $J = J(V)$ or $V = V(J)$. In order to find these functions, we have to define the boundary conditions describing the mechanisms of carrier injection from the anode $x = 0$ and the cathode $x = L$ into the bulk [14]. In what follows, we will assume that the mean free path of the electron is very small and that the additional portion of the kinetic energy, which is given to the trapped electron by en external electric field, is too small. For the trapped holes and electrons, we additionally introduce into our analysis the following assumption

$$\mu_n = \mu_{t1} = \mu_{t2} = 0; \quad C_t = 0; \quad \tau_{t1}^{-1} = c_{t12}P_{t2} = c_{t21}P_{t1}; \quad \tau_1^{-1} = c_{21}N_{t1} = c_{12}N_{t2}$$

Under these conditions, the equation for the electric field distribution $E(x)$ in a solid is of the following form [15]

$$\frac{\varepsilon\mu_p E}{J}\frac{dE}{dx} = a_0 - a_1\frac{E}{J} - a_2\frac{E^2}{J^2} \tag{14}$$

where

$$a_0 = 1 + \frac{2c_{t2}P_{t2}}{v_{t2}}; \quad a_1 = \left(\frac{1}{c_n N_{t1}} + \frac{\tau_1 v_{n1}}{c_n N_{t1}} + \tau_1 - \tau_{t1} - \frac{2}{v_{t2}}\right)\cdot q\mu_p v_{t1}P_{t1};$$
$$a_2 = \left(2 + \frac{v_{n1}}{c_n N_{t1}}\right)\frac{q^2\mu_p^2}{C_p}\left(v_{p2}N_{t2} + v_{t1}P_{t1}\right) \tag{15}$$

Using a new variable $z = E/J$, (14) becomes

$$\varepsilon\mu_p J z\frac{dz}{dx} = a_0 - a_1 z - a_2 z^2; \quad z = E/J \tag{16}$$

The general integral of (16) is

$$|z - z_1|^A |z - z_2|^B = C\cdot\exp\left(-\frac{a_2 x}{\varepsilon\mu_p J}\right) \tag{17}$$

where

$$z_1 = \frac{1}{2a_2}\left(-a_1 + \left(a_1^2 + 4a_0a_2\right)^{\frac{1}{2}}\right); \quad z_2 = \frac{-1}{2a_2}\left(a_1 + \left(a_1^2 + 4a_0a_2\right)^{\frac{1}{2}}\right);$$
$$A = \frac{z_1}{z_1 - z_2}; \quad B = \frac{z_2}{z_2 - z_1} \tag{18}$$

and C is a constant of integration. The most general form of a function $V_1 = V_1(J)$ can be written as

$$V_1 = J\int_0^d z\,dx = J\int_{z(0)}^{z(d)} z\left(\frac{dx}{dz}\right)dz = \varepsilon\mu_p J^2 \int_{z(0)}^{z(d)} \frac{z^2}{a_0 - a_1 z - a_2 z^2}dz \tag{19}$$

Combining (19), we have

$$V_1 = \frac{\varepsilon\mu_p J^2}{a_2}(z(0) - z(d)) + \frac{\varepsilon\mu_p J^2}{a_2^2}\int_{z(0)}^{z(d)} \frac{a_1 z - a_0}{(z - z_1)(z - z_2)}dz \tag{20}$$

Next, combining (20) and (17), we can find another equivalent form of (20), that is

$$V_1 = \frac{\varepsilon\mu_p J}{a_2}[E(0) - E(d)] - \left(a_1 - \frac{a_0}{z_2}\right)\frac{Jd}{a_2} - \frac{\varepsilon\mu_p a_0 J^2}{z_2 a_2^2}\cdot \ln\left|\frac{E(0) - Jz_1}{E(d) - Jz_1}\right|; \quad z_2 < 0 \tag{21}$$

