NEW POWER QUALITY INDICES

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ABSTRACT

The author shows that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices (such as inter/harmonic groups and subgroups). The comparison of the frequency and amplitude estimation error, based on numerical simulations is presented. Presentation of selected power quality indices is then followed by comparison of estimation error in the case of application of FFT-based algorithms and parametric methods. Investigated waveforms are typical for dc arc furnace plant. MUSIC and ESPRIT high-resolution methods are used to analyze waveforms in a supply system of a DC arc furnace.

KEY WORDS

Power quality, harmonics, interharmonics, parametric spectral estimation, MUSIC, ESPRIT

1 Introduction

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The proliferation of nonlinear loads connected to power systems has triggered a growing concern with power quality issues. The inherent operation characteristics of these loads deteriorate the quality of the delivered energy, and increase the energy losses as well as decrease the reliability of a power system [1, 2, 3, 4]. The methods of power quality assessment in power systems are almost exclusively based on Fourier Transform. The crucial drawback of the Fourier Transform-based methods is that the length of the window is related to the frequency resolution. Moreover, to ensure the accuracy of Discrete Fourier Trans-form, the sampling interval of analysis should be an exact integer multiple of the waveform fundamental period [5]. Parametric spectral methods, such as ESPRIT or MUSIC [5] do not suffer from such inherent limitations of resolution or dependence of estimation error on the window length (phase dependence of the estimation error). The resolution of these methods is to high degree independent on signal-to-noise ratio and on the initial phase of the harmonic components. The author argues that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices [6].

The paper is composed as follows: After the description of parametric methods (ESPRIT and MUSIC), the comparison of its performance (estimation error), based on numerical simulation is presented. Next part presents basics of selected power quality indices (harmonic sub/groups), followed by comparison of estimation error in the case of application of FFT-based algorithms and parametric methods.

2 Error of estimation of parametric spectral methods

The performance (error of estimation) of the subspace methods has been extensively investigated in the literature, especially in the context of the Direction-of-Arrival (DOA) estimation. Based on [7] and [8], the derivation of variance in the case of frequency component estimation is presented.

Comparison of mean square error is useful for theoretical assessment of accuracy of both methods with emphasis to root-MUSIC and ESPRIT. Both methods are similar in the sense that they are both eigendecomposition-based methods which rely on decomposition of the estimated correlation matrix into two subspaces: noise and signal subspace. On the other hand, MUSIC uses the noise subspace to estimate the signal components while ESPRIT uses the signal subspace. In addition, the approach is in many points different. Numerous publications were dedicated to the analysis of the performance of the aforementioned methods (e.g. [7, 8, 9, 10, 11, 12, 13]). Unfortunately, due to many simplifications, different assumptions and the complexity of the problem, published results often appear to be contradictory and sometimes misleading.

When roughly summarizing different results form the literature, a resume of basic parameters can be established, as shown in Table 1.

To the best authors knowledge, the comparison of accuracy to such extent of two different parametric methods

Method	CC	AC	RFE
Periodogram	small	medium	medium
MUSIC	high	high	medium
ESPRIT	medium	very high	none

Table 1. Comparison of basic performance characteristics of spectral methods (CC-computational cost, AC-accuracy, RFE-risk of false estimates).

based on numerical simulation of real-like signals is for the first time presented in this work.

3 Performance analysis of MUSIC

From the available N data samples the autocorrelation sequence $r_x[k]$ is computed for a chosen number of delays k. The autocorrelation matrix is then formed and then eigen-decomposed as: $\mathbf{R}_x = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{*T}$, where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k]$. In one of possible approaches the polynomials are built from eigenvectors spanning the noise subspace. The roots of each of such polynomials correspond to signal zeros. Now the following expression can be defined [8]:

$$D(z) = \sum_{i=K+1}^{M} [U_i(z)][U_i^*(1/z^*)] \quad (1)$$

The idea of MUSIC (Multiple Signal Classification) was developed in [14] where the *averaging* was proposed for improvement of the performance of Pisarenko estimator [15]. Instead of using only one noise eigenvector, the MUSIC method uses many *noise eigenfilters*. The number of computed eigenvalues $M > K+1$. All eigenvalues can be partitioned as follows:

$$\underbrace{\lambda_1 \geq \lambda_2 \geq \ldots \lambda_K}_{K \text{ signal eigenvalues}} \geq \underbrace{\lambda_{K+1} \geq \lambda_{K+2} \geq \ldots \lambda_M}_{M-K \text{ noise eigenvalues}} \quad (2)$$

Instead of one annihilating filter (as in Pisarenko's estimator), MUSIC method uses $M-K$ noise eigenfilters.

$$U_i(z) = \sum_{m=0}^{M-1} u_i[m] z^{-m}; i = K+1, \ldots, M \quad (3)$$

Every eigenfilter has $M-1$ roots, K roots are common for all eigenfilters. The common K roots can be found by averaging.

