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Further theoretical description for double injection in the metal-solid-metal system

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A problem of the current flow in a solid containing an arbitrary distribution of defects is presented. The different analytical forms of the electric field intensity distributions are found. The conditions in which the system can act as an *n-p* or a *p-n* blocking diode are determined. Also, it was found that the current density can decreases with the voltage. Usually, the current-voltage characteristic is strongly non-linear.

1. INTRODUCTION

Numerous investigations of electric conduction in solids [1 – 5] demonstrate that the current-voltage dependence $J(V)$ can have the different form such as the power function or the Schottky function or the Pool function. Also, for thin amorphous or impure crystalline structure films the current-voltage dependence can be decreasing [6 – 10]. Generally, we remark that the electrode materials have an influence on electric conduction. In the works [11 – 16] we analysed the transport of bipolar space charge in a solid with two given boundary conditions. In this paper we further continue to present our theoretical analysis of double injection which takes into account the two boundary functions describing the mechanisms of carrier injection from the electrodes into a solid.

The purpose of this paper is to find new electric field distributions for which the analytical forms of the current-voltage characteristics can be determined.

2. MODEL SYSTEM AND BASIC EQUATIONS

In this work we will consider a more general case of electric conduction which occurs in amorphous or impure crystalline solids. To this end we make the following assumptions:

(I) As the model system, the planar capacitor with the anode $x=0$ and the cathode $x=L$ will be taken into consideration,

(II) In a solid the atoms are packed closely together and the concentration of atoms is possibly maximum,

(III) Many of the discrete trapping states between the valence level and the zero reference level are available [17–19],

(IV) In the given trapping level the trapped electron can pass from trap to trap with the mobility μ_{ti} where subscript i denotes the number of the trapping level [20],

(V) The hole mobility μ_p and the electron mobility μ_n as well as μ_{ti} are independent of the electric field intensity E,

(VI) The carrier diffusion is negligible.

With these assumptions the bipolar space charge transport will be analysed. The basic equations such as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral are written as:

$$\varepsilon\frac{\partial E(x,t)}{\partial x}=q\left\{(p(x,t)-p_0)-(n(x,t)-n_0)-\sum_{i=1}^{m}(n_{ti}(x,t)-n_{ti,0})\right\}; \tag{1}$$

$$p_0=n_0+\sum_{i=1}^{m}n_{ti,0},$$

$$\begin{gathered}\frac{\partial}{\partial x}\left\{\left[\mu_n n(x,t)+\mu_p p(x,t)+\sum_{i=1}^{m}\mu_{ti}n_{ti}(x,t)\right]E(x,t)\right\}+\\+\frac{\partial p(x,t)}{\partial t}-\frac{\partial n(x,t)}{\partial t}-\sum_{i=1}^{m}\frac{\partial n_{ti}(x,t)}{\partial t}=0,\end{gathered} \tag{2}$$

$$\sum_{i=1}^{m}\frac{\partial n_{ti}(x,t)}{\partial t}=\sum_{i=1}^{m}\left\{c_{ni}N_{ti}n(x,t)-v_{ni}n_{ti}(x,t)+v_{pi}N_{ti}-C_{pi}p(x,t)n_{ti}(x,t)+\frac{1}{q}\frac{\partial J_{ti}}{\partial x}\right\}; \tag{3}$$

$$J_{ti}=q\mu_{ti}n_{ti}(x,t)E(x,t);\quad N_{ti}\gg n_{ti},$$

$$\frac{\partial n(x,t)}{\partial t}=\sum_{i=1}^{m}\{v_{ni}n_{ti}(x,t)-c_{ni}N_{ti}n(x,t)\}+\frac{\partial}{\partial x}[\mu_n n(x,t)E(x,t)];\quad N_{ti}\gg n_{ti}, \tag{4}$$

$$\int_0^L E(x,t)\mathrm{d}x=V;\quad V=\text{const.}>0, \tag{5}$$

where q is electric charge, ε — dielectric constant, x — distance from the electrode, t — time, p — hole concentration, n — electron concentration, n_{ti} — concentration of trapped electrons in the i-th trapping level, p_0; n_0; $n_{ti,0}$ are the equilibrium concentrations of free and trapped carriers, v_{pi}; c_{ni}; v_{ni}; C_{pi} are the generation-recombination parameters of the trapping levels, m — number of the trapping levels, N_{ti} — concentration of traps in the i-th trapping level, and V is the voltage applied to the electrodes. The above equations are written for the constant temperature conditions.

