Advanced Spectrum Estimation Methods for Signal Analysis in Power Electronics

Zbigniew Leonowicz, Tadeusz Lobos, and Jacek Rezmer

***Abstract*—Modern frequency power converters generate a wide spectrum of harmonic components. Large converters systems can also generate noncharacteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long. A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. Min-norm harmonic retrieval method is an example of high-resolution eigenstructure-based methods. The Prony method as applied for signal analysis was also tested for this purpose. Both high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonics frequencies. To investigate the methods several experiments were performed using simulated signals, current waveforms at the output of a simulated frequency converter, and current waveforms at the output of an industrial frequency converter. For comparison, similar experiments were repeated using the fast Fourier transform (FFT). The comparison proved the superiority of the new methods. However, their computation is much more complex than FFT**

***Index Terms*—AC motor drives, discrete Fourier transform, power system harmonics, Prony method, spectral analysis, subspace methods.**

I. Introduction

MODERN frequency power converters generate a wide spectrum of harmonic components, which deteriorate the quality of the delivered energy, increase the energy losses, as well as decrease the reliability of a power system. In some cases, large converter systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of noncharacteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage [1], [2]. Interharmonics are defined as noninteger harmonics of the main fundamental under consideration. The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

Interharmonics are considered more damaging than characteristic harmonics components of the distorted signals. Their emission is specified lower than those are for the harmonics.

There are many different approaches for measuring harmonics, including fast Fourier transform (FFT), application of adaptive filters, artificial neural networks, **<AUTHOR: DOES "SVD" NEED TO BE DEFINED?>** SVD, higher order spectra, etc. [3]–[8]. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels. The linear methods of system spectrum estimation (Blackman–Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in these methods, the estimated spectrum can be a smeared version of the true spectrum [9].

These methods usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long [1]. It is very important to develop better tools of interharmonic estimation to avoid possible damages due to its influence.

In this paper the frequencies of signal components are estimated using the Prony model and min-norm method.

The Prony method is a technique for modeling sampled data as a linear combination of exponentials [8]. Although it is not a spectral estimation technique, the Prony method has a close relationship to the least-squares linear prediction algorithms used for **<AUTHOR: DO "AR" AND "ARMA" NEED TO BE DEFINED?>** AR and ARMA parameter estimation. The Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics. The paper [10] presents a new method of real-time measurement of power system frequency based on the Prony model.

The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods" [11]. Their resolution is theoretically independent of the signal-to-noise ratio (SNR). The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function. Pisarenko first observed that the zeros of the z transform of the eigenvector, corresponding to the minimum eigenvalue of the covariance matrix, lie on the unit circle, and their angular positions correspond to the frequencies of the sinusoids. In a later development, it was shown that the eigenvectors might be divided into two groups, namely, the eigenvectors spanning the signal space and eigenvectors spanning the orthogonal noise space. The eigenvectors

Manuscript received March 20, 2001; revised February 7, 2003. Abstract published on the Internet March 4, 2003.

Z. Leonowicz is with the Laboratory for Advanced Brain Signal Processing, Brain Science Institute, RIKEN, Saitama 351-0198, Japan, on leave from the Department of Electrical Engineering, Wroclaw University of Technology, 50-370 Wroclaw, Poland (e-mail: leon@bsp.brain.riken.go.jp).

T. Lobos and J. Rezmer are with the Department of Electrical Engineering, Wroclaw University of Technology, 50-370 Wroclaw, Poland.

Digital Object Identifier 10.1109/TIE.2003.812361

spanning the noise space are the ones whose eigenvalues are the smallest and equal to the noise power. One of the most important techniques, based on the Pisarenko's approach of separating the data into signal and noise subspaces is the min-norm method.

To investigate the ability of the methods several experiments were performed. Simulated signals, current waveforms at the output of a simulated three-phase frequency converter as well as current waveforms at the output of an industrial frequency converter were investigated. For comparison, similar experiments were repeated using the FFT.