3.1 Errors of estimation

The root-MUSIC algorithm uses the estimated covariance matrix to compute the signal zeros from (1). Also from (1) we can obtain the relation between the error of the signal zeros and the estimated $D(z)$ [8]. When analyzing the mean squared error (MSE) of the signal zeros estimates, the relationship between the errors in signal zeros and the estimated $D(z)$ is as follows:

$$D(z) = c\sum_{l=1}^{L-1}(1-(z_l+\Delta z_l)z^{-1})(1-(z_l+\Delta z_l)^*z) \quad (4)$$

When evaluating the errors of $D(z)$ on the unit circle $(D(z)_{|z=e^{j\omega}} = D(e^{j\omega}))$:

$$\begin{aligned} D(e^{j\omega_i}) &= c|\Delta z_i|^2 \prod_{l=1,l\neq i}^{L-1} |(1-(z_l+\Delta z_l)z_i^{-1}|^2 \quad (5) \\ &\approx c|\Delta z_i|^2 \prod_{l=1,l\neq i}^{L-1} |(1-z_l z_i^{-1})|^2 \end{aligned}$$

Taking the expected value on both sides, we obtain:

$$\begin{aligned} \mathcal{E}\{|\Delta z_i|^2\} &= \frac{\mathcal{E}\{D(e^{j\omega_i})\}}{c\prod_{l=1,l\neq i}^{L-1}|(1-z_l z_i^{-1})|^2} = \quad (6) \\ &= S_{\text{MUSIC}}\frac{\mathcal{E}\{D(e^{j\omega_i})\}}{L} \end{aligned}$$

where L is the number of samples and S_{MUSIC} can be seen as a sensitivity parameter of the root-MUSIC method and is equal to [8]:

$$\begin{aligned} S_{\text{MUSIC}} &= \frac{L}{c\prod_{l=1,l\neq i}^{L-1}|(1-z_l z_i^{-1})|^2} = \quad (7) \\ &= L\lim_{\omega\to\omega_i}\frac{|1-e^{j\omega_i}e^{-j\omega}|^2}{D(e^{j\omega})} \end{aligned}$$

After introduction of the derivative of $\mathbf{V}(\omega)$:

$$\mathbf{V}^{'T}(\omega) = \frac{1}{\sqrt{L}}\left(0, je^{j\omega}, 2je^{2j\omega}, ..., j(L-1)e^{(j(L-1)\omega)}\right) \quad (8)$$

and taking into account, that $D(j\omega) = \mathbf{V}^H(\omega)\mathbf{P_{noise}}\mathbf{V}(\omega)$, S_{MUSIC} becomes:

$$S_{\text{MUSIC}} = \frac{L}{\mathbf{V}'^H(\omega_i)\mathbf{P_{noise}}\mathbf{V}'(\omega_\mathbf{i})} \quad (9)$$

where: $\mathbf{P_{noise}} = \mathbf{I} - \mathbf{P_{signal}}$.

Considering, that:

$$\begin{aligned} D(j\omega) &= \mathbf{V}^H(\omega)(\mathbf{I}-\mathbf{P_{signal}})\mathbf{V}(\omega) = \quad (10) \\ &= 1 - \mathbf{V}^H(\omega)\left(\sum_{l=1}^{M}\mathbf{e}_l\mathbf{e}_l^H\right)\mathbf{V}(\omega) \end{aligned}$$

and, that estimated $\hat{\mathbf{e}}_l = \mathbf{e}_l + \eta_l$, where η is the respective estimation error , it is possible to formulate the MSE of the roots in root-MUSIC [8], as (see (6)):

$$\mathcal{E}\{|\Delta z_i|^2\} = \frac{S_{\text{MUSIC}}}{L}\cdot\frac{(L-M)\sigma_{noise}^2}{N}\cdot \quad (11)$$
$$\cdot\left(\sum_{k=1}^{M}\frac{\lambda_k}{(\lambda_k-\sigma_{noise}^2)^2}\right)\left|\mathbf{V}^H(\omega_i)\mathbf{e}_k\right|^2$$

where N is the dimension of the covariance matrix and M is the dimension of signal subspace.

