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A bipolar space charge problem for solids including a secondary electron emission

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In this paper a new mathematical model for a space charge transport through a solid placed between the two electrodes is presented. Using new equations for allowed electron-hole transitions, the effect of the light on the electric field distribution and on the shape of current - voltage characteristic is determined. For a space charge distribution some new singular solutions are obtained. Also, some new shapes of current - voltage characteristic with negative resistance are determined. In this paper it is found that the system can act as an n-p-n or p-n or n-n blocking diode.

Keywords: double injection, current flow problem, trapping levels

1. INTRODUCTION

One of the fundamental problems of a macroscopic theory of electric conduction is to find the total concentration of charge carriers in a solid placed between the two electrodes. In this paper, we will assume that the divergence of the electric field distribution will be defined by the total concentration of carriers. Also, we will suppose that the contact processes have an influence on the shape of the electric field distribution, which corresponds to a current – voltage characteristic. Additionally, we will determine the effect of the light on the space charge density distributions.

The purpose of this work is to find a relation between the space charge distributions and a current-voltage characteristic of the metal – solid – metal system.

2. THE NEW MATHEMATICAL MODEL

In this paper, we will consider a solid in which the system of atoms is very chaotic and the different structural defects (pollutants and impurities, the Frenkl defects) and dislocations occur. This property will be characterised by the Zeemann internal effect (the splitting of the total energy of an orbital electron). For an orbital electron in the given atom, we will assume that the total energy (the sum of the positive kinetic energy and the negative potential energy of the electric field of the positive nucleus) is negative and that the zero reference level is at a finite distance from the nucleus. These properties are shown in Fig. 1 and in Fig. 2. In the case of an isolated atom (here in Fig. 1a it is symbolised by the A atom) the total energy of an orbital electron is negative for any distance from the nucleus. When an orbital electron absorbs a portion of the kinetic energy of a photon or a phonon, the total energy of the electron increases. The inverse case is when an orbital electron can lose a portion of the total energy. This is caused by the Coulomb force interaction between the orbital electron and the positive nucleus. With such the physical assumptions we will determine some internal and boundary processes (which are shown in Fig. 1-3). First let us return to Fig. 1a. Here, let us take into account the two valence electrons of the isolated atom A.

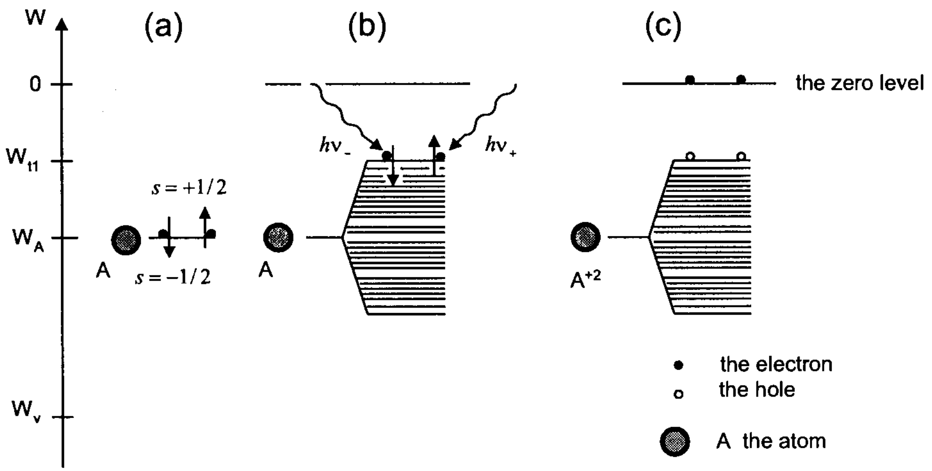

Fig. 1. The splitting of the energy state and an electron-hole generation: (a) two valence electrons of the A isolated atom; (b) the A atom is packed closely together with many other adjacent atoms and two valence electrons fill the W_{t1} higher energy level as well as these electrons absorb the energy $h\nu_+$ and $h\nu_-$ of incident photons; (c) the same situation for the A atom in a solid and two empty states are left by two valence electrons. W is the total energy of an electron, h is the Planck constant and $\nu_+; \nu_-$ denote the frequencies of photons

These electrons occupy the W_A energy state with the spin number $s = \pm 1/2$. When this atom is packed into a solid (that is, the A atom presents a pollutant or an impurity in Fig. 1b), these electrons can occupy the higher energy state. Such the property can be

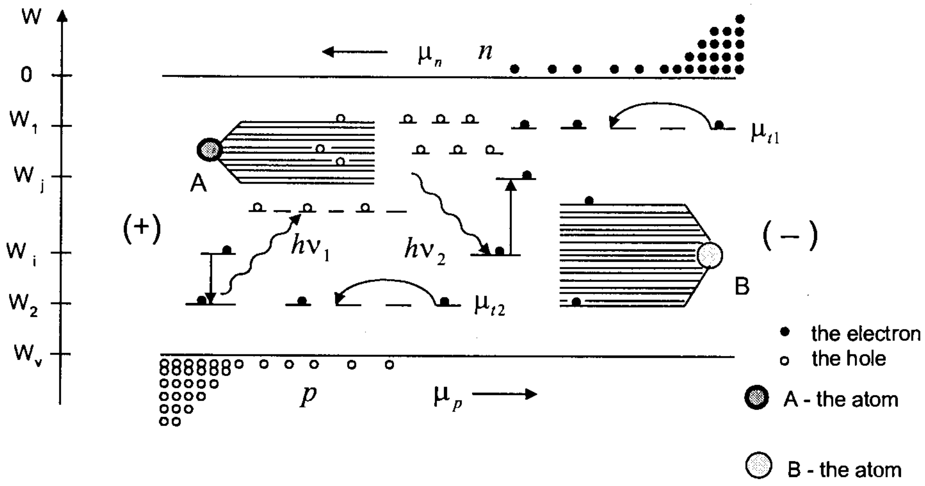

Fig. 2. The energy diagram for the internal and boundary processes in a solid placed between the (+) anode and the (−) cathode. W denotes the total energy of an electron