Now, according to (9a)–(12a), we can find the electric field distribution $E_0(x)$

$$E_0(x) = \left(\frac{6mJ^2}{q\varepsilon_0^2}(L - x) + E_0^3(L)\right)^{\frac{1}{3}} \quad \text{and} \quad E_0(\vartheta = 0) = 0 \tag{22}$$

for which the integral (12a) results in

$$V_2 = \frac{q\varepsilon_0^2}{8mJ^2}\left\{\left[E_0^3(L) + \frac{6mJ^2}{q\varepsilon_0^2}(L - d)\right]^{\frac{4}{3}} - E_0^4(L)\right\} \tag{23}$$

Thus, the current-voltage characteristic $V = V(J)$ has the following parametric form [16, 17]

$$V = V_1[J, E(0)] + V_2[J, E_0(L)] \quad \text{and} \quad J = f_0[E(0)]; \quad J = f_L[E_0(L)] \tag{24}$$

Here, the boundary functions $f_0[E(0)]$ and $f_L[E_0(0)]$ describe the mechanisms of carrier injection from the electrodes into the interior of the system. In what follows, we will consider some cases of the current flow through the system. As the first case, we will take into consideration the following internal conditions

$$E(d) = 0; \quad A = B = \frac{1}{2}; \quad z_1 = -z_2 \tag{25}$$

under of which, we get a boundary function

$$E(0) = Jz_1\sqrt{1 + \exp\left(\frac{2J_g}{J}\right)}; \quad J_g = \frac{a_2 d}{\varepsilon\mu_p} \tag{25a}$$

describing the hole injection from the anode into the bulk. In the particular case, there is

$$\text{if } \frac{J_g}{J} > 4 \text{ then } E(0) = Jz_1\exp\left(\frac{J_g}{J}\right) \text{ and } V_1 = \frac{\varepsilon\mu_p z_1}{a_2}J^2\exp\left(\frac{J_g}{J}\right) \tag{25b}$$

Next according to (23) and (24), as an example, for the linear boundary function $J = \sigma_L E_0(L)$, where σ_L is the boundary parameter, (24) is of the form

$$V = \frac{z_1 d}{J_g}J^2\exp\left(\frac{J_g}{J}\right) + \frac{q\varepsilon_0^2 J^2}{8m\sigma_L^4}\left\{\left[1 + \frac{6m(L - d)\sigma_L^3}{q\varepsilon_0^2 J}\right]^{\frac{4}{3}} - 1\right\} \tag{26}$$

Additionally, if there is $\frac{1}{4}J_g << \frac{6m(L - d)\sigma_L^3}{q\varepsilon_0^2}$ (a case of high level of electron emission from the cathode into a vacuum) then (26) is reduced to

$$V = \frac{z_1 dJ^2}{J_g}\exp\left(\frac{J_g}{J}\right) + \beta J^{\frac{2}{3}} \quad \text{and} \quad \beta = \frac{3}{4}\left(\frac{6m}{q\varepsilon_0^2}\right)^{\frac{1}{3}}(L - d)^{\frac{4}{3}} \tag{26a}$$

Another case occurs for $J >> 2J_g$, that is $E(0) = Jz_1\sqrt{2}$. Here, (26) can be replaced by

$$V = \beta_2 J^2 - \beta_1 J \tag{27}$$

where

$$\beta_2 = \frac{z_1 d}{J_g}\left(\sqrt{2} + \ln(\sqrt{2} - 1)\right); \quad \beta_1 = z_1 d - \frac{L - d}{\sigma_L} \tag{27a}$$

In (27), there can be $\beta_1 \geqslant 0$ (a case of a voltage source) or $\beta_1 < 0$ (a case of a current stabiliser). Now, let us consider a case of the high level of hole injection from the anode into the bulk, that is

$$E(0) << Jz_1; \quad A = B = \frac{1}{2}; \quad z_1 = -z_2 \tag{28}$$

Using a new parameter $w = \frac{E(d)}{Jz_1}$ for (21), we have

$$V_1 = \frac{\varepsilon\mu_p J}{a_2}[E(0) - E(d)] - \left(a_1 - \frac{a_0}{z_2}\right)\frac{dJ}{a_2} - \frac{\varepsilon\mu_p a_0 J^2}{z_2 a_2^2}\left(w + \frac{w^2}{2} + \frac{w^3}{3} + \ldots\right)$$
$$\text{for } w < 1 \text{ and } w = \sqrt{1 - \exp\left(\frac{-2J_g}{J}\right)} \tag{28a}$$