II. Prony Method

Assuming the N complex data samples $x[1], \ldots, x[N]$ the investigated function can be approximated by M exponential functions

$$y[n] = \sum_{k=1}^{M} A_k e^{(\alpha_k + j\omega_k)(n-1)T_p + j\psi_k} \tag{1}$$

where $n = 1, 2, \ldots, N$; T_p is the sampling period; A_k is the amplitude; α_k is the damping factor; ω_k is the angular velocity; and ψ_k is the initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n] = \sum_{k=1}^{M} h_k z_k^{n-1} \tag{2}$$

where $h_k = A_k e^{j\psi_k}$ and $z_k = e^{(\alpha_k + j\omega_k)T_p}$.

The estimation problem bases on the minimization of the squared error over the N data values

$$\delta = \sum_{n=1}^{N} |\varepsilon[n]|^2 \tag{3}$$

where

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{M} h_k z_k^{n-1}. \tag{4}$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data may be made.

Consider the discrete-time function consisting of M exponents

$$x[n] = \sum_{k=1}^{M} h_k z_k^{n-1}. \tag{5}$$

The M equations of (5) may be expressed in matrix from as

$$\begin{bmatrix} z_1^0 & z_2^0 & \ldots & z_M^0 \\ z_1^1 & z_2^1 & \ldots & z_M^1 \\ \vdots & \vdots & & \vdots \\ z_1^{M-1} & z_2^{M-1} & \ldots & z_p^{M-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[M] \end{bmatrix}. \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the z_k exponents as its roots

$$F(z) = \prod_{k=1}^{M} (z - z_k) = (z - z_1)(z - z_2) \cdots (z - z_p). \tag{7}$$

The polynomial may be represented as the sum

$$F(z) = \sum_{m=0}^{M} a[m] z^{M-m} = a[0] z^M + a[1] z^{M-1} + \cdots + a[M-1] z + a[M]. \tag{8}$$

Shifting the index in (5) from n to $n - m$ and multiplying by the parameter $a[m]$ yields

$$a[m] x[n-m] = a[m] \sum_{k=1}^{M} h_k z_k^{n-m-1}. \tag{9}$$

Equation (9) can be modified to

$$\sum_{m=0}^{M} a[m] x[n-m] = \sum_{k=1}^{M} h_k z_k^{n-M} \left\{ \sum_{m=0}^{M} a[m] z_k^{M-m-1} \right\}. \tag{10}$$

The right-hand summation in (10) may be recognized as polynomial defined by (8), evaluated at each of its roots z_k yielding the zero result

$$\sum_{m=0}^{M} a[m] x[n-m] = 0. \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e., $N > 2M$, In the overdetermined data case, the linear equation (11) must be modified to

$$\sum_{m=0}^{M} a[m] x[n-m] = e[n]. \tag{12}$$

In this case, the estimation problem bases on the minimization of the total squared error

$$E = \sum_{n=M+1}^{N} |e[n]|^2. \tag{13}$$

III. Min-Norm Method

The min-norm method involves projection of the signal vector

$$\boldsymbol{S}_i = [1 \quad e^{j\omega_i} \quad \ldots \quad e^{j(N-1)\omega_i}]^T \tag{14}$$

onto the entire noise subspace.

The model of the signal $\boldsymbol{x}$ consists of M independent complex exponentials in noise

$$\boldsymbol{X} = \sum_{i=1}^{M} A_i \boldsymbol{S}_i + \eta; A_i = |A_i|\, e^{j\phi_i}. \tag{15}$$

If the noise is white, the correlation matrix is

$$\mathbf{R_x} = \sum_{i=1}^{M} E\left\{A_i A_i^*\right\} \boldsymbol{S}_i \boldsymbol{S}_i^T + \sigma_0^2 \mathbf{I}. \tag{16}$$

$N-M$ smallest eigenvalues of the correlation matrix (matrix dimension $N > M+1$) correspond to the noise subspace and M largest (all greater than σ_0^2-noise variance) corresponds to the signal subspace.