Referring to equation (3), it is necessary to notice that this formula presents only the total trapping rate. It is very well known that the electron can absorb or emit a portion of energy, passing between the allowed discrete energy levels. Electron passage between the i-th and j-th trapping levels is characterised by coefficients c_{ji} and c_{ij}, which define the rate of change of concentration by the equation

$$\frac{\partial n_{ti}(x,t)}{\partial t} = c_{ji} n_{tj}(x,t) N_{ti} - c_{ij} n_{ti}(x,t) N_{tj}. \tag{6}$$

For the internal processes described by (1) – (6) we will characterise the steady state of electric conduction by a relation between the current density J and the voltage V in the form $J = J(V)$ or $V = V(J)$.

3. SOLUTION OF THE PROBLEM

From (1) – (5) it follows that the stationary state is described by the following equations:

$$\frac{\varepsilon}{q}\frac{\mathrm{d}E(x)}{\mathrm{d}x} = p(x) - n(x) - \sum_{i=1}^{m} n_{ti}(x), \tag{1a}$$

$$J = qE(x)\left[\mu_n n(x) + \mu_p p(x) + \sum_{i=1}^{m} \mu_{ti} n_{ti}(x)\right] = \text{const.} \tag{2a}$$

$$\sum_{i=1}^{m}\left\{c_{ni} N_{ti} n(x) + v_{pi} N_{ti} - v_{ni} n_{ti}(x) - C_{pi} p(x) n_{ti}(x) + \frac{\mathrm{d}}{\mathrm{d}x}(\mu_{ti} n_{ti}(x) E(x))\right\} = 0, \tag{3a}$$

$$\sum_{i=1}^{m}\{v_{ni} n_{ti}(x) - c_{ni} N_{ti}\, n(x)\} + \frac{\mathrm{d}}{\mathrm{d}x}[\mu_n n(x) E(x)] = 0, \tag{4a}$$

$$\int_0^L E(x)\mathrm{d}x = V. \tag{5a}$$

In order to find the current-voltage relation we have to give the boundary conditions describing the mechanisms of carrier injection from the elecrodes into the bulk. In this work we shall continue to consider the cases of electric conduction in which the electric field intensity is of the form $E(x) = E(x,A,B)$, where A and B are constants of integration. In what follows, we will assume that the potential barrier width at the electrodes is much smaller in comparison with the mean free path [21 – 24]. Under these conditions, the boundary conditions can be written in the form $J = \mathrm{f}_0[E(0)]$ and $J = \mathrm{f}_L[E(L)]$ where f_0 and f_L are the boundary functions describing carrier emission from the anode $x = 0$ and the cathode $x = L$ into the bulk, respectively. On this basis, the constant values of A and B can be defined by the boundary values $E(0) = E(x = 0, A, B)$ and $E(L) = E(x = L, A, B)$ as well as by

$$V = \int_0^L E(x,A,B)\mathrm{d}x = V(A,B) \tag{7}$$

in order to obtain the current-voltage characteristic in the parametric form

$$V=V[E(0),E(L)];\quad J=f_0[E(0)];\quad J=f_L[E(L)], \tag{7a}$$

or

$$V=V[E(L)];\quad J=f_L[E(L)] \text{ and } E(0)\to 0. \tag{7b}$$

In this paper, we shall find the characteristics $J=J(V)$ or $V=V(J)$, making use of (7a) and (7b).

3.1. Carrier generation

In this section we shall consider a case of electric conduction in which electron transition from the valence band to the conduction band via trapping levels is dominant. Under these conditions, we can assume that $c_{ni}=C_{pi}=0$ in (1a)–(4a). In what follows, we will assume that $\mu_{ti}=\mu_t=\text{const}$. Moreover, we additionally suppose that the generation parameters satisfy the condition $v_{ni}\leqslant v_{pi}$, so that the hole generation is dominant. After introducing the new variables described by

$$\lambda_1=q\mu_n pE/J;\quad \lambda_2=q\mu_n n_t E/J;\quad \rho_1=\mu_p/\mu_n;\quad \rho_2=\mu_t/\mu_n, \tag{8}$$

where $n_t=\sum_{i=1}^{m} n_{ti}$, equations (1a)–(4a) can be written as

$$\frac{\varepsilon\mu_n E}{J}\frac{\mathrm{d}E}{\mathrm{d}x}=(\rho_1+1)\lambda_1-(1-\rho_2)\lambda_2-1, \tag{9}$$

$$\lambda_1=\frac{q}{\rho_1 J}\left\{\sum_{i=1}^{m} v_{pi}N_{ti}\right\}x+A_1, \tag{10}$$

$$\lambda_2=-\frac{q}{\rho_2 J}\left\{\sum_{i=1}^{m} v_{pi}N_{ti}\right\}x+A_2, \tag{11}$$

where A_1 and A_2 are constants of integration. Hence, we get the following electric field distribution

$$E^2(x)=\alpha x^2+C_1x+C_2;\quad \alpha=\left(\frac{1}{\mu_p}+\frac{1}{\mu_t}\right)\frac{q}{\varepsilon}\sum_{i=1}^{m} v_{pi}N_{ti}, \tag{12}$$

or in the equivalent form

$$E(x)=\sqrt{\alpha\left(x+\frac{C_1}{2\alpha}\right)^2+K};\quad K=\frac{4\alpha C_2-C_1^2}{4\alpha}. \tag{12a}$$