Next, for $J >> 2J_g$ and $E(0) = 0$ (a case of a pn junction), limiting the power series to the third power, (28a) becomes

$$V_1 = \frac{\varepsilon \mu_p a_0 J^2}{z_1 a_2^2} \cdot \frac{w^3}{3} \text{ and } w = \sqrt{\frac{2J_g}{J}} \tag{28ab}$$

Next, for (23), using the quadratic boundary function $J = a_L E_0^2(L)$ (here $a_L > 0$ is the boundary parameter), a function (24) takes the following form

$$V = \alpha J^{\frac{1}{2}} + \frac{q\varepsilon_0^2}{8ma_L^2}\left\{\left[1 + \frac{6ma_L^{\frac{3}{2}} J^{\frac{1}{2}}}{q\varepsilon_0^2}(L-d)\right]^{\frac{4}{3}} - 1\right\}; \quad \alpha = \frac{1}{3}\left(\frac{8a_0 d^3}{\varepsilon \mu_p}\right)^{\frac{1}{2}} \tag{29}$$

When the cathode injects an infinite amount of electrons, (29) takes the simplier form

$$V = \alpha J^{\frac{1}{2}} + \beta J^{\frac{2}{3}}; \quad \beta = \frac{3}{4}\left(\frac{6m}{q\varepsilon_0^2}\right)^{\frac{1}{3}} (L-d)^{\frac{4}{3}} \tag{29a}$$

Also, in a solid a negative space charge can be distributed when

$$E(d) >> |z_{1,2} J| \text{ and } E(0) >> |z_{1,2} J| \text{ and } E_0(x = L) = 0 \tag{30}$$

From (17) we get

$$E(d) = E(0) \cdot \exp(-J_g/J) \text{ and } J_g = \frac{a_2 d}{\varepsilon \mu_p} \tag{30a}$$

Under these conditions, according to (21)–(23), a function (24) becomes

$$V = \frac{JdE(0)}{J_g}\left(1 - \exp\left(-J_g/J\right)\right) + \beta J^{\frac{2}{3}} - \gamma J; \quad \gamma = \frac{a_1 d}{a_2} \tag{31}$$

For example, when the boundary function $J = f_0[E(0)]$ is quadratic in the form $J = \alpha_0^2 E^2(0)$, the current voltage – characteristic (31) is of the following form

$$V = \frac{d}{\alpha_0 J_g} J^{\frac{3}{2}}\left(1 - \exp(-J_g/J)\right) + \beta J^{\frac{2}{3}} - \gamma J \tag{31a}$$

Here $\alpha_0 > 0$ is the boundary parameter. If $J_g/J > 4$, then (31a) can be replaced by

$$V = \frac{d}{\alpha_0 J_g} J^{\frac{3}{2}} + \beta J^{\frac{2}{3}} - \gamma J \tag{31aa}$$

Another example, when the boundary function is linear $J = \sigma_0 E(0)$, where $\sigma_0 > 0$ denotes the boundary conductivity parameter, a function (31) has the following form

$$V = \frac{d}{\sigma_0 J_g} J^2\left(1 - \exp\left(-J_g/J\right)\right) + \beta J^{\frac{2}{3}} - \gamma J \tag{31b}$$