The matrix of eigenvectors is defined as

$$\mathbf{E}_{\text{noise}} = \left[\mathbf{e}_{M+1} \quad \mathbf{e}_{M+2} \quad \ldots \quad \mathbf{e}_N\right]. \tag{17}$$

The min-norm method uses one vector $\mathbf{d}$ for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and its first element equal to one. These conditions are expressed by the following equations:

$$\begin{aligned} \mathbf{d} &= \mathbf{E}_{\text{noise}} \mathbf{E}_{\text{noise}}^{*T} \mathbf{d} \\ \mathbf{d}^{*T} \ell &= 1. \end{aligned} \tag{18}$$

Equation (18) can be expressed in one equation

$$\mathbf{d}^{*T}\ell = \left(\mathbf{E}_{\text{noise}} \mathbf{E}_{\text{noise}}^{*T} \mathbf{d}\right)^{*T} \ell = \mathbf{d}^{*T} \mathbf{E}_{\text{noise}} \mathbf{E}_{\text{noise}}^{*T} \ell = 1. \tag{19}$$

The Lagrangian has the form

$$\begin{aligned} L = \mathbf{d}^{*T}\mathbf{d} &+ \mu\left(1 - \mathbf{d}^{*T}\mathbf{E}_{\text{noise}}\mathbf{E}_{\text{noise}}^{*T}\ell\right) \\ &+ \mu^*\left(1 - \ell^{T}\mathbf{E}_{\text{noise}}\mathbf{E}_{\text{noise}}^{*T}\mathbf{d}\right). \end{aligned} \tag{20}$$

The gradient of (20) has the form

$$\nabla_{\mathbf{d}^*} L = \mathbf{d} - \mu \mathbf{E}_{\text{noise}} \mathbf{E}_{\text{noise}}^{*T} \ell = \mathbf{0} \tag{21}$$

where μ is chosen in such way that the first element of the vector is equal to one.

For this purpose $\mathbf{E}_{\text{noise}}$ is transformed to

$$\mathbf{E}_{\text{noise}} = \begin{bmatrix} \mathbf{c}^{*T} \\ \mathbf{E}'_{\text{noise}} \end{bmatrix} \tag{22}$$

where $\mathbf{c}^{*T}$ is the upper row of the matrix. Hence, $\mathbf{c} = \mathbf{E}_{\text{noise}}^{*T}\ell$.

From (21) and (22), it results that the first element of the vector $\mathbf{d}$ is equal to $\mu\mathbf{c}^{*T}\mathbf{c}$.

Finally, vector $\mathbf{d}$ is equal to

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T}\mathbf{c}} \mathbf{E}_{\text{noise}}\mathbf{c} = \begin{bmatrix} 1 \\ \frac{\left(\mathbf{E}'_{\text{noise}}\mathbf{c}\right)}{\left(\mathbf{c}^{*T}\mathbf{c}\right)} \end{bmatrix}. \tag{23}$$

The pseudospectrum defined with the help of $\mathbf{d}$ is defined as

$$\hat{P}\left(e^{j\omega}\right) = \frac{1}{\left|\mathbf{w}^{*T}\mathbf{d}\right|^2} = \frac{1}{\mathbf{w}^{*T}\mathbf{d}\mathbf{d}^{*T}\mathbf{w}} \tag{24}$$