Here, C_1 and C_2 are new constants of integration. Next, using the boundary values of $E(0)$ and $E(L)$ as well as the voltage condition (5a), we obtain

$$V=\frac{1}{2\sqrt{\alpha}}\left\{K\ln\left|\frac{E(l)+z_1}{E(0)+z_0}\right|+z_1E_1(L)-z_0E(0)\right. \tag{13}$$

and

$$z_0=\frac{C_1}{2\sqrt{\alpha}};\quad z_1=L\sqrt{\alpha}+z_0;\quad C_1=\frac{E^2(L)-E^2(0)-\alpha L^2}{L};\quad K=E^2(0)-z_0^2. \tag{13a}$$

Thus, referring to (7a), we remark that the two mechanisms of carrier injection from the electrodes $x=0$ and $x=L$ into the bulk which are described by the boundary functions $J=f_0[E(0)]$ and $J=f_L[E(L)]$ deifne the current-voltage relation in the parametric form. In the case when carrier injection is symmetrical, that is $f_0[E(0)]\equiv f_L[E(L)]$, then we have $E(0)=E(L)$ and $z_1=-z_0=\frac{L}{2}\sqrt{\alpha}$. In this case the space charge density $q_v=\varepsilon\frac{dE}{dx}$ has the form

$$q_v(x)=\frac{\varepsilon\alpha(x-L/2)}{E(x)};\quad E(x)>0, \tag{14}$$

Hence, it follows that a negative charge $q_v(x)<0$ is distributed in the region $0\leqslant x<L/2$ and a positive charge $q_v(x)>0$ occurs in the region $L/2<x\leqslant L$. Therefore, the metal-solid-metal system acts as an $n-p$ junction and the current-voltage characteristic is defind by

$$V=\frac{1}{2\sqrt{\alpha}}\left(E^2(L)-\frac{\alpha L^2}{4}\right)\ln\left|\frac{E(L)+\frac{1}{2}L\sqrt{\alpha}}{E(L)-\frac{1}{2}L\sqrt{\alpha}}\right|+\frac{1}{2}LE(L);\quad J=f_L[E(L)]. \tag{15}$$

From (15) it follows that $V\to V_1=L^2\sqrt{\alpha}/4$ and $\frac{dV}{dE(L)}\to\infty$ as $E(L)\to L\sqrt{\alpha}/2$. If the boundary function $J=f_L[E(L)]$ is monotonic, then $\frac{dJ}{dV}\to 0$ and $J\to J_0=f_L(2V_1/L)$ as $V\to V_1$. Thus, the function $J=J(V)$ is defined and differentiable for all values of $V\geqslant 0$. If $E(L)\ll\frac{1}{2}L\sqrt{\alpha}$, then (15) results in

$$E(L)=\sqrt[3]{\frac{3}{4}\alpha LV}\quad\text{and}\quad J=f_L[E(L)]. \tag{15a}$$

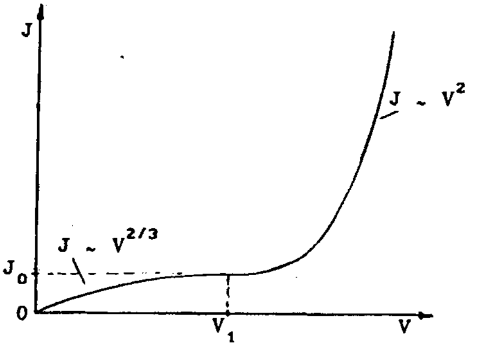

Fig. 1. The shape of the current-voltage characteristic (15) determined by the quadratic boundary function $J\propto E^2(L)$. The scale is arbitrary for clarity

If $E(L) \gg \frac{1}{2} L\sqrt{\alpha}$, that is, the electric field becomes uniform, then $J = f_L(V/L)$. In Fig. 1 the current-voltage characteristic (15) is illustrated for the quadratic boundary function $J \propto E^2(L)$ (a particular case of the tunnel effect).