explained by the splitting of the W_A energy state. In other words, this is caused by the different internal interactions (for example, the Zeemann internal effect) between the A atom and many adjacent atoms. Next, when a portion of the kinetic energy is given to the two valence electrons, these electrons can become free. The two empty energy states (which are left by the electrons) represent the two holes (this effect is shown in Fig. 1c). In Fig. 2 some internal and boundary processes are presented when a solid is placed between the anode and the cathode. Here the pollutants or the impurities are symbolised by the A or B atoms. Between the zero and valence levels many energy states (the trapping states) are available for the holes and for the electrons. When an orbital electron absorbs a photon, this electron can pass from the valence level to the zero level via trapping levels. In Fig. 2 this is symbolised by the photon energy $h\nu_2$. The inverse case is when an orbital electron can pass from the higher trapping level to the lower trapping level and an energy portion $h\nu_1$ is emitted. Here, h is the Planck constant and $\nu_1 ; \nu_2$ denote the photon frequencies. Analogously, we are known as the allowed transitions for the trapped holes. Such the allowed electron – hole transitions are called carrier generation - recombination processes. When an external electric field supplies the different kinetic energy to the two adjacent atoms (in Fig. 2 this property is characterised by the mobilities μ_{t1} and μ_{t2}), the trapped electron can pass from trap to trap in the given trapping level. When the external electric field is applied the electron can pass from the cathode into a solid. Moreover, this electron can become free and have the mobility μ_n. Similarly, the hole injection occurs when the valence electron can pass from the bulk into the anode and the valence state is empty. When the external electric field gives a portion of kinetic energy to the valence electron of an adjacent atom, this electron can pass from the atom to the given atom and can fill the empty state on the valence level. Such the transport is characterised by the mobility μ_p in Fig. 2.

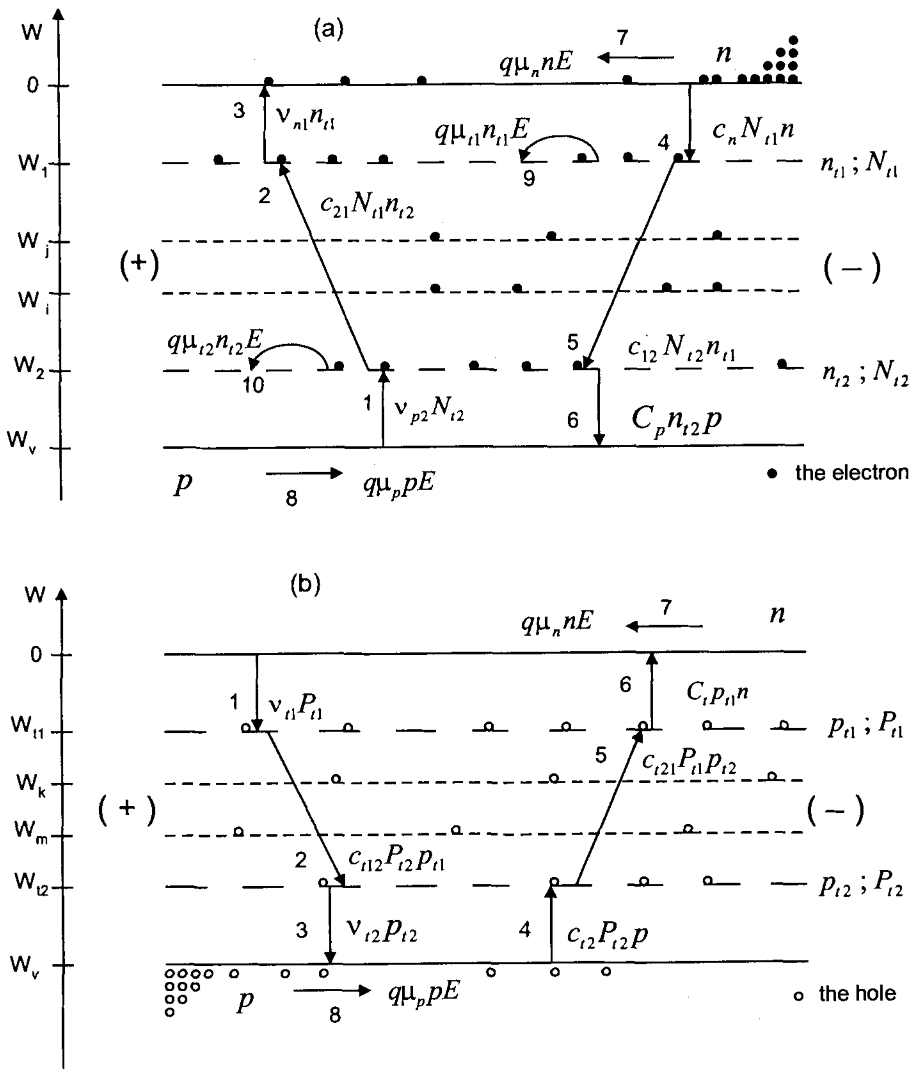

Fig. 3. The energy diagram illustrating a model (1)-(7): (a) allowed electron transitions; (b) allowed hole transitions. W is the total energy of an electron

For our mathematical considerations, the trapping levels will be grouped into the four permissible energy levels (this concept is shown in Fig. 3). With this assumption, the so-called effective parameters such as the frequency parameters c_{21} and c_{t12} as well as the recombination parameters c_{12} and c_{t21} will be used. For the trapped electrons, the concentrations of traps in the first and second trapping level will be represented