In the case of single signal source with following parameters: power P_1, $\lambda_1^{signal} = L \cdot P_1$, $\lambda_1 = \lambda_1^{signal} + \sigma_{noise}^2$, and $\mathbf{e}_1 = \mathbf{V}(\omega_1)$, the sensitivity of root-MUSIC is given by [8] (see (9)):

$$S_{\mathrm{MUSIC}} = \frac{L}{\mathbf{V}_1^H(\omega_1)\mathbf{P}_{noise}\mathbf{V}_1(\omega_1)} = \frac{12L}{(L-1)(L+1)} \quad (12)$$

Using (11), the expected error of estimation will be [9]:

$$\mathcal{E}\{|\Delta z_1|^2\} = \frac{12L}{(L-1)(L+1)} \cdot \cdot \frac{\lambda_1 \sigma_{noise}^2 (L-1)}{LN(LP_1)^2} \approx \frac{12\sigma_{noise}^2}{L^2 P_1 N} \quad (13)$$

The analysis of more than one sources case is analytically very difficult (see [8]) and demands more arbitrary assumptions about the SNR and other signal parameters. Although reported results of numerical simulations show good correspondence to derived analytical expressions, their usefulness is quite limited.

4 Performance analysis of ESPRIT

The original ESPRIT (Estimation of Signal Parameter via Rotational Invariance Technique) was described by Paulraj, Roy and Kailath and later developed, for example, in [16]. It is based on a naturally existing shift invariance between the discrete time series which leads to rotational invariance between the corresponding signal subspaces. The shift invariance is illustrated below.

After the eigen-decomposition of the autocorrelation matrix as:

$$\mathbf{R}_x = \mathbf{U}^{*T}\mathbf{\Lambda}\mathbf{U} \quad (14)$$

it is possible to partition a matrix by using special *selector matrices* which select the first and the last $(M-1)$ columns of a $(M \times M)$ matrix, respectively:

$$\begin{aligned} \mathbf{\Gamma}_1 &= [\mathbf{I}_{M-1} | \mathbf{0}_{(M-1)\times 1}]_{(M-1)\times M} \\ \mathbf{\Gamma}_2 &= [\mathbf{0}_{(M-1)\times 1} | \mathbf{I}_{M-1}]_{(M-1)\times M} \end{aligned} \quad (15)$$

By using of matrices $\mathbf{\Gamma}$ two subspaces are defined, spanned by two subsets of eigenvectors as follows:

$$\begin{aligned} \mathbf{S}_1 &= \mathbf{\Gamma}_1 \mathbf{U} \\ \mathbf{S}_2 &= \mathbf{\Gamma}_2 \mathbf{U} \end{aligned} \quad (16)$$

For the matrices defined as $\mathbf{S}_1$ and $\mathbf{S}_2$ in (16), for every $\omega_k; k \in \mathbf{N}$, representing different frequency components, and matrix $\mathbf{\Phi}$, defined as:

$$\mathbf{\Phi} = \begin{bmatrix} e^{j\omega_1} & 0 & \cdots & 0 \\ 0 & e^{j\omega_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\omega_k} \end{bmatrix} \quad (17)$$

the following relation can be proven [10]:

$$[\mathbf{\Gamma}_1 \mathbf{U}]\mathbf{\Phi} = \mathbf{\Gamma}_2 \mathbf{U} \quad (18)$$

The matrix $\mathbf{\Phi}$ contains all information about frequency components. In order to extract this information, it is necessary to solve (18) for $\mathbf{\Phi}$. By using a unitary matrix (denoted as $\mathbf{T}$), the following equations can be derived:

$$\begin{aligned} \mathbf{\Gamma}_1(\mathbf{U}\mathbf{T})\mathbf{\Phi} &= \mathbf{\Gamma}_2(\mathbf{U}\mathbf{T}) \\ \mathbf{\Gamma}_1 \mathbf{U} \underbrace{(\mathbf{T}\mathbf{\Phi}\mathbf{T}^{*\mathbf{T}})}_{\text{eig. of } \mathbf{\Phi}} &= \mathbf{\Gamma}_2 \mathbf{U} \end{aligned} \quad (19)$$

In the further considerations the only interesting subspace is the *signal subspace*, spanned by signal eigenvectors $\mathbf{U}_s$. Usually it is assumed that these eigenvectors correspond to the largest eigenvalues of the correlation matrix and $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K]$. ESPRIT algorithm determines the frequencies $e^{j\omega_K}$ as the eigenvalues of the matrix $\mathbf{\Phi}$.