3.2. Generation-recombination processes in the deep traps

In amorphous or very impure crystalline structures there can exist a very large concentration of structural defects (traps). Under these conditions, the mean free path of an electron may be very small. Thus, we can suppose that the carrier transport occurs in the valence band and in the trapping levels, that is $\mu_n = 0$. In what follows, we will assume that the energy separation between the trapping levels is very small. With this assumtpion, introducing the following parameters

$$\theta_{0i} = \frac{v_{ni}}{c_{ni} N_{ti}}; \quad \theta_{0i} \ll 1 \tag{16}$$

we can assume that $\theta_{01} \approx \theta_{02} \approx \ldots \theta_{0m} = \theta_0$ and $C_{p1} = C_{p2} = \ldots C_{pm} = C_p$ (this is possible since the recombination parameters depend on the atomic number and on the temperature). In this case, consolidation of (1a)–(4a) leads to the following differntial equations

$$\rho \frac{d\lambda}{dx} = \frac{q}{J} \sum_{i=1}^{m} v_{pi} N_{ti} - \frac{C_p J \lambda (1 - \rho\lambda)}{q(\mu_t E)^2}, \tag{17}$$

$$\frac{\varepsilon \mu_t E}{J} \frac{dE}{dx} = \lambda(1 + \rho\theta) - \theta; \quad \theta = 1 + \left(\sum_{i=1}^{m} \theta_{0i}^{-1} \right)^{-1} \approx 1 + \theta_0/m, \tag{18}$$

where

$$\lambda = q\mu_t pE/J; \quad \rho = \mu_p/\mu_t; \quad \mu_t = \mu_{ti}; \quad i = 1, 2, \ldots, m. \tag{19}$$

Combining (17)–(19), we obtain the differential equation describing the electric field distribution $E(x)$ in the form

$$\frac{\varepsilon \mu_p \theta_1}{2J} \frac{d}{dx} \left(\frac{dE^2}{dx} \right) = \frac{q}{J} \sum_{i=1}^{m} v_{pi} N_{ti} - \frac{C_p J}{q(\mu_t E)^2} \left\{ \theta_1 \left(\frac{\varepsilon \mu_t}{2J} \frac{dE^2}{dx} + \theta \right) \times \right.$$
$$\left. \times \left[1 - \rho\theta_1 \left(\frac{\varepsilon \mu_t}{2J} \frac{dE^2}{dx} + \theta \right) \right] \right\}; \quad \theta_1 = (1 + \rho\theta)^{-1}. \tag{20}$$

When $\rho = 1$ $(\mu_t = \mu_p = \mu)$ and $\theta_0/m \ll 1$ (a case of deep traps), equation (20) takes the simpler form

$$\frac{\varepsilon\mu}{4J} \frac{d}{dx} \left(\frac{dE^2}{dx} \right) = \frac{q}{J} \sum_{i=1}^{m} v_{pi} N_{ti} - \frac{C_p J}{4q(\mu E)^2} \left\{ 1 - \left(\frac{\varepsilon\mu}{2J} \frac{dE^2}{dx} \right)^2 \right\}. \tag{20a}$$

This equation may be expressed by a first-order Bernoulli-type differential equation for which the general integral is of the form

$$\frac{\varepsilon\mu}{2J}\frac{\mathrm{d}E^2}{\mathrm{d}x}=\sqrt{1-CE^{2k}+\frac{\gamma E^2}{1-k}};\quad k=\frac{\varepsilon C_p}{2q\mu};\quad \gamma=\frac{2q\varepsilon\mu}{J^2}\sum_{i=1}^{m}\nu_{pi}N_{ti},\tag{20b}$$

where C is a constant of integration. Equation (20b) is easily integrated for $k\approx 0$ ($C_p \ll 2q\mu/\varepsilon$), $k=1/2$ and $k=2$. When $k\approx 0$, the function $E(x)$ is of the form of (12) and (12a)

$$E^2(x)=\alpha x^2+C_1x+C_2;\quad \alpha=\frac{2q}{\varepsilon\mu}\sum_{i=1}^{m}\nu_{pi}N_{ti},$$

where C_1 and C_2 are constants of integration. Therefore, in this case the current-voltage characteristic (13) occurs. If $k=1/2$ then equation (20b) can be integrated in the elementary manner, but the constants of integration satisfy a transcendental equation. Thus, the determination of a function $J(V)$ with the two given boundary functions becomes a difficult problem, which we have omitted. In the case when $k=2$, two solutions exist:

