Power Quality Evaluation using Advanced Spectrum Estimation Methods

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***Abstract*— The authors show that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices (such as inter/harmonic groups and subgroups) .**

The comparison of the frequency and amplitude estimation error, based on numerical simulations is presented. Presentation of selected power quality indices is then followed by comparison of estimation error in the case of application of FFT-based algorithms and parametric methods.

Investigated waveforms are typical for dc arc furnace plant. MUSIC and ESPRIT high-resolution methods are used to analyze waveforms in a supply system of a DC arc furnace. Reduction of variance of results is achieved by using robust averaging procedure (winsorized mean).

***Index Terms*-- distortion, eigenvalues and eigenfunctions, electrical engineering, estimation, frequency domain analysis, furnaces, industrial power system harmonics, measurement, parameter estimation, quality control.**

I. Introduction

THE quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The proliferation of nonlinear loads connected to power systems has triggered a growing concern with power quality issues. The inherent operation characteristics of these loads deteriorate the quality of the delivered energy, and increase the energy losses as well as decrease the reliability of a power system [6,2,1].

The methods of power quality assessment in power systems are almost exclusively based on Fourier Transform. The crucial drawback of the Fourier Transform-based methods is that the length of the window is related to the frequency resolution. Moreover, to ensure the accuracy of Discrete Fourier Transform, the sampling interval of analysis should be an exact integer multiple of the waveform fundamental period [4]Parametric spectral methods, such as ESPRIT or MUSIC [7] do not suffer from such inherent limitations of resolution or dependence of estimation error on the window length (phase dependence of the estimation error).

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The resolution of these methods is to high degree independent on signal-to-noise ratio and on the initial phase of the harmonic components.

The author argues that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices [6].

Investigated signals, originating from a power supply of a typical for dc arc furnace plant [1], contain significant stochastic components, due to stochastic nature of electric arc. As a consequence, measurements of selected power quality parameters show quite high variability. In order to alleviate this problem, the use of robust averaging [5] is proposed.

The paper is composed as follows: After the short description of parametric methods (ESPRIT and MUSIC) and selected robust location measures, the comparison of the frequency and amplitude estimation error, based on numerical simulation is presented. Next part presents basics of selected power quality indices (harmonic sub/groups), followed by comparison of estimation error in the case of application of FFT-based algorithms and parametric methods as well as presentation of advantages of use of robust averaging.

II. Parametric Methods

The ESPRIT and the root-Music spectrum estimation methods are based on the linear algebraic concepts of subspaces and so have been called “subspace methods” [7]; the model of the signal in this case is a sum of sinusoids in the background of noise of a known covariance function.

A. MUSIC

The MUSIC method assumes the model of the signal as:

$$\mathbf{x} = \sum_{i=1}^{p} A_i \mathbf{s}_i + \eta \;; \quad A_i = |A_i| e^{j\phi_i} \tag{1}$$

where $\mathbf{s}_i = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$, A_i –amplitudes of the signal components, N – number of signal samples, p – number of the components, η - noise, ϕ_i - components’ frequencies. The autocorrelation matrix of the signal is estimated from signal samples as:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathrm{E}\left\{ A_i A_i^* \right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} \tag{2}$$

N-p smallest eigenvalues of the correlation matrix (matrix dimension *N>p+1*) correspond to the noise subspace and *p* largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

The matrix of noise eigenvectors of the above matrix (2) is used

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{p+1} & \mathbf{e}_{p+2} & \cdots & \mathbf{e}_N \end{bmatrix} \quad (3)$$

to compute the projection matrix for the noise subspace:

$$\mathbf{P}_{noise} = \mathbf{E}_{noise}\mathbf{E}_{noise}^{*T} \quad (4)$$

which, by using an auxiliary vector $\mathbf{w} = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$ allows computation of projection of vector **w** onto the noise subspace as:

$$\begin{aligned} &\mathbf{w}^{*T}\mathbf{P}_{noise}\mathbf{w} = \mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w} = \\ &= \sum_{i=p+1}^{N} E_i\left(e^{j\omega}\right)E_i^*\left(e^{j\omega}\right) \xrightarrow{Z} \sum_{i=p+1}^{N} E_i(z)E_i^*\left(1/z^*\right) \end{aligned} \quad (5)$$

The last polynomial in (5) has *p* double *roots* lying on the unit circle, which angular positions correspond to the frequencies of the signal components. This method of finding the frequencies is therefore called *root-MUSIC*.