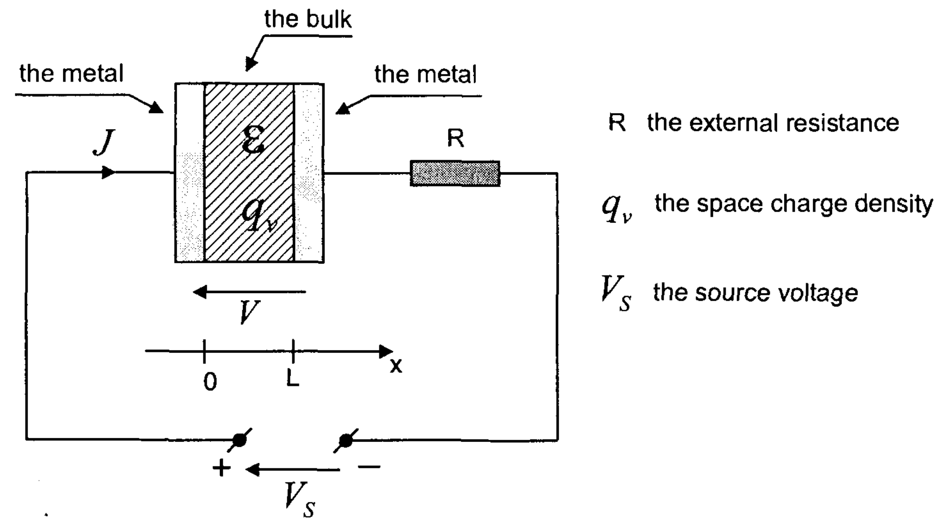

Fig. 4. A planar capacitor is connected with an external resistance and a voltage source. Here,(+);(−) denote the terminals of a voltage source

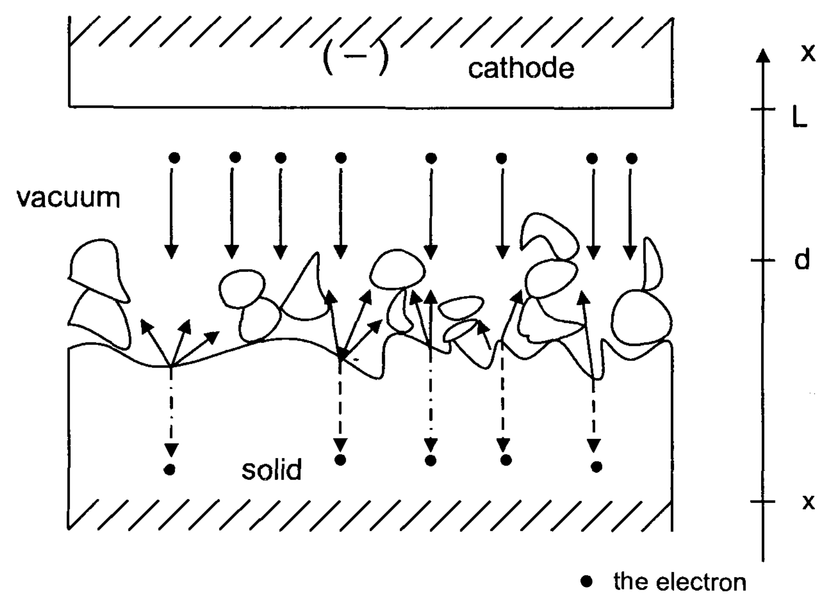

Fig. 5. An electron transport between the cathode and a solid plane makes a model (9)-(12)

by N_{t1} and N_{t2} , respectively. Analogously, for the trapped holes, the concentrations of traps in the first and second trapping level will be equal to P_{t1} and P_{t2}, respectively. The system of atoms will be treated as an unlimited reservoir of traps, that is $P_{t1} >> p_{t1}$; $P_{t2} >> p_{t2}$; $N_{t1} >> n_{t1}$ and $N_{t2} >> n_{t2}$. The metal-solid-metal system will be represented by a planar capacitor system with the anode $x = 0$ and the cathode $x = L$ (the system is shown in Fig. 4). Also, L denotes the distance between the electrodes. Moreover, we will assume that the diffusion current is negligible [1-6]. For a solid, we will assume that the polarisation effect is characterised by the dielectric constant ε. Additionally, the carrier mobilities are independent of the electric field intensity E [7-9]. For the $x = L$ contact, we will consider a special case of electron emission from the cathode into a solid (this property is shown in Fig. 5). Here, for a solid, we assume that the contact structure is strongly nonhomegenous. On these conditions a secondary electron emission occurs [10-14]. This property will be represented by a vacuum capacitor with the contact voltage V_2 and a distance parameter denoted by $L-d$ [15]. In our considerations, for the planar capacitor system, the basic equations such as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral will be used. On this basis, the space charge transport through the bulk is described by

$$\frac{\varepsilon}{q}\frac{\partial E(x,t)}{\partial x} = p(x,t) + p_{t1}(x,t) + p_{t2}(x,t) - (n(x,t) + n_{t1}(x,t) + n_{t2}(x,t)) \quad (1)$$

$$\frac{\partial}{\partial x}\left\{\left[\mu_n n(x,t) + \mu_p p(x,t) + \mu_{t1} n_{t1}(x,t) + \mu_{t2} n_{t2}(x,t)\right] E(x,t)\right\} + \frac{\partial p(x,t)}{\partial t}$$
$$+\frac{\partial p_{t1}(x,t)}{\partial t} + \frac{\partial p_{t2}(x,t)}{\partial t} - \frac{\partial n(x,t)}{\partial t} - \frac{\partial n_{t1}(x,t)}{\partial t} - \frac{\partial n_{t2}(x,t)}{\partial t} = 0 \quad (2)$$

$$\frac{\partial n(x,t)}{\partial t} = \nu_{t1} P_{t1} + \nu_{n1} n_{t1}(x,t) - c_n N_{t1} n(x,t) - C_t n(x,t) p_{t1}(x,t) + \frac{\partial}{\partial x}\left[\mu_n n(x,t) E(x,t)\right] \quad (3)$$

$$\frac{\partial p_{t1}(x,t)}{\partial t} = \nu_{t1} P_{t1} + c_{t21} P_{t1} p_{t2}(x,t) - c_{t12} P_{t2} p_{t1}(x,t) - C_t n(x,t) p_{t1}(x,t) \quad (4)$$