1) The first solution is

$$E^2(x)=\frac{\gamma}{2C_1}+2C_2^2\exp\left(\frac{2J\sqrt{C_1}}{\varepsilon\mu}x\right)\pm\sqrt{\left(E^2(x)-\frac{\gamma}{2C_1}\right)^2+K_1};\tag{21}$$

$$K_1=\frac{4C_1-\gamma^2}{4C_1^2};\quad C_1>0.$$

Here C_1 and C_2 are suitably selected constants of integration. In particular case when $E^2(x)\gg\gamma/2C_1$, the solution (21) is reduced to the form

$$E(x)=E(0)\left(\frac{E(L)}{E(0)}\right)^{x/L}.\tag{21a}$$

Hence, on the basis (5a), we obtain the function $J=J(V)$ in the following parametric form

$$V\ln\left(\frac{E(L)}{E(0)}\right)=L(E(L)-E(0));\quad J=\mathrm{f}_0[E(0)];\quad J=\mathrm{f}_L[E(L)].\tag{21b}$$

For example, with the Schottky injection at the boundaries $x=0$ and $x=L$ so that $J=J_0\exp(b_L\sqrt{E(L)})$ and $J=J_0\exp(b_0\sqrt{E(0)})$ where J_0, b_0 and b_L are the boundary parameters, then the current-voltage characteristic is the Schottky type function $J\propto\exp(\mathrm{const}\cdot\sqrt{V})$ (this is acceptable for mean values of V). Another example, when the boundary functions f_0 and f_L are quadratic or linear (this is acceptable for the high and low voltage conditions, respectively), we have $J\propto V^2$ or $J\propto V$, respectively.

2) the second solution is

$$E^2(x)+\frac{\gamma}{2C}=A\cos\left(\frac{2J\sqrt{C}}{\varepsilon\mu}x-C_1\right);\quad A=\frac{\sqrt{\gamma^2+4C}}{2C},\tag{22}$$

where C_1 is a new constant of integration. Since $\cos\delta=\cos(-\delta)$, therefore here can occur the space charge transport for which $E(0)=E(L)$. This condition is possible only when $C_1=\frac{JL\sqrt{C}}{\varepsilon\mu}$, that is

$$E^2(x)+\frac{\gamma}{2C}=A\cos\left(\frac{J\sqrt{C}}{\varepsilon\mu}(2x-L)\right).\tag{22a}$$

In the case when the electrodes $x=0$ and $x=L$ inject an infinite quantity of carriers, that is $E(0)=E(L)\rightarrow 0$, we have

$$\cos C_1=\frac{1}{\sqrt{1+\frac{4C}{\gamma^2}}}.\tag{22b}$$

If $\frac{2\sqrt{C}}{\gamma}\rightarrow\infty$, then $C_1\rightarrow\frac{\pi}{2}$ and the integration constant C is

$$C=\left(\frac{\pi\varepsilon\mu}{2JL}\right)^2.\tag{22c}$$

In particular case when $\frac{2C}{\gamma}\rightarrow\infty$, that is, a case in which the recombination processes are dominant, the function $E(x)$ described by (22a) becomes

$$E(x)=\sqrt{\frac{2JL}{\pi\varepsilon\mu}\cos\left(\frac{\pi}{2L}(2x-L)\right)}.\tag{23}$$

Hence on the basis (5a), we obtain Child's law in the form

$$J=\frac{\pi^3\varepsilon\mu V^2}{8a^2L^3};\quad a=\int_0^{\pi/2}\sqrt{\cos t}\,dt.\tag{24}$$

From (23) it follows that a positive space charge is distributed in the region $0<x<\frac{1}{2}L$ and a negative space charge is distributed in the region $\frac{1}{2}L<x<L$. Thus, in this case the metal-solid-metal system acts as a $p-n$ junction. When only the electron -hole recombination processes exist ($v_{pi}=0$) in the bulk with deep traps ($\theta_0/m\ll 1$), then equations (17)–(19) lead to

$$\varepsilon\mu E\frac{dE}{dx}=\pm\sqrt{J^2-BE^{\chi}};\quad \mu=\mu_t=\mu_p;\quad \chi=\frac{\varepsilon C_p}{q\mu},\tag{25}$$

where B is a constant of integration. Substituting (25) into (5a) and limiting our attention only to the (+) sign, we obtain

$$V=\varepsilon\mu\int_{E(0)}^{E(L)}\frac{E^2}{\sqrt{J^2-BE^{\chi}}}\,\mathrm{d}E \tag{26}$$

and

$$\int_{E(0)}^{E(L)}\frac{E}{\sqrt{J^2-BE^{\chi}}}\,\mathrm{d}E=\frac{L}{\varepsilon\mu}. \tag{27}$$

From (25) and (26) it follows that there exists a function $V=V[E(0),E(L)]$. Thus, according to (7a), we see that a function $J=J(V)$ is defined. In what follows, we will determine some current-voltage characteristics by the use of (26) and (27). For $\chi\approx 0$, that is $C_p\ll q\mu/\varepsilon$, we have

$$V=\frac{2L(E^3(L)-E^3(0))}{3(E^2(L)-E^2(0))};\quad J=\mathrm{f}_0[E(0)];\quad J=\mathrm{f}_L[E(L)]. \tag{28}$$