In theory, the equation (18) is satisfied exactly. In practice, matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ are derived from an estimated correlation matrix, so this equation does not hold exactly, it means that (18) represents an over-determined set of linear equations.

4.1 Errors of estimation

In the case of ESPRIT algorithm, the main source of errors is the estimate of the matrix $\mathbf{\Phi}$. The equation (18) can be solved for $\mathbf{\Phi}$ using Least Squares or Total Least Squares approach. The choice of approach has no influence on asymptotical performance of ESPRIT as shown in [8]).

The error in the matrix $\mathbf{\Phi}$, denoted as $\mathbf{\Delta}_\Phi$, causes errors in the eigenvalues of $\mathbf{\Phi}$. The error of an eigenvalue (here denoted as Δz_i), which can be regarded as a performance index of ESPRIT and can be approximated by:

$$\Delta z_i = \mathbf{p}_i \mathbf{\Delta}_\Phi \mathbf{e}_i \quad (20)$$

where $\mathbf{e}_i$ is the eigenvector of $\mathbf{\Phi}$ corresponding to the eigenvalue z_i, whereas $\mathbf{p}_i$ is the corresponding *left* eigenvector, so that $\mathbf{\Phi}\mathbf{e}_i = z_i \mathbf{e}_i$ and $\mathbf{p}_i \mathbf{\Phi} = z_i \mathbf{p}_i$.

From (18), the approximation of error $\Delta\mathbf{\Phi}$ can be derived using:

$$(\mathbf{S}_1 + \mathbf{\Delta}_{S_1})(\mathbf{\Phi} + \mathbf{\Delta}_\Phi) \approx (\mathbf{S}_2 + \mathbf{\Delta}_{S_2}) \quad (21)$$

as:

$$\mathbf{\Delta}_\Phi \approx \mathbf{S}_1^+ \mathbf{\Delta}_{S_2} - \mathbf{S}_1^+ \mathbf{\Delta}_{S_1} \mathbf{\Phi} \quad (22)$$

In the case of single signal source with following parameters: power P_1, $\lambda_1^{signal} = L \cdot P_1$, $U_1 = \mathbf{V}(\omega_1) = \frac{1}{\sqrt{L}}\left[1, e^{j\omega_1}, \ldots, e^{j(L-1)\omega_1}\right]^T$, the dominant term of MSE of ESPRIT is given by [9]:

$$\mathcal{E}\{|\Delta z_1|^2\} \approx \frac{2\sigma_{noise}^2}{L^2 P_1 N} \quad (23)$$

It can be noted that, approximately, the mean square error of MUSIC (13) is six times higher than the MSE of ESPRIT (23) in the case of a single signal source.

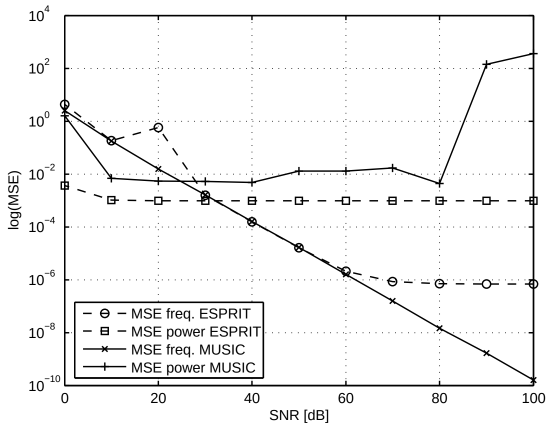

Figure 1. MSE of frequency and power estimation (ESPRIT, MUSIC) depending on SNR. Averaged 1000 independent runs.

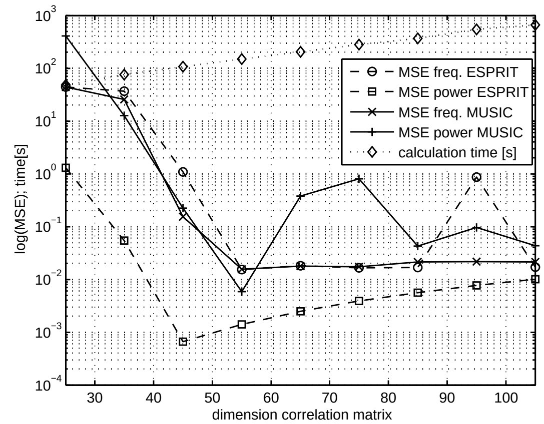

Figure 2. MSE of frequency and power estimation (ESPRIT, MUSIC) depending on the size of correlation matrix. Averaged 1000 independent runs.