$$\frac{\partial p_{t2}(x,t)}{\partial t} = c_{t12} P_{t2} p_{t1}(x,t) - c_{t21} P_{t1} p_{t2}(x,t) - \nu_{t2} p_{t2}(x,t) + c_{t2} P_{t2} p(x,t) \quad (5)$$

$$\frac{\partial n_{t1}(x,t)}{\partial t} = c_{21} N_{t1} n_{t2}(x,t) - \nu_{n1} n_{t1}(x,t) - c_{12} N_{t2} n_{t1}(x,t) + c_n N_{t1} n(x,t)$$
$$+\frac{\partial}{\partial x}\left[\mu_{t1} n_{t1}(x,t) E(x,t)\right] \quad (6)$$

$$\frac{\partial n_{t2}(x,t)}{\partial t} = \nu_{p2} N_{t2} + c_{12} N_{t2} n_{t1}(x,t) - c_{21} N_{t1} n_{t2}(x,t) - C_p p(x,t) n_{t2}(x,t)$$
$$+\frac{\partial}{\partial x}\left[\mu_{t2} n_{t2}(x,t) E(x,t)\right] \quad (7)$$

with the voltage condition for a solid

$$\int_0^d E(x,t)dx = V_1 \quad (8)$$

And also, the electron transport between the electrode and a solid is as follows

$$\varepsilon_0 \frac{\partial E_0(x,t)}{\partial x} = -q n_0(x,t) \quad (9)$$

$$\frac{\partial}{\partial x}\left[n_0(x,t)\vartheta(x,t)\right] = \frac{\partial n_0(x,t)}{\partial t} \quad (10)$$

$$-2q\int_L^x E_0(x,t)dx = m\vartheta^2(x,t) - m\vartheta^2(L,t) \quad (11)$$

$$\int_d^L E_0(x,t)dx = V_2 \quad (12)$$

Here, $q = 1.602 \cdot 10^{-19} C$, $\varepsilon_0 = 8.85 \cdot 10^{-12} F/m$; E_0 is the electric field intensity in a vacuum, m is the effective mass of an electron, x is the distance from the electrode, t is the time, ϑ is the velocity of an electron; n, n_0 and p are the free hole and electron concentrations, respectively; $n_{t1}, n_{t2}, p_{t1}, p_{t2}$ are the concentrations of the trapped holes and electrons, respectively; μ_{t1} and μ_{t2} are the mobilities of trapped electrons; $\nu_{p2}, \nu_{n1}, \nu_{t1}, \nu_{t2}$ denote the frequency parameters; c_n, C_p, C_t, c_{t2} denote the recombination parameters. The applied voltage V between the electrodes of a planar capacitor is

$$V = V_1 + V_2 \quad \text{and} \quad V = const. > 0 \quad (13)$$

With the above equations (1)-(13) we shall define the stationary state and we shall find different current-voltage characteristics.

3. THE STATIONARY STATE

Letting $(\partial/\partial t) = 0$ in (1)-(12), the stationary state of the space charge transport is determined by the following equations

$$\frac{\varepsilon}{q}\frac{dE(x)}{dx} = p(x) + p_{t1}(x) + p_{t2}(x) - (n(x) + n_{t1}(x) + n_{t2}(x)) \tag{1a}$$

$$J = qE(x)\left[\mu_n n(x) + \mu_p p(x) + \mu_{t1} n_{t1}(x) + \mu_{t2} n_{t2}(x)\right]; J = const. \tag{2a}$$

$$\nu_{t1}P_{t1} + \nu_{n1}n_{t1}(x) - c_n N_{t1} n(x) - C_t n(x) p_{t1}(x) + \frac{d}{dx}\left[\mu_n n(x)E(x)\right] = 0 \tag{3a}$$

$$\nu_{t1}P_{t1} + c_{t21}P_{t1}p_{t2}(x) - c_{t12}P_{t2}p_{t1}(x) - C_t n(x) p_{t1}(x) = 0 \tag{4a}$$

$$c_{t12}P_{t2}p_{t1}(x) - c_{t21}P_{t1}p_{t2}(x) - \nu_{t2}p_{t2}(x) + c_{t2}P_{t2}p(x) = 0 \tag{5a}$$

$$c_{21}N_{t1}n_{t2}(x) - \nu_{n1}n_{t1}(x) - c_{12}N_{t2}n_{t1}(x) + c_n N_{t1} n(x) + \frac{d}{dx}\left[\mu_{t1} n_{t1}(x)E(x)\right] = 0 \tag{6a}$$

$$\nu_{p2}N_{t2} + c_{12}N_{t2}n_{t1}(x) - c_{21}N_{t1}n_{t2}(x) - C_p p(x) n_{t2}(x) + \frac{d}{dx}\left[\mu_{t2} n_{t2}(x)E(x)\right] = 0 \tag{7a}$$

$$\int_0^d E(x)dx = V_1 \tag{8a}$$

$$\varepsilon_0 \frac{d\,E_0(x)}{dx} = -qn_0(x) \tag{9a}$$

$$J = qn_0(x)\vartheta(x) \tag{10a}$$

$$-2q\int_L^x E_0(x)dx = m\vartheta^2(x) - m\vartheta^2(L) \tag{11a}$$

$$\int_d^L E_0(x)dx = V_2 \tag{12a}$$

Here, J is the current density. The space charge transport through the system will be characterised by a current density-voltage function in the form $J = J(V)$ or voltage – current density $V = V(J)$. In order to find these functions, we have to define the boundary conditions describing the mechanisms of carrier injection from the anode $x = 0$ and the cathode $x = L$ into the bulk.