For example, when the boundary functions f_0 and f_L are linear or quadratic, then we have $J\propto V$ or $J\propto V^2$, respectively. In the case when $\chi=1$, that is $C_p=q\mu/\varepsilon$, (26) and (27) yield

$$V=\frac{2\varepsilon\mu}{5B}\{E^2(0)\sqrt{J^2-BE(0)}-E^2(L)\sqrt{J^2-BE(L)}\}+\frac{4J^2L}{5B}, \tag{29}$$

$$3J^2(E^2(L)-E^2(0))+B(E^3(L)-E^3(0))=\left(\frac{3BL}{2\varepsilon\mu}\right)^2+ \\ +\frac{3L}{\varepsilon\mu}(2J^2+BE(L))\sqrt{J^2-BE(L)}. \tag{30}$$

Therefore, the relations (29), (30) and the boundary functions f_0 and f_L define the current-voltage characteristic in the parametric form. In particular, when the anode $x=0$ injects an infinite quantity of holes, that is $E(0)\to 0$, the function $J(V)$ is described by

$$V=\frac{4J^2L}{5B}-\frac{2\varepsilon\mu E^2(L)}{5B}\sqrt{J^2-BE(L)}, \tag{29a}$$

$$\left(\frac{3L}{2\varepsilon\mu}\right)^2B^2+BE^3(L)+3J^2E^2(L)-\frac{6LJ^3}{\varepsilon\mu}=0 \tag{30a}$$

and $J=\mathrm{f}_L[E(L)]$. From (30a) it follows that the integration constant B is of the form

$$B=-\frac{2(\varepsilon\mu)^2E^3(L)}{(3L)^2}\left\{1-\sqrt{1+\frac{(3L)^2J^2(6LJ-3\varepsilon\mu E^2(L))}{(\varepsilon\mu)^3E^6(L)}}\right\}. \tag{30b}$$

For example, when the boundary functions is quadratic $J=\frac{\varepsilon\mu}{bL}E^2(L)$ and $0<b<2$, then the current-voltage relation is $J\propto V^2/L^3$. As a next example, let us take into account the linear boundary function $J=\sigma_L E(L)$ where σ_L is the boundary parameter. First, let us additionally assume that $J\gg\frac{\varepsilon\mu}{2L}E^2(L)$. Under this condition the integration constant B becomes

$$B=2\sqrt{\frac{2\varepsilon\mu J^3}{3L}}. \tag{30c}$$

Hence, on the basis (29a), we obtain a function $V=V(J)$ in the form

$$V=\alpha_1 J^{1/2}-\alpha_2 J^{3/2}\sqrt{1-\alpha_3 J^{1/2}};\quad \alpha_1=\frac{1}{5}\sqrt{\frac{6L^3}{\varepsilon\mu}};\quad \alpha_2=\frac{1}{5\sigma_L^2}\sqrt{\frac{3\varepsilon\mu L}{2}};\quad \alpha_3=\frac{2}{\sigma_L}\sqrt{\frac{2\varepsilon\mu}{3L}}. \tag{31}$$

For $J\ll 1/\alpha_3^2=\frac{3L\sigma_L^2}{8\varepsilon\mu}$ the function (31) takes the simpler form

$$V=\alpha_1 J^{1/2}-\alpha_2 J^{3/2}. \tag{31a}$$

Since there must be $J\geqslant\frac{\varepsilon\mu}{2L}E^2(L)$, thus, the function $V=V(J)$ described by (31a) is positive. Therefore, in the case of the linear boundary function f_L the current-voltage relation is strongly nonlinear. Now, let us return to (26) and (27). For the value of $\chi=3$, the formula (26) resutls in

$$V=\frac{2\varepsilon\mu}{3}\cdot\frac{E^3(L)-E^3(0)}{\sqrt{J^2-BE^3(L)}+\sqrt{J^2-BE^3(0)}}. \tag{32}$$

An interesting case occurs when $E(0)\to 0$. In this case (32) becomes

$$V=\frac{2\varepsilon\mu}{3}\cdot\frac{E^3(L)}{J+\sqrt{J^2-BE^3(L)}}. \tag{32a}$$