5 Numerical performance comparison of MUSIC and ESPRIT

Several experiments with simulated, stochastic signals were performed, in order to compare different performance aspects of both parametric methods MUSIC and ESPRIT, compared to commonly used power spectrum (FFT based method). Testing signals are designed to belong to a class of waveforms commonly present in power systems. Each run of spectrum and power estimation is repeated many times (Monte Carlo approach) and then the mean-square error (MSE) is computed.

Parameters of test signals are as follows:

- one main 50 Hz harmonic with unit frequency and amplitude,
- random number of higher odd harmonic components with random amplitudes (lower than 0.5) and random initial phases (from 0 to 8 higher harmonics); if not otherwise specified,
- sampling frequency 5000 Hz,
- each signal generation repeated 1000-100000 times with re-initialization of random number generator,
- SNR=20 dB if not otherwise specified,
- size of the correlation matrix = 50 if not otherwise specified,
- signal length 200 samples if not otherwise specified.

Selected results are presented below:

The relation to signal-to-noise ratio (Fig. 1) reveals strong dependence of the accuracy of the frequency estimation on SNR and almost no dependence of amplitude estimation (with exception to MUSIC which shows higher errors for very low and very high noise levels).

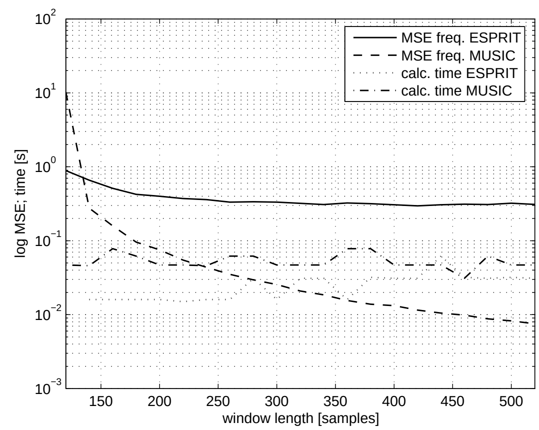

Figure 3. MSE of frequency and power estimation (ESPRIT, MUSIC) and average calculation time depending on the data window length. Averaged 10000 independent runs.

The size of the correlation matrix must be chosen optimally, as can be seen from Fig. 2. In the case of both methods, there exists an optimum of the size (relative to the data length) which assures the lowest estimation error. Most probably, there exists a trade-off between increasing accuracy of the estimated correlation matrix and increasing numerical errors with the matrix size.

The data sequence length influences the accuracy of MUSIC method than ESPRIT stronger (Fig. 3). For shorter data lengths ESPRIT method is faster to calculate; this advantage vanishes with increasing number of data samples taken into calculation.

In Fig. 4 the results are shown where the amplitude of higher harmonics was gradually increased from 0.1 to

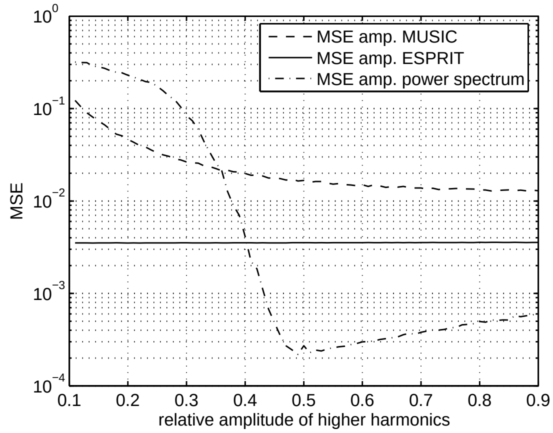

Figure 4. MSE of amplitude estimation (ESPRIT, MUSIC, power spectrum) depending on the relative amplitude of higher harmonics amplitudes. Averaged 10000 independent runs.

0.9 of the fundamental 50 Hz component. In such way the problem of masking of the higher low-amplitude harmonics components by a strong fundamental component was investigated. The results show an extremely high masking effect in the case of power spectrum, while MUSIC and ESPRIT methods show very little dependence (almost no dependence in the case of ESPRIT method). This is a very important feature which partially explains excellent performance of parametric methods in the task of calculation of power quality indices.