First let us combine (9a)-(12a) in order to find a function $E_0(x)$. From (9a) and (11a) it follows that the derivative $dE_0/d\vartheta$ is

$$\frac{dE_0}{d\vartheta} = \frac{dE_0/dx}{d\vartheta/dx} = \frac{mJ}{q\varepsilon_0 E_0} \tag{14}$$

Hence, with the assumption $E_0(\vartheta = 0) = 0$ we obtain the electron velocity in a vacuum

$$\vartheta = \frac{q\varepsilon_0}{2mJ}\cdot E_0^2 \tag{14a}$$

Next, on this basis we can find a space charge density distribution $\varepsilon_0\frac{dE_0}{dx}$. Using (9a) and (10a), we have

$$\varepsilon_0\frac{dE_0}{dx} = -\frac{J}{\vartheta} = -\frac{2mJ^2}{q\varepsilon_0 E_0^2} \tag{14b}$$

Thus, a function $E_0(x)$ is in the form of

$$E_0(x) = \left(\frac{6mJ^2}{q\varepsilon_0^2}(L-x) + E_0^3(L)\right)^{\frac{1}{3}} \tag{15}$$

Substituting (15) into the voltage integral (12a), the contact voltage V_2 can be expressed by the boundary value $E_0(L)$ in the form

$$V_2 = \frac{q\varepsilon_0^2}{8mJ^2}\left\{\left[\frac{6mJ^2}{q\varepsilon_0^2}(L-d) + E_0^3(L)\right]^{\frac{4}{3}} - E_0^4(L)\right\} \tag{16}$$

Now, for a solid region $x \in <0, d>$, we will assume that generation processes are dominant. According to (1a)-(8a), this property will be expressed by the following condition

$$c_n = c_{12} = C_p = C_t = 0 \tag{17}$$

Under conditions of (17), as a function of x, the hole current density distribution is linear

$$q\mu_p p(x)E(x) = q\left(\nu_{p2}N_{t2} + \nu_{t1}P_{t1}\right)\cdot x + C_1 \tag{17a}$$

where C_1 is the constant of integration. For the trapped holes we will introduce into our analysis the following time parameters

$$c_{t21}P_{t1} = c_{t12}P_{t2} = \tau_{t12}^{-1}; \quad c_{t2}P_{t2} = \tau_{t2}^{-1} \tag{18}$$

Hence, on the basis of (4a) and (5a) we ascertain that the trapped hole concentration depends on the free hole concentration

$$p_{t1}(x) + p_{t2}(x) = \frac{2\tau_{t2}^{-1}}{\nu_{t2}} \cdot p(x) + \nu_{t1}P_{t1}\left(\frac{2}{\nu_{t2}} + \tau_{t12}\right) \tag{18a}$$

For carrier generation processes, we additionally assume that the trapped electrons in the second trapping level are immobile, that is $\mu_{t2} = 0$. With this assumption, from (7a) and from (17) it follows that the concentration $n_{t2}(x)$ is uniform

$$n_{t2}(x) = \nu_{p2}N_{t2}\tau_{21}; \tau_{21}^{-1} = c_{21}N_{t1} \tag{19}$$

For our carrier generation problem, we will consider a case in which the time parameters satisfy the following equation

$$\tau_{21} = \frac{\nu_{t1}P_{t1}}{\nu_{p2}N_{t2}}\left(\frac{2}{\nu_{t2}} + \tau_{t12}\right) \tag{20}$$

With (20) we will determine the right hand side of (1a). First, let us notice that there is

$$p(x) + p_{t1}(x) + p_{t2}(x) - n_{t2}(x) = \left(1 + \frac{2\tau_{t2}^{-1}}{\nu_{t2}}\right) \cdot p(x) \tag{20a}$$

Now, instead of (1a) we have

$$\frac{\varepsilon}{q}\frac{dE(x)}{dx} = \left(1 + \frac{2\tau_{t2}^{-1}}{\nu_{t2}}\right) \cdot p(x) - n(x) - n_{t1}(x) \tag{20b}$$

In order to determine a function $E(x)$, we introduce into (20b) the new variables

$$\lambda_1 = \frac{q\mu_p p(x)E(x)}{J}; \ \lambda_2 = \frac{q\mu_p n(x)E(x)}{J}; \ \lambda_3 = \frac{q\mu_p n_{t1}(x)E(x)}{J} \tag{21}$$

Thus, (1a) and (2a) are respectively written as

$$\frac{\varepsilon\mu_p}{J}E\frac{dE}{dx} = \left(1 + \frac{2\tau_{t2}^{-1}}{\nu_{t2}}\right) \cdot \lambda_1 - \lambda_2 - \lambda_3 \tag{22}$$

$$1 = \lambda_1 + r_1\lambda_2 + r_2\lambda_3; \ r_1 = \frac{\mu_n}{\mu_p}; \ r_2 = \frac{\mu_{t1}}{\mu_p} \tag{23}$$

For our analysis we assume that $\nu_{n1} \approx \nu_{t1}$ and $P_{t1} \approx N_{t1}$. With such internal parameters we have a function $\lambda_2(x)$

$$\lambda_2 = -\frac{q\nu_{t1}P_{t1}}{r_1 J} \cdot x + C_2 \tag{24}$$

Here C_2 is a constant of integration. Combining (22)-(24), we get

$$\frac{\varepsilon\mu_p}{J}E\frac{dE}{dx} = \left(1 + \frac{1}{r_2} + \frac{2\tau_{t2}^{-1}}{\nu_{t2}}\right) \cdot \lambda_1 - \frac{r_2 - r_1}{r_2}\lambda_2 - \frac{1}{r_2} \tag{25}$$

Next, taking into account (24) and (21) as well as (17a), (25) is written as

$$E\frac{dE}{dx} = \alpha_0 x + C_3 \tag{26}$$

where

$$\alpha_0 = \frac{q}{\varepsilon\mu_p} \cdot \left(1 + \frac{1}{r_2} + \frac{2\tau_{t2}^{-1}}{\nu_{t2}}\right)\left(\nu_{p2}N_{t2} + \nu_{t1}P_{t1}\right) + \frac{q\,(r_2 - r_1)}{\varepsilon\mu_p r_1 r_2}\nu_{t1}P_{t1} \tag{27}$$

and C_3 denotes a constant of integration. We can notice that there is $\alpha_0 > 0$. In the particular case, when $\mu_p = \mu_n = \mu_{t1}$, (27) results in

$$\alpha_0 = \frac{2q\theta}{\varepsilon\mu_p}\left(\nu_{p2}N_{t2} + \nu_{t1}P_{t1}\right); \theta = 1 + \frac{\tau_{t2}^{-1}}{\nu_{t2}} \tag{27a}$$