The relation between B, $E(0)$ and $E(L)$ follows from (27). Assuming that $J^2\gg BE^3(L)$, which is possible when $E(L)\gg E(0)=0$, we have

$$V=\frac{\varepsilon\mu E^3(L)}{3J}\quad \text{for}\quad 2JL\gg\varepsilon\mu E^2(L) \tag{33}$$

with the boundary condition $J=f_L E(L)$. For example, if the boundary function is quadratic, then the current-voltage relation is $J\propto V^2$. Another example, if the linear boundary function occurs $f_L=\sigma_L E(L)$, where σ_L is the boundary conductivity parameter, then (33) results in $J\propto V^{1/2}$ for $J\ll 2L\sigma_L^2/\varepsilon\mu$. For the power boundary function (a particular case of the Schottky or Pool or Fowler-Nordheim function) $J=a_L E^k(L)$, where a_L denotes the boundary parameter, (33) can be written as

$$V \cdot J^{\frac{k-3}{k}} = \text{const.} \tag{34}$$

Thus, for $k>3$, the $J(V)$ curve is decreasing, that is $\frac{dV}{dJ}<0$. When $k=3$, we have $\frac{dV}{dJ}=0$, that is, the system acts as a voltage stabiliser. The relationship (34) is illustrated in Fig. 2 and 3.

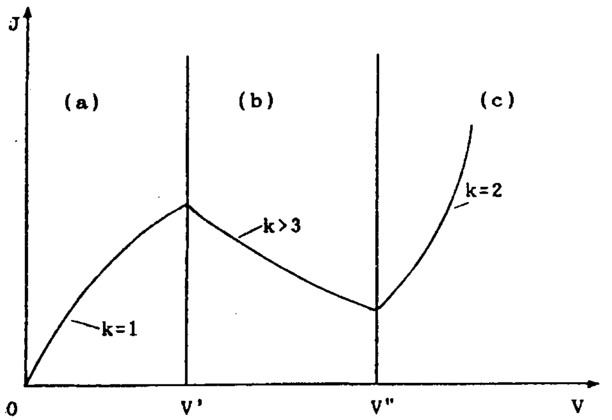

Fig. 2. Interpretation of the relationship (34) and three voltage regions: (a) low voltage region with linear boundary function $k=1$, (b) mean voltage region with the Schottky or Pool boundary function $k>3$, (c) high voltage region with quadratic boundary function $k=2$. The voltage values of V' and V'' are determined by the continuity condition of the $J(V)$ curve. The scale is arbitrary for clarity

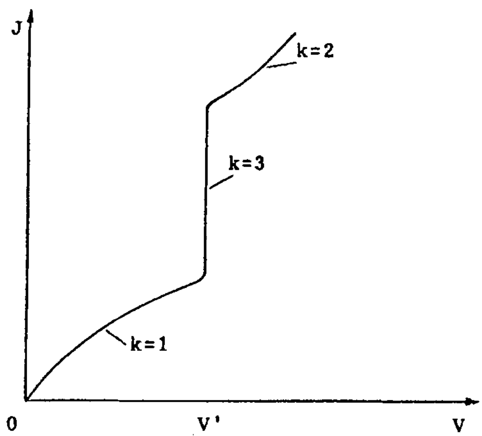

Fig. 3. Other interpretation of the relationship (34). The scale is arbitrary for clarity. Such $J(V)$ curve is typical for the amorphous anthracene structure

4. DISCUSSION

In [11 – 16] we have compared our methodology with a regional approximation method and with a small signal theory. In this section we further wish to refer to some other concepts which have been presented in the last two decades. On this basis we can compare our analysis with the others and, as well we can indicate the practical importance of the space charge theory.

Considering a $p-n$ junction, Fletcher assumed that the boundary values of the hole and electron concentrations can be expressed by constant electrochemical potentials (quasi-Fermi levels). These conditions are called the Fletcher or Boltzmann boundary conditions. Nussbaum [25] showed that these conditions, in particular, in the space charge region, are not universally valid. This work shows that the boundary problem is one of the difficulties of the bipolar space charge theory.

A theory of double injection under conditions of recombination and diffusion for thin insulating crystals has been developed by Torpey [26]. The analysis of the current flow shows that Child's law $J \propto V^2/L^3$ is possible. Also, in that work individual effects of recombination, diffusion, contacts and crystal thickness have been discussed. Generally, we ought to observe that the diffusion problem with the two mobilities defines the general solution $E(x)$ containing constants of integration for which the number of the boundary or Cauchy initial conditions is higher than 2. This is the fundamental difficulty of the diffusion problem. Moreover, we ought to note that one cannot separate the internal and contact processes. The boundaries and interior together describe the current flow in the metal-solid (insulator or semiconductor)-metal system.

A theory explaining a mechanism for the switching effect has been presented by Hindley [8]. The Schockley hot carrier model has been considered. The analysis leads to a relation $V(J)$ in the form $V^2J = \text{const.}$ According to (34), we see that, for the power boundary function with $k=6$, this relationship is obtained.