6 New power quality indices

A number of power system applications require an accurate knowledge of the spectral components of current and voltage waveforms. Especially, the power quality field attracts increasing interest. The main application of spectral components in the field of Power Quality refers to the calculation of waveform distortion indices. Several indices are in common use for the characterization of waveform distortions. However, they generally refer to periodic signals which allow an "exact" definition of harmonic components and require only a numerical value to characterize them. The waveforms obtained from a power supply of a typical DC arc furnace plant are analyzed. The IEC groups and subgroups [17] are estimated by using FFT and the results are compared with advanced methods: the ESPRIT and the root-MUSIC methods.

6.1 Experimental setup and preprocessing

The simulated DC arc furnace plant, consists of a DC arc furnace connected to a medium voltage ac busbar with two parallel thyristor rectifiers that are fed by transformer secondary windings with Δ and Y connections, respectively. The power supply of arc furnace is modelled using Power System Blockset in Matlab. The electric arc was simulated with a Chua's circuit, which shows good similarity with real measurements [2].

The medium voltage busbar is connected to the high voltage busbar with a HV/MV transformer whose windings are Δ-Y connected. The power of the furnace is 80 MW. The other parameters are: Transformer T_1 - 80 MVA, 220kV/21kV; Transformer T_2 - 87 MVA, 21kV/0.638kV/0.638kV.

The evaluation of harmonic and interharmonic subgroups has been made using the following assumptions: window length - 200 ms non overlapping. For each window, the n^{th} harmonic subgroup includes all spectral components inside the frequency interval $[n \cdot f_1 - 7.5, n \cdot f_1 + 7.5]$ Hz. The interharmonic subgroup includes all the spectral components inside the frequency interval $]n \cdot f_1 + 7.5, (n+1) \cdot f_1 - 7.5[$ Hz [18]. When applying parametric methods filters have been applied for pre-processing of data. In particular: a bandstop Butterworth IIR filter blocking the main (50Hz) component; a lowpass (40 Hz) Butterworth IIR filter applied for analyzing interharmonics groupings for $n = 0.5$ and bandpass Butterworth IIR filters for other subgroups,

The amplitudes of the harmonic and interharmonic subgroups $C_{n-200ms}$ and $C_{n+0.5-200ms}$ can be evaluated, respectively, as:

$$C^2_{n-200ms} = \sum_{k=-1}^{1} C^2_{10n+k} \quad (24)$$

$$C^2_{n+0.5-200ms} = \sum_{k=-2}^{8} C^2_{10n+k} \quad (25)$$

where C_{10n+k} are the spectral components (RMS value) of the spectral (DFT) output.

According to the cited norms the relations (24) and (25) are computed on 15 successive 200 ms windows in order to obtain values of the progressive average inside a 3

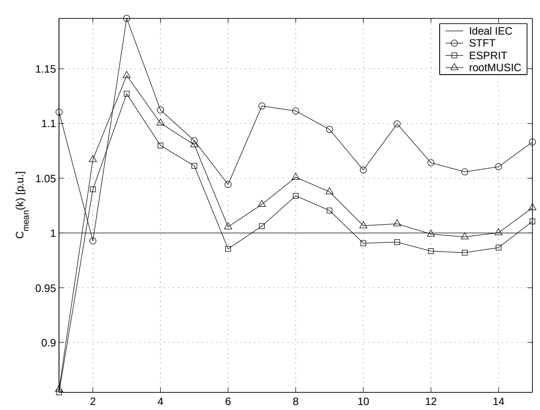

Figure 5. Progressive average of the fifth harmonic subgroup of the voltage.

seconds interval. Obtained results were compared to the "Ideal IEC" which is a value of interharmonic or subharmonic subgroups computed over the whole interval of 3 seconds [1] of the waveform under investigation.

Selected results of the progressive average of harmonic subgroups calculation of the waveforms of voltage and current are presented in Fig. 5. From the analysis of other results it can be noted that the results obtained by using "Ideal IEC" give a very high value of the progressive average in the neighbourhood of the fundamental harmonic referred to the IEC interharmonic subgroups. This phenomenon can be explained by the problem of spectral leakage present in the FFT based algorithms (STFT) and therefore the high energy content leaking into the neighborhood of the fundamental component of the voltage waveform. As shown in the Fig. 5, the high resolution methods give results closer to the "Ideal IEC" than the ones obtained with STFT for the evaluation of the progressive average.

7 Conclusions

Performed analysis and experiments allow concluding that parametric spectral estimation methods are reliable and accurate tool for the analysis of waveforms in power systems and its properties can be used for diagnostic and power quality applications.

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