Let us notice that the equivalent form of (26) is

$$\frac{dE}{dx} = \frac{\alpha_0(x - x_0)}{E}; C_3 = -\alpha_0 \cdot x_0 \tag{28}$$

Here x_0 denotes a new constant of integration. From (28) we obtain two singular solutions, namely

$$E_1(x) = -\sqrt{\alpha_0} \cdot (x - x_0) \quad \text{and} \quad E_2(x) = \sqrt{\alpha_0} \cdot (x - x_0) \tag{29}$$

Another solution follows from (26), that is

$$E(x) = \sqrt{\alpha_0 \cdot x^2 + b \cdot x + E^2(0)} \tag{30}$$

where b is a constant of integration. Using (29)-(30) and (16) and the applied voltage $V = V_1 + V_2$, we can define the current-voltage characteristic in the following parametric form

$$V = V_1[J, E(0)] + V_2[J, E_0(L)] \text{ and } J = f_0[E(0)] \text{ and } J = f_L[E_0(L)] \tag{31}$$

The boundary function f_0 and f_L describe the mechanisms of carrier injection from the anode and the cathode into the interior of the system. First, let us consider a singular solution $E_1(x)$. The integral (8a) is

$$V_1 = \int_0^d E_1(x)dx = -\frac{\sqrt{\alpha_0}}{2}\left(d^2 - 2x_0 d\right) \quad \text{for} \quad x_0 \geq d \tag{32}$$

that is

$$V_1 = -V_{10} + E_1(0)\cdot d \;\; ; \;\; E_1(0) = \sqrt{\alpha_0}\cdot x_0 \;\; ; \;\; J = f_0[E_1(0)] \tag{32a}$$

where

$$V_{10} = \frac{\sqrt{\alpha_0}}{2}d^2 \tag{32b}$$

A formula (32a) is equivalent to

$$J = f_0\left[(V_1 + V_{10})/d\right] \;\; ; \;\; \frac{E_1(0)}{\sqrt{\alpha_0}} \geq d \tag{33}$$

Now, we will define a current density – voltage characteristic (31) when a boundary function $J = f_0[E_1(0)]$ is strongly monotonic. Using (16) for (31), we have (a case of an n-n junction)

$$V = -V_{10} + f_0^{-1}(J)\cdot d + \frac{q\varepsilon_0^2}{8mJ^2}\left\{\left[\frac{6mJ^2}{q\varepsilon_0^2}(L-d) + E_0^3(L)\right]^{\frac{4}{3}} - E_0^4(L)\right\} \tag{34}$$

$$\text{and} \quad J = f_L[E_0(L)]$$

Here $f_0^{-1}(J)$ denotes the inverse function. A boundary function $J = f_L[E_0(L)]$ describes the mechanism of electron injection from the cathode into a vacuum. Analogously, proceeding with a singular solution $E_2(x)$, (33) and (34) are replaced by (a case of a p-n junction)

$$J = f_0\left[(V_1 - V_{10})/d\right] \;\; ; \;\; x_0 \leq 0 \tag{35}$$

as well as

$$V = V_{10} + f_0^{-1}(J)\cdot d + \frac{q\varepsilon_0^2}{8mJ^2}\left\{\left[\frac{6mJ^2}{q\varepsilon_0^2}(L-d) + E_0^3(L)\right]^{\frac{4}{3}} - E_0^4(L)\right\} \tag{36}$$

$$\text{and} \quad J = f_L[E_0(L)]$$

For singular solutions (29), another case is when there is $0 \leq x_0 \leq d$. Thus, in a solid the electric field distribution can be defined as follows (a case of an n-p-n junction)

$$E(x) = \begin{cases} E_1(x) \; ; \; 0 \leq x \leq x_0 \\ E_2(x) \; ; \; x_0 \leq x \leq d \end{cases} \tag{37}$$

Consequently, the integral (8a) is written as

$$V_1 = \int_0^{x_0} E_1(x)dx + \int_{x_0}^{d} E_2(x)dx$$

Hence, on the basis of (37) and (29), we have

$$V_1 = \frac{E^2(0)}{\sqrt{\alpha_0}} - d\cdot E(0) + V_{10} \;\; ; \;\; \frac{1}{2}V_{10} \leq V_1 \leq V_{10} \tag{38}$$

A voltage parameter V_{10} is expressed by (32b). Taking into account (38) and (16), a function (31) results in

$$V = V_{10} - E(0)\cdot d + \frac{E^2(0)}{\sqrt{\alpha_0}} + \frac{q\varepsilon_0^2}{8mJ^2}\left\{\left[\frac{6mJ^2}{q\varepsilon_0^2}(L-d) + E_0^3(L)\right]^{\frac{4}{3}} - E_0^4(L)\right\} \tag{39}$$

$$\text{and} \quad J = f_0[E(0)] \quad \text{and} \quad J = f_L[E_0(L)]$$

Here $J = f_0[E(0)]$ describes the mechanism oh hole injection from the anode into the bulk and $J = f_L[E_0(L)]$ describes the mechanism of electron injection from the cathode into a vacuum. Now, let us return to (30). Another case of an n-p-n junction occurs when there is $E(0) = E(d)$, namely

$$E(x) = \sqrt{\alpha_0\left(x - \frac{d}{2}\right)^2 + K} \;\; ; \;\; K = E^2(0) - \frac{1}{4}\alpha_0 d^2 \geq 0 \tag{40}$$

Here K is a constant of integration. Substituting (40) into the integral (8a), we obtain

$$V_1 = \frac{1}{2\sqrt{\alpha_0}}\left\{\sqrt{\alpha_0}d\cdot E(0) + K\cdot \ln\left|\frac{E(0) + \frac{d}{2}\sqrt{\alpha_0}}{E(0) - \frac{d}{2}\sqrt{\alpha_0}}\right|\right\} \;\; ; \;\; K \geq 0 \;\; ; \;\; K < 0 \tag{41}$$