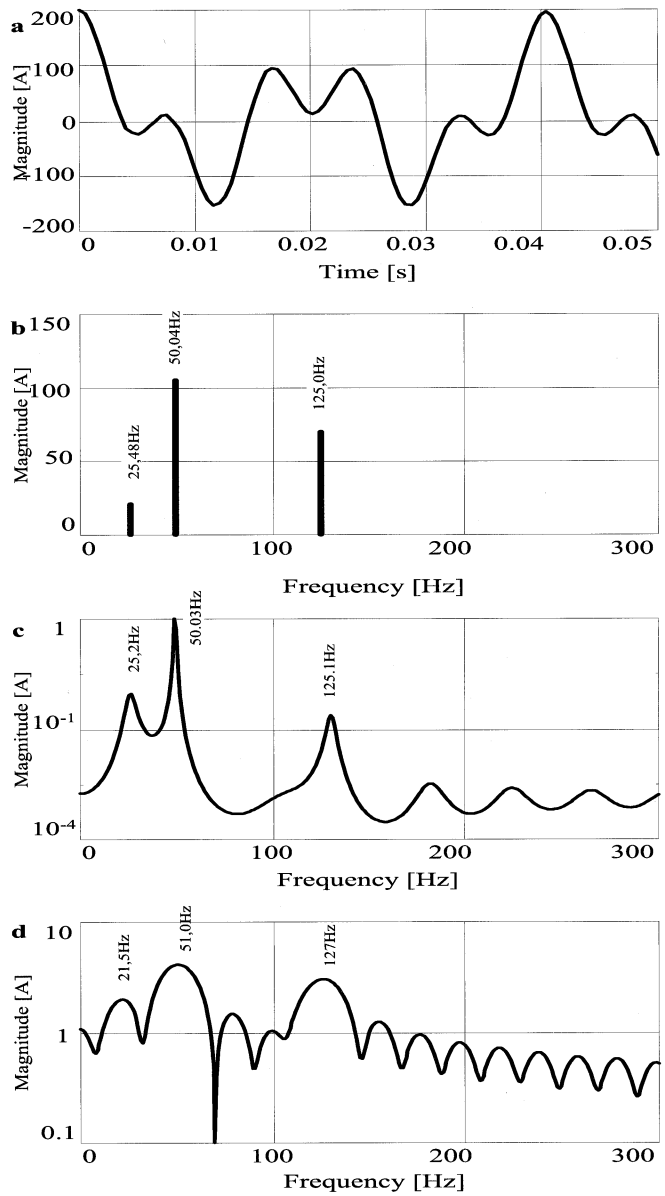

Fig. 1. Voltage waveform at the output of a simulated dc arc furnace power supply installation. (a) Investigation results: Prony, $M = 30$. (b) Min-norm. (c) FFT. (d) $f_p = 2000$ Hz, $N = 100$.

where $\mathbf{W}$ is defined similarly to $\boldsymbol{s}$ in (14).

IV. Experiments With Simulated Waveforms

Several experiments were performed with the signal waveform described in [2]. The investigated signal is characteristic for dc arc furnace installations without compensation. It consists of the basic harmonic (50 Hz), one higher harmonic (125 Hz), one interharmonic (25 Hz), and is additionally distorted by 5% random noise. The sampling interval was 0.5 ms. The signal was investigated using the Prony and min-norm methods. Both methods enable us to detect all the signal components already using 100 samples (Fig. 1). For detection of the 25-Hz component using the Fourier algorithm, many more samples were needed. When using the same number of samples (100)

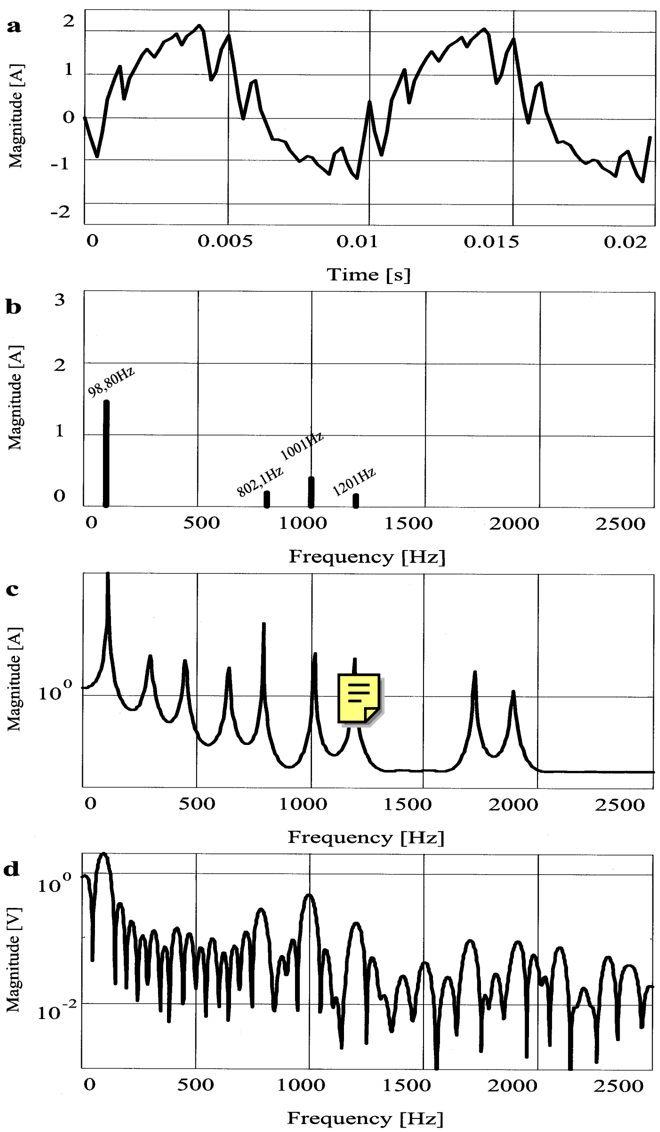

Fig. 2. Current waveform at the output of a simulated frequency converter. (a) Investigation results: Prony, $N = 80$, $M = 40$. (b) Min-norm, $N = 100$. (c) FFT, $N = 80$. (d) $f_p = 5000$ Hz.