For $J_g/J > 4$, instead of (31b), there is

$$V = \frac{d}{\sigma_0 J_g} J^2 + \beta J^{\frac{2}{3}} - \gamma J \tag{31ba}$$

Now, still, in order to find the other case of the current flow, let us return to (17)-(18). Here, we ascertain that the electric field can be uniform

$$E(x) \equiv z_1 J \text{ and } V_1 = z_1 dJ \tag{32}$$

Next, referring to (23) and (24), for example, using the linear boundary function $J = \sigma_L E_0(L)$ or the quadratic boundary function $J = a_L E_0^2(L)$ for the cathode, we obtain (respectively)

$$V = z_1 dJ + \frac{q\varepsilon_0^2 J^2}{8m\sigma_L^4}\left\{\left[1 + \frac{6m(L-d)\sigma_L^3}{q\varepsilon_0^2 J}\right]^{\frac{4}{3}} - 1\right\} \tag{32a}$$

or

$$V = z_1 dJ + \frac{q\varepsilon_0^2}{8ma_L^2}\left\{\left[1 + \frac{6ma_L^{\frac{3}{2}} J^{\frac{1}{2}}}{q\varepsilon_0^2}(L-d)\right]^{\frac{4}{3}} - 1\right\} \tag{32b}$$

where σ_L and a_L are the boundary parameters. Generally, for (23), when the electron emission from the cathode into a vacuum is described by $1 >> \frac{6mJ^2(L-d)}{q\varepsilon_0^2 E_0^3(L)}$ (this is acceptable for the low level of electron injection), and under conditions of (32), there is

$$V = z_1 dJ + (L-d) f_L^{-1}(J) \tag{32c}$$

Here $f_L^{-1}(J)$ denotes the inverse function of the boundary function $f_L[E_0(L)]$ describing the electron emission from the metal into the vacuum. For example, in the case of linear or quadratic boundary function and of the Fowler-Nordheim function, a formula (32c) exists. The inverse case occurs when the cathode injects an infinite amount of electrons $1 << \frac{6mJ^2(L-d)}{q\varepsilon_0^2 E_0^3(L)}$ (this is acceptable for the Poole or Schottky boundary function, and also, there can be $E_0(L) = 0$). Now, (32c) is replaced by a function

$$V = z_1 dJ + \beta J^{\frac{2}{3}} \tag{32d}$$

and β is expressed by (26a).

4. DISCUSSION and CONCLUSIONS

In the above we have analysed the current flow between the two electrodes $x = 0$ and $x = L$ when the hole mobility $\mu_p = \mu_p(x)$ and the electron mobility $\mu_n = \mu_n(x)$ are discontinuous for $x = d$ and $0 \leqslant d \leqslant L$. With this assumption, the plane $x = d$ can be characterised by a surface charge density q_s in the form

$$q_S = \varepsilon_0 E_0(d) - \varepsilon E(d) \quad \text{and} \quad q_S = q_S(J) \tag{33}$$

under of these conditions, the metal-solid-metal system can act as a p-n or an n-n junction or an Schottky barrier (32)–(32d). In the case of an n-n blocking diode, which is determined by (25), the surface charge density is equal to $q_S = \varepsilon_0 E_0(d)$. Moreover, from (26)–(27a) it follows that a function (24) has the *minimum* (which can be denoted by V_{th}) and there is

$$V(J = 0+) = V(J = \infty) = \infty \tag{34}$$

Thus, the function (24) defines the "V" type curve with the negative differential resistance. This property denotes that there exists a set of the applied voltage $0 < V < V_{th}$ (a case of the voltage source) for which the current density does not determine. Here, a formula (25a), which corresponds to the given internal processes of the bulk, presents a new mechanism describing carrier injection from the metal into the bulk. The existence of the threshold voltage V_{th} can explain the switching effect for a solid [6–8, 17, 18]. Using the mechanism (25a), we ascertain that the voltage $V = V(J)$ or the current $J = J(V)$ can be stabilised. Also, for an n-n blocking diode described by (30)–(31ba), the similar property is observed. In the case of a p-n junction, the voltage $V = V(J)$ is presented by (28)–(29a), the system can act as the voltage stabiliser. Moreover, the negative differential resistance is not observed.
Now, let us compare our methodology with other ones. A regional approximation method has been presented by Lampert [6]. Here, for a p-n junction (and others), using the boundary condition $E(0) = E(L) = 0$ and quasi-neutrality assumption, the electric field intensity distribution is strongly monotonic at the electrode regions as well as this function is quasi-uniform in the bulk. On this basis, Lampert showed that the current – voltage characteristic can be shaped as $J \propto V^4$ or the "S" type curve. Investigating the current flow through the metal-imperfect insulator- metal system, the similar concept has been used by Kao [10]. He showed that the system could act as Schottky barrier. Also, using a quadratic boundary function, Kao showed that the negative differential resistance could occur. Generally, quasi – neutrality assumption is not quite mathematically clear. This condition denotes that the electric field divergence is almost equal to zero. In other words, with this assumption, the total concentration of carriers is given. In our space charge problem, this function is determined by the mechanisms of carrier injection from the two electrodes into the bulk.