We can show that a function (41) is defined for $K < 0$. Some mathematical properties of (41) are as follows

$$V_1 = \begin{cases} \dfrac{4E^3(0)}{3\alpha_0 d} \;\; ; \;\; E(0) << \dfrac{d}{2}\sqrt{\alpha_0} \\ E(0)\cdot d \;\; ; \;\; E0) >> \dfrac{d}{2}\sqrt{\alpha_0} \end{cases} \tag{41a}$$

$$V_1 \to \frac{1}{2}V_{10} \;\; as \;\; E(0) \to \frac{d}{2}\sqrt{\alpha_0} \tag{41b}$$

$$\frac{dV_1}{dE(0)} > 0 \;\; ; \;\; E(0) \neq \frac{d}{2}\sqrt{\alpha_0} \tag{41c}$$

$$\frac{dV_1}{dE(0)} \to +\infty \;\; as \;\; E(0) \to \frac{d}{2}\sqrt{\alpha_0} \tag{41d}$$

Thus, substituting a function (41) and a function (16) into (31), we ascertain that a function $V = V(J)$ is defined. As an example, let us consider a case in which the cathode injects an infinite amount of electrons. For example, this is acceptable when there is $E_0(L) = 0$. With this boundary condition, a function (31) becomes

$$V = \beta J^{\frac{2}{3}} + \frac{1}{2\sqrt{\alpha_0}}\left\{\sqrt{\alpha_0}d \cdot E(0) + K \cdot \ln\left|\frac{E(0) + \frac{d}{2}\sqrt{\alpha_0}}{E(0) - \frac{d}{2}\sqrt{\alpha_0}}\right|\right\} \;\; ; \;\; \beta = \frac{3}{4}\left(\frac{6m}{q\varepsilon_0^2}\right)^{\frac{1}{3}}(L-d)^{\frac{4}{3}} \tag{42}$$

$$\text{and } J = f_0[E(0)]$$

And a boundary function f_0 describes the mechanism of hole injection from the anode into the bulk. A special case occurs when $d \to L$ and $\varepsilon E(d) = \varepsilon_0 E_0(d)$. Now, instead of (42), a function (31) takes the following limiting form

$$V = \frac{1}{2\sqrt{\alpha_0}}\left\{\sqrt{\alpha_0}d \cdot E(0) + K \cdot \ln\left|\frac{E(0) + \frac{d}{2}\sqrt{\alpha_0}}{E(0) - \frac{d}{2}\sqrt{\alpha_0}}\right|\right\} \;\; ; \;\; J = f_0[E(0)] \tag{42a}$$

Here, there must be $f_0[E(0)] \equiv f_L[\varepsilon_r E(L)]$ and $\varepsilon_r = \varepsilon/\varepsilon_0$.

4. DISCUSSION and CONCLUSIONS

In the above, we have determined the divergence of the electric field distribution. For the bulk, when carrier generation processes are dominant and the condition (20) is valid, the electric field intensity satisfies a homogenous equation (26) or (28). Upon these conditions there exist two singular particular solutions $E_1(x)$ and $E_2(x)$ (which (29) expresses) as well as the general integral (30). First let us discuss the shape of a current density – voltage characteristic $J = J(V)$, which is determined by (34) and by (36). Here, let us assume that a boundary function $f_0[E(0)]$ is strongly increasing and $f_0(0) = 0$. Similarly, the same assumption is for a boundary function $f_L[E_0(L)]$. According to (33)-(36), we see that the $J(V)$ curve is displaced and strongly increasing. For (34), we ascertain that the $J(V)$ curve can exist only for $V > V_{10}$ and for $f_0^{-1}(J) \geq d \cdot \sqrt{\alpha_0}$. In other words, there exists a set of values of applied voltage V in which the current density J is not defined. This property denotes that the whole system acts as a solar cell. In the case of (36), we see that the $J(V)$ curve is strongly increasing for $V > V_{10}$ and $J(V) \equiv 0$ for $0 \leq V \leq V_{10}$. Therefore, the system acts as a perfect blocking diode and a voltage stabiliser. Also, this mathematical property corresponds to a solar cell. The third case of singular solutions is expressed by (37)-(39). Here, in what follows, we take into consideration the same as above boundary functions $f_0[E(0)]$ and $f_L[E_0(L)]$. On the basis of (38), we ascertain that a function (39) can exist only in a set of $0 \leq J \leq f_0(2E_{10})$, where there is $E_{10} = \frac{d}{2}\sqrt{\alpha_0}$. For (34) and (36) as well as (39), some example of the shape of the $J(V)$ curve is illustrated in Fig. 6. Now, let us return to the particular integral (40). In this case, at the anode region $0 \leq x \leq \frac{d}{2}$, the space charge density $q_v = \varepsilon\frac{dE}{dx} < 0$ is negative. Analogously, for the bulk, at the $x = d$ contact region, that is $\frac{d}{2} \leq x \leq d$, the space charge density is positive

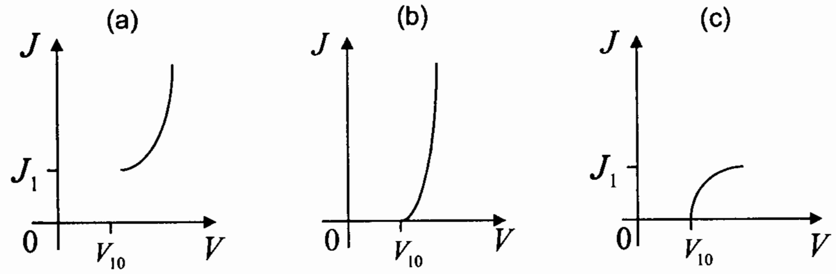

Fig. 6. The $J(V)$ curves shaped by the singular solutions: (a) the curve corresponds to $E_1(x)$ or to (34); (b) the curve corresponds to $E_2(x)$ or to (36); (c) the curve corresponds to (37) or to (39)