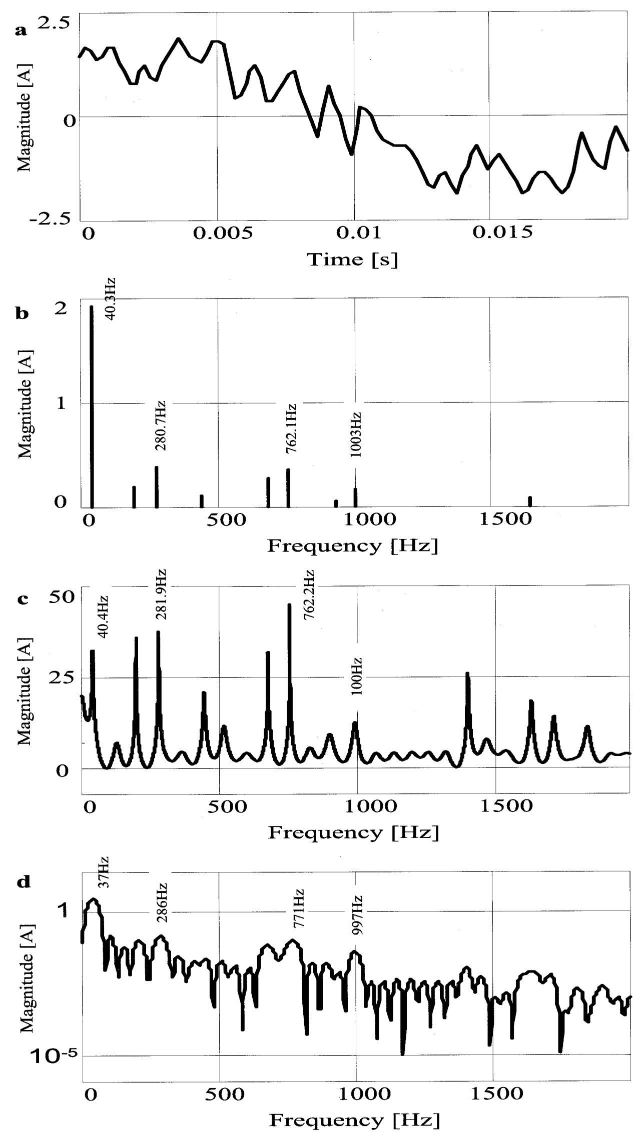

Fig. 3. Current waveform at the output of a real frequency converter. (a) Investigation results: Prony, $M = 40$. (b) Min-norm. (c) FFT. (d) $N = 100$, $f_p = 5000$ Hz.

the Fourier algorithm indicates the frequencies ca. 21.5, 51, and 127 Hz.

V. Simulation of a Frequency Converter

In recent years, simulation programs for complex electrical circuits and control systems have been essentially improved. The simulation of characteristic transient phenomena concerning the electrical quantities becomes feasible without any arrangement of hardware. Among many available simulation programs, the EMTP-ATP as a Fortran-based program adapted to DOS/Windows serves for modeling complex one- and three-phase networks occurring in drive, control, and power systems.

In this paper, we show investigation results of a 3-kVA PWM converter with a modulation frequency of 1 kHz supplying a two-pole 1-kW asynchronous motor ($U = 380$ V, $I = 2.8$ A). The simulated converter can change the output frequency within a range $0.1 \div 150$ Hz.

Fig. 2 shows the current waveform at the converter output for the frequency 100 Hz. The sampling interval was 0.2 ms. The Prony and min-norm methods enable us to estimate the frequencies of all the signal components using 100 samples. The following frequencies have been detected: ca. 100, 800, 1000, and 1200 Hz. The estimation accuracy is better than when using the Fourier algorithm.

VI. Industrial Frequency Converter

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter composed of a single-phase half-controlled bridge rectifier and a voltage-source converter.