After the calculation of the frequencies, the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relations:

$$\mathbf{e}_i^{*T}\mathbf{R}_x\mathbf{e}_i = \lambda_i \quad \text{and} \quad \mathbf{R}_x = \sum_{i=1}^{p} P_i s_i s_i^{*T} + \sigma_0^2\mathbf{I} \quad (6)$$

and solving for P_i – components' powers.

B. ESPRIT

The original ESPRIT algorithm [7] is based on naturally existing *shift invariance* between the discrete time series, which leads to rotational invariance between the corresponding signal subspaces.

The assumed signal model is as in (1). The eigenvectors **E** of the autocorrelation matrix of the signal define two subspaces (signal and noise subspaces) by using two selector matrices $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$.

$$\mathbf{S}_1 = \mathbf{\Gamma}_1\mathbf{E} \quad \text{and} \quad \mathbf{S}_2 = \mathbf{\Gamma}_2\mathbf{E} \quad (7)$$

The rotational invariance between both subspaces leads to the equation:

$$\mathbf{S}_1 = \mathbf{\Phi}\mathbf{S}_2 \quad (8)$$

where:

$$\mathbf{\Phi} = \begin{bmatrix} e^{j\omega_1} & 0 & \cdots & 0 \\ 0 & e^{j\omega_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\omega_p} \end{bmatrix} \quad (9)$$

The matrix $\mathbf{\Phi}$ contains all information about p components' frequencies. Additionally, the TLS (total least-squares) approach assumes that both estimated matrices **S** can contain errors and finds the matrix $\mathbf{\Phi}$ as minimization of the Frobenius norm of the error matrix. Amplitudes of the components can be found in similar way as with MUSIC method using (6).

III. Accuracy Of Parametric Methods

The comparison of mean square error of the frequency and amplitude estimation is useful for practical assessment of accuracy of both methods: root-MUSIC and ESPRIT. Both methods are similar in the sense that they are both eigendecomposition-based methods, which rely on decomposition of the estimated correlation matrix into two subspaces: noise and signal subspace. On the other hand, MUSIC uses the noise subspace to estimate the signal components while ESPRIT uses the signal subspace. In addition, the approach is in many points different. Numerous publications were dedicated to the analysis of the performance of the aforementioned methods. Unfortunately, due to many assumed simplifications, and the complexity of the problem, published results are often contradictory and sometimes misleading.

Several experiments with simulated, stochastic signals were performed, in order to compare performance aspects of both parametric methods MUSIC and ESPRIT. Testing signal is designed to belong to a class of waveforms often present in power systems [4,1,2]. Each run of spectrum and power estimation is repeated many times (Monte Carlo approach) and the mean--square error (MSE) is computed.

Parameters of test signals:

- one 50 Hz main harmonic with unit amplitude.
- random number of higher odd harmonic components with random amplitude (lower than 0.5) and random initial phase (from 0 to 8 higher harmonics).
- sampling frequency 5000 Hz.
- each signal generation repeated 1000 times with re-initialization of random number generator.
- SNR=40 dB if not otherwise specified.
- size of the correlation matrix = 50 if not otherwise specified.
- signal length 200 samples if not otherwise specified.

In Fig. 1 it can be seen that the performance of both methods is similar for frequency estimation, only MUSIC performs better for SNR higher than 60 dB and lower than 20 dB. The error of power estimation is significantly lower for ESPRIT algorithm in the whole SNR range. From the Fig. 2 it appears that there exists an optimal size of the correlation matrix which assures the lowest possible estimation error (tradeoff between accuracy of estimation of the correlation matrix and increase of numerical errors with the size of the correlation matrix). In Fig. 3 appears a sharp decrease of the estimation error for a specific length of the data sequence (for ESPRIT method, MUSIC results are similar). ESPRIT method performs better for both problems shown in Figs. 2 and 3.

IV. Power Quality Indices

A number of power system applications require an accurate knowledge of the spectral components of non-stationary current and voltage waveforms.

The main application of spectral components in the field of Power Quality refers to the calculation of waveform distortion indices [2,3].

Several indices are in common use for the characterization of waveform distortions. However, they generally refer to periodic signals, which allow an „exact" definition of harmonic components and deliver only one numerical value to characterize them.

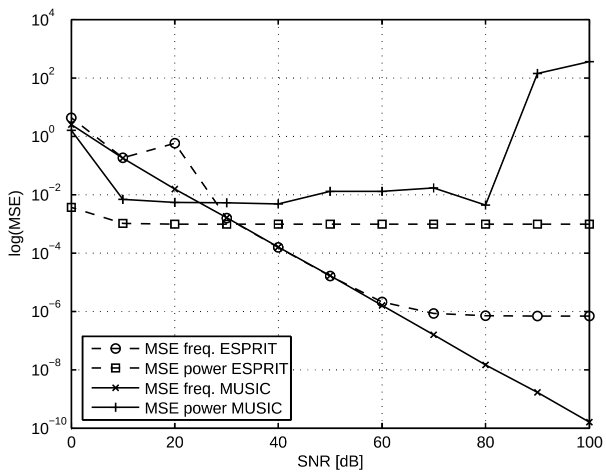

Fig. 1. MSE of frequency and power estimation depending on SNR.