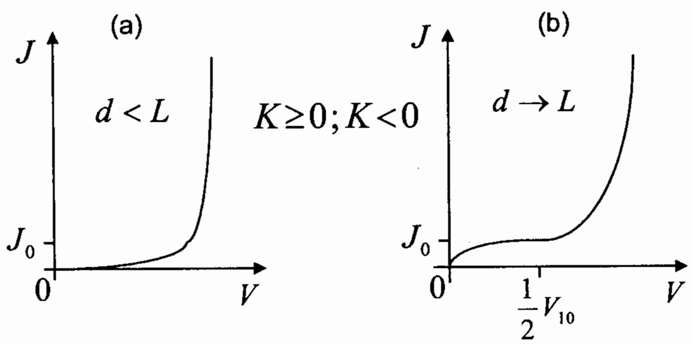

Fig. 7. The $J(V)$ curves correspond to (40) for $K \geq 0$ or $K < 0$: (a) the effect of a secondary electron emission is important; (b) a secondary electron emission is weak

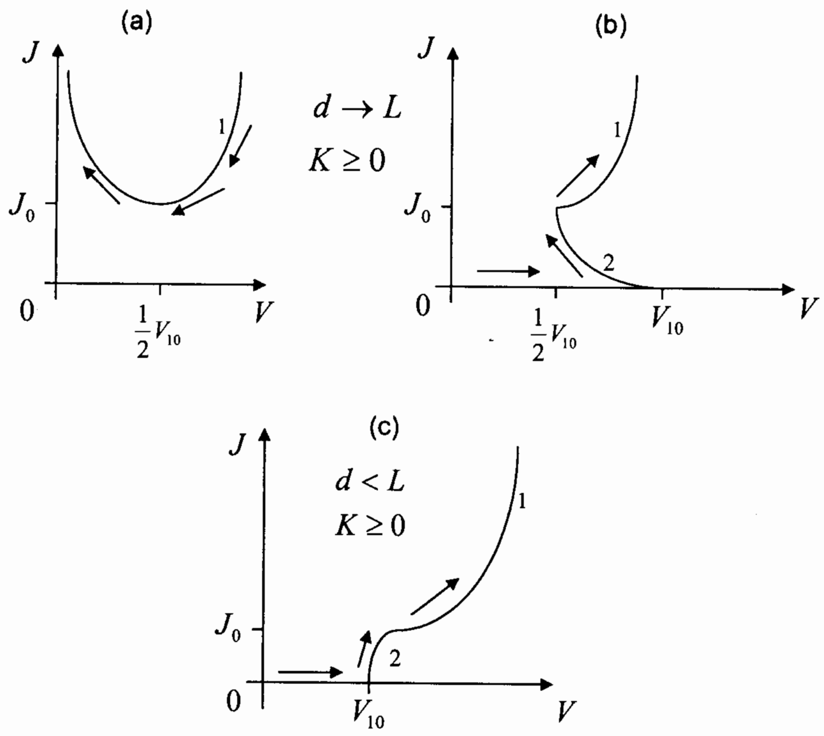

Fig. 8. The $J(V)$ curves correspond to (40) or to (37) for $K \geq 0$: (a) a boundary function f_0 is decreasing for $2V < V_{10}$ or f_0 is increasing for $2V \geq V_{10}$ as well as a secondary electron emission is weak; (b) f_0 is increasing and $f_0(0) = 0$ as well as a secondary electron emission is weak; (c) a secondary electron emission is important and f_0 is increasing and $f_0(0) = 0$. Here 1 – denotes (42)-(42a) and 2 – is (39)

$q_v = \varepsilon \dfrac{dE}{dx} > 0$. Thus, the whole system acts as an n-p-n junction and the $J(V)$ curve can be of the form (42). Moreover, we see that the bulk absorbs a portion of external solar energy in order to form the internal electric field (this physical property is expressed by $K < 0$). Some example of the shape of (42) is illustrated in Fig. 7a, when a boundary function $f_0[E(0)]$ is strongly increasing and $f_0(0) = 0$. If there is $\beta \to 0$ as $d \to L$, then (42) can be replaced by (42a) (an example is illustrated in Fig. 7b). A function from Fig. 7 is typical for the SiC structure [1, 12, 13]. In Fig. 6-8 there is $J_0 = f_0(E_{10})$ and $J_1 = f_0(2E_{10})$.

5. SUMMARY

In the above we have supposed that the divergence of the electric field can be described in terms of the total energy of an orbital electron.

The main idea of a space charge theory is to identify the internal and boundary **processes that** occur in the metal – solid – metal system. In general, this problem can **be solved by the** following electric field conditions [1-3, 5, 6, 13, 14]:
(a) the transient state of the discharging capacitor, characterised by

$$\int_0^d E(x,t)dx + \int_d^L E_0(x,t)dx = 0 \quad \text{for} \quad t \geq 0 \tag{43}$$

(aa) the transient state or the stationary state of the charging capacitor, described by

$$\int_0^d E(x,t)dx + \int_d^L E_0(x,t)dx = V(t) \neq 0 \quad \text{for} \quad t \geq 0 \tag{44}$$

where the voltage function is usually of the form $V(t) = const.$ or $V(t) = V_m \sin \omega t$, **and the parameters** V_m and ω are given; as well as
(aaa) the open system, in which the total current density J_t is

$$J_t(t) = \varepsilon \partial E(x,t)/\partial t + J_c(x,t) \equiv 0 or J_t(t) = \varepsilon_0 \partial E_0(x,t)/\partial t + J_{c0}(x,t) \equiv 0 \tag{45}$$

and

$$-\frac{\partial q_s}{\partial t} = J_{c0}(d,t) - J_c(d,t) \tag{45a}$$

where J_c and J_{c0} are respectively the convection current densities in a solid and in a vacuum, and q_s is the surface charge density on the plane $x = d$.

In this work, by making use of the current density – voltage characteristic $J(V)$, we have identified the interior and the boundaries together.

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