The waveforms of the converter output current under normal conditions were investigated using the Prony, min-norm, and

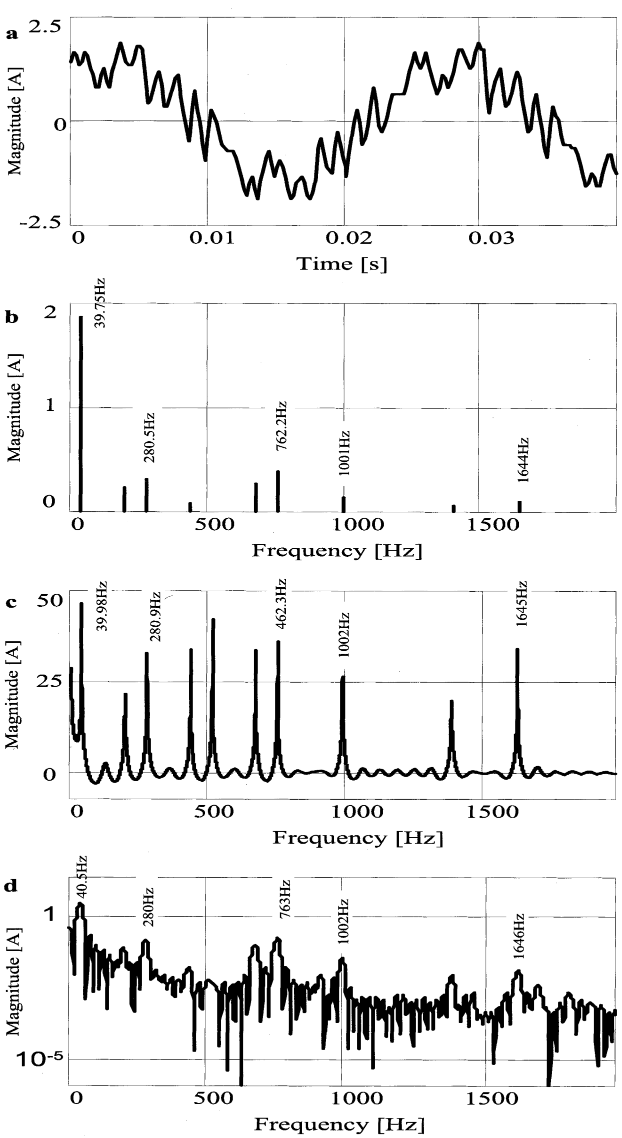

Fig. 4. Current waveform at the output of a real frequency converter. (a) Investigation results: Prony, $M = 80$. (b) Min-norm. (c) FFT. (d) $N = 200$, $f_p = 5000$ Hz.

FFT methods for the sampling windows equal to 20 ms (Fig. 3) and 40 ms (Fig. 4). The main frequency of the waveform was 40 Hz. When using the 40-ms window the estimation results are a little more accurate than for the 20-ms window. However, the smaller window makes it possible to detect the main harmonic components. When using the FFT method, the window length has a greater influence on the estimation accuracy.

Using the Prony and min-norm methods, the following harmonics have been detected: 5th, 7th, 17th, 19th, 25th, 35th, and and 41st. It was also possible to estimate the frequency of the fundamental component.

VII. Conclusions

It has been shown that a high-resolution spectrum estimation method, such as min-norm could be effectively used for parameter estimation of distorted signals. The Prony method could also be applied for estimation the frequencies of signal components. The accuracy of the estimation depends on the signal distortion, the sampling window, and the number of samples taken into the estimation process.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make possible the estimation of interharmonics.

When using both high-resolution methods, Prony and min-norm, the estimation accuracy in most cases is better than when using the Fourier algorithm. Application of the proposed advanced methods makes possible the estimation of the components, which frequencies differ insignificantly. However, their computation is much more complex than FFT.

References

[1] R. Carbone, D. Menniti, N. Sorrentino, and A. Testa, “Iterative harmonics and interhamonics analysis in multiconverter industrial systems,” in *Proc. 8th Int. Conf. Harmonics and Quality of Power*, Athens, Greece, 1998, pp. 432–438.

[2] P. Mattavelli, L. Fellin, P. Bordignon, and M. Perna, “Analysis of interharmonics in DC arc furnace installation,” in *Proc. 8th Int. Conf. Harmonics and Quality of Power*, Athens, Greece, 1998, pp. 1092–1099.