5. SUMMARY

In this work we have assumed that the allowed electron transitions between the lower and the higher energy levels are stimulated only by the electric – magnetic force interactions between adjacent atoms and by incident photons. In a solid, the additional trapping levels (for positive and negative charge carriers) can be caused by impurities or by pollutants [18]. On this basis we have determined some physical interpretations. As a singular case of electric conduction in a solid, we have analysed an electric field distribution corresponding to the low level of hole injection from the anode into the bulk. Under these conditions a negative space charge with the density $q_v = \varepsilon \frac{dE}{dx} < 0$ can be distributed in the bulk, while we have assumed that the holes are mobile with the mobility μ_p. In consequence, with this assumption, we have to give the boundary condition describing the mechanism of hole injection from the anode into the bulk. From (1)–(8) it follows that this particular integral of (14) is physically acceptable when an initial space charge is negative (which must be given in order to determine the transient state), that is $q_v(x,0) = \varepsilon \frac{\partial E(x,0)}{\partial x} < 0$. Practically, using an electron radiation or an implantation method, this condition can be realised. For example, in a solid, which is implanted by the O_8^{16} and N_7^{14} atoms at the $x = d$ solid/vacuum contact, the electrons can be absorbed by these impurities (or pollutants) from the $x = d$ plane. Next, such the injected electrons can be absorbed by internal phonons (rotators and oscillators). Under these conditions an initial negative space charge can be formed in the anode-solid/vacuum-cathode system. For a solid, we can show that the negative space charge is distributed when the recombination parameter C_p satisfies the following condition [15]

$$\tau_1^{-2} \ll \frac{1}{18} v_{p2} N_{t2} C_p \quad \text{or} \quad \tau_1^{-2} \ll \frac{1}{18} v_{t1} P_{t1} C_p \tag{35}$$

From (35) it follows that the free positive space charge is neutralised by the trapped electrons (which are localised in the second trapping level) when the concentration of traps N_{t2} and the recombination parameter C_p are sufficiently great (the left formula). Also, from (35) it follows that the negative space charge is induced when the hole generation in a solid is not too strong. In other words, a negative space charge in the bulk can occur when the atomic number Z_2 corresponding to the second trapping level (for the trapped electrons) is less than the atomic number Z_v corresponding to the valence level [16], that is

$$Z_2 \ll 3 \cdot Z_v; \qquad \tau_1^{-1} \simeq 0.02 \cdot v_{p2} \tag{35a}$$

$$Z_2 \ll 2 \cdot Z_v; \qquad \tau_1^{-1} \simeq 0.028 \cdot v_{p2} \tag{35b}$$

$$Z_2 \ll 1.5 \cdot Z_v; \qquad \tau_1^{-1} \simeq 0.04 \cdot v_{p2} \tag{35c}$$

For example, if there is $Z_2 = 6; 7; 8$, then conditions (35a)–(35c) are acceptable for the material such as ZnO, CdS, GaAs, ZnS, ZnTe, Se^{79}_{34}, $BaTiO_3$, J^{127}_{58} and others. With (35a)–(35c) the negative resistance, which is expressed by (34), can occur. According to the different experiments [6–8, 17], the current-voltage characteristics, which are expressed by (24), are observed.

6. REFERENCES

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