A Dumke's theory explaining the switching effect has been presented by Lampert [1] and Milnes [2]. In this theory, a solid with deep impurities has been presented. It was assumed that the total current density can be defined by the photoelectron current density and by the equilibrium hole current density. Under these conditions the current-voltage characteristic has been written as

$$J = aV^2 + bJV$$

or

$$J = aV^2 + b_1 J^2 V$$

or

$$VJ = \text{const.}$$

where a, b and b_1 are the material parameters. This theory is not mathematically clear since the boundary problem has not been considered.

Different analytical methods for the current flow in the metal-insulator-metal system have been presented by Lampert [1]. He considered the different interactions between carriers such as:

(a) diffusion model,

(b) recombination model with the two lifetimes τ_n and τ_p,
(c) model of the minority carrier flow,
(d) bimolecular recombination model,
(e) generation-recombination processes via the trapping levels,
with the boundary condition $E(0)=E(L)$. He also presented Parmenter and Ruppel's model for the contacts, that is $n(L)=f_1[E(L)]$ and $p(0)=f_2[E(0)]$. He considered the different insulator junctions such as $p-n$, $p-i-n$, $p-n-p$, $n-p-n$ and made use of a regional approximation method. The mathematical analysis can lead to the "S" type function $J(V)$ with NDR (Negative Differential Resistance), that is $\mathrm{d}V/\mathrm{d}J<0$.

A study of the minority carrier flow in perfect and imperfect crystal structure with non-ohmic contacts has been presented by Kao [6, 21, 22]. The contacts have been characterized by a potential barrier. For the defect crystal, the time constant (the lifetime) of minority carrier depends on the occupancy of the recombination centers. Moreover, the carrier injection efficienty depends on the potential barrier $V(x)$ at the electrodes. Under these conditions, it was found that NDR regime occurs only for the imperfect crystals.

In general, a main idea of classical analysis presented by Lampert and Kao is to assume the space charge regions. From our considerations it follows that the space charge regions are determined by the mechanism of carrier injection from the electrodes into a solid and by interactions between carriers. The conduction model (1)–(6) can explain the experiment characteristics $J(V)$ or $V(J)$ which occur for the typical insulator materials such as anthracene, teflon, polyethylene, halogen glass, SiO_2, Al_2O_3, TiO_2, ZnO, ZnS, CdS, ZnTe, SiC, Sb_2S_3, SbS_3 and Se (also, selenium is the typical photo-conductor). The current-voltage characteristic (15) which is shown in Fig. 1 is typical for the SiC structure. Similarly, the function showed in Fig. 2 is typical for the SiO_2 or Al_2O_3 structure.

5. CONCLUSIONS

In the above we have considered some possible cases of generation-recombination processes in a solid and a few conditions describing carrier injection from the electrodes into the bulk. From our considerations it follows that the $J(V)$ curves can have the different form such as $J\propto V$, $J\propto V^2/L^3$, $J\propto \exp(\mathrm{const.}\,V)$, $J\propto \exp(\mathrm{const.}\,V^{1/2})$ or the other described by (13), (15), (21b), (28), (29a) and (30b), (31), (33), (34). The relation (33) can lead to NDR.

We ascertain that the generation processes determine the electric field intentisy in the form $E^2(x)=\alpha x^2+Ax+B$ where α is the material parameter and A and B are constants of integration. In this case the symmetric injection of carriers is possible and the system can act as an n-p blocking diode with the current-voltage characteristic (15) (Fig. 1). Under conditions of the recombination processes the system can act as a p-n blocking diode with the current-voltage characteristic (24).

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DALSZA ANALIZA PODWÓJNEGO WSTRZYKIWANIA W UKŁADZIE METAL-CIAŁO NIEMETALICZNE-METAL

W niniejszym opracowaniu przedstawiono problem przepływu prądu elektrycznego w nieuporządkowanej strukturze krystalicznej. Wyznaczono analityczne formy rozkładu natężenia pola elektrycznego. Określono warunki w których układ działa jak dioda blokująca typu *n-p* oraz *p-n*. Stwierdzono, że charakterystyka prądowo-napięciowa jest zazwyczaj nieliniowa i może być funkcją malejącą.

ДАЛЬШЫЙ АНАЛИЗ ДВОЙНОЙ ИНЖЕКЦИИ В СИТСТЕМЕ МЕТАЛ – ТВЕРДОЕ ТЕЛО – МЕТАЛ

В настоящей статье представляется проблема электропроводимости в неупорядоченных твердых телах. Получается разные решения для напряженности электрического поля.

Определяется условия электрического транспорта в которых распределение объёмного заряда соотвествует „*n-p*" или „*p-n*" типа переходом. В статье утверждается что вольтамперная характеристика обычно нелинейна и может быть представлена в виде убывающей функции.