[3] T. Lobos and K. Eichhorn, “Recursive real-time calculation of basic waveforms of signals,” *Proc. Inst. Elect. Eng.*, pt. C, vol. 138, no. 6, pp. 469–470, Nov. 1991.

[4] [illegible]ocki and T. Lobos, “Artificial neural networks for real-time estimation of basic waveforms of voltages and currents,” *IEEE Trans. Power Syst.*, vol. 9, pp. 612–618, Sept. 1994.

[5] Z. Leonowicz, T. Lobos, P. Ruczewski, and J. Szymanda, “Application of higher-order spectra for signal processing in electrical power engineering,” *COMPEL*, vol. 17, no. 5/6, pp. 602–611, 1998.

[6] T. Lobos, Z. Leonowicz, and J. Rezmer, “Harmonics and interharmonics estimation using advanced signal processing methods,” in *Proc. 9th IEEE Int. Conf. Harmonics and Quality of Power*, vol. I, Orlando, FL, 2000, pp. 335–340.

[7] T. Lobos, T. Kozina, and H.-J. Koglin, “Power systems harmonics estimation using linear least squares methods and SVD,” *Proc. IEE—Gen. Transmission Distrib.*, vol. 148, no. 6, pp. 567–572, 2001.

[8] M. Meunier and F. Brouage, “Fourier transform, wavelets, prony analysis: Tool for harmonics and quality of power,” in *Proc. 8th Int. Conf. Harmonics and Quality of Power*, Athens, Greece, 1998, pp. 71–76.

[9] S. M. Kay, *Modern Spectral Estimation: Theory and Application*. Englewood Cliffs, NJ: Prentice-Hall, 1988, pp. 224–225.

[10] T. Lobos and J. Rezmer, “Real time determination of powersystem frequency,” *IEEE Trans. Instrum. Meas.*, vol. 46, pp. 877–881, Aug. 1997.

[11] C. W. Therrien, *Discrete Random Signals and Statistical Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1992, pp. 614–655.

Zbigniew Leonowicz <AUTHOR: PLEASE LIST YEAR(S) AND GRADE(S) OF IEEE MEMBERSHIP, IF APPLICABLE> received the M.Sc. and Ph.D. degrees in electrical engineering from Wroclaw University of Technology, Wroclaw, Poland, in 1997 and 2001, respectively.

Since 1997, he has been with the Department of Electrical Engineering, Wroclaw University of Technology. He is currently on leave at the Laboratory for Advanced Brain Signal Processing, Brain Science Institute, RIKEN, Saitama, Japan. His current research interests include modern digital signal processing methods applied to power system analysis.

Dr. Leonowicz received the NATO Advanced Fellowship in 2001 and spent it at the Technical University of Dresden, Germany.

Tadeusz Lobos <AUTHOR: PLEASE LIST YEAR(S) AND GRADE(S) OF IEEE MEMBERSHIP, IF APPLICABLE> received the M.Sc., Ph.D., and Habilitate Doctorate (Dr. Sc.) degrees in electrical engineering from Wroclaw University of Technology, Wroclaw, Poland, in 1960, 1967, and 1975, respectively.

Since 1960, he has been with the Department of Electrical Engineering, Wroclaw University of Technology, where he became a Full Professor in 1989. From 1982 to 1986, he was with the University of Erlangen-Nuremberg, Germany. His current research interests are in the areas of transients in power systems, control and protection, and, in particular, applications of neural networks and signal processing methods in power systems.

Dr. Lobos was awarded a Research Fellowship in 1976 by The Alexander von Humboldt Foundation, Germany, and he spent this fellowship at the Technical University of Darmstadt. He received the Humboldt Research Award, Germany, in 1998.

Jacek Rezmer <AUTHOR: PLEASE LIST YEAR(S) AND GRADE(S) OF IEEE MEMBERSHIP, IF APPLICABLE> received the M.Sc. and Ph.D. degrees from Wroclaw University of Technology, Wroclaw, Poland, in 1987 and 1995, respectively.

He is currently **<AUTHOR: CURRENT POSITION?>** with the Department of Electrical Engineering, Wroclaw University of Technology. His fields of interests include digital signal processing and computer measurement techniques, in particular, their applications in electrical engineering.