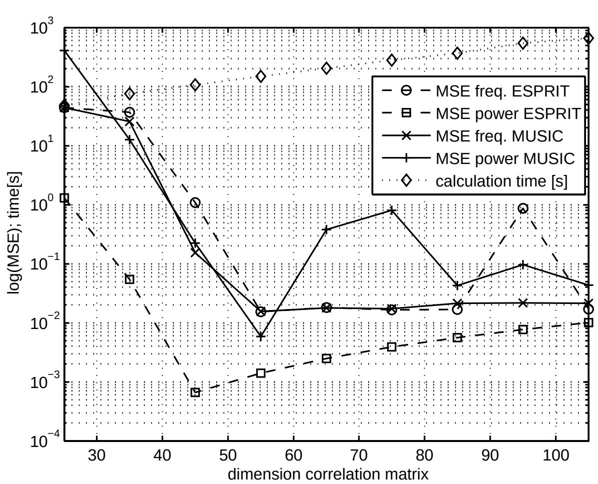

Fig. 2. MSE of frequency and power estimation depending on the size of the correlation matrix.

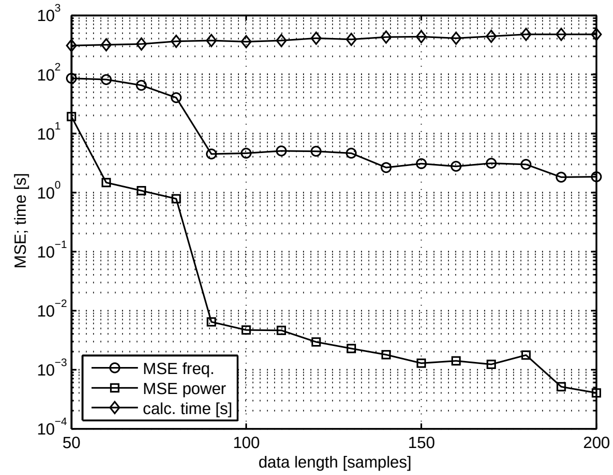

Fig. 3. MSE of frequency and power estimation (ESPRIT) depending on the data window length.

When the spectral components are time varying in amplitude and/or in frequency (as in case of non-stationary signals), a wrong use of the term harmonic can arise and several numerical values are needed to characterize the time-varying nature of each spectral component of the signal.

I this paper we compare the IEC harmonic and interharmonic subgroups calculation as introduced by the IEC Standard drafts (IEC Std 61000-4-7, 61000-4-30).

Cited IEC Standard drafts - with reference to DFT with 5 Hz resolution in frequency (200 ms of window length for 50 Hz fundamental frequency) - introduce the concept of harmonic and interharmonic groupings and characterize the waveform distortions with the amplitudes of these groupings, as shown in Fig. 5.

V. Robust Averaging

Averaging is probably the most common basic statistical procedure in experimental science [5]. It is used for estimating the location of data (or "central tendency") in the presence of random variations among the observations, which can be removed by this procedure. Data variations can be a result of variations in the phenomenon of interest or of some unavoidable measuring errors. In many cases, variations are caused by other phenomena, which occur simultaneously. In signal processing terms, this can be considered as contamination of useful "signal", such as presence of characteristic harmonics in the waveform, by useless "noise", caused by stochastic fluctuations of the electric arc. Averaging allows the cancellation of the noise by averaging of spectra – in such way only repetitive part of the waveform spectrum remains in the averaged result. Averaging is typically done using arithmetic mean, which is the most widely known estimator of the location of the data.

Numerous publications are devoted to the problem of robustness of estimation of the data location (for details, see [5]). In the presence of outliers, i.e. "the data that deviate from the pattern set by the majority of the data set", has lead to the development of *robust* location measures. Robustness of an estimator is measured by the *breakdown value*, which tells us how many data points need to be replaced by arbitrary values in order to make the estimator explode (tend to infinity) or implode (tend to zero). The arithmetic mean is not robust and has 0% breakdown value whilst median is very robust with breakdown value of 50%.

Many location estimators can be presented in unified way by ordering the values of data and then applying the weight function:

$$\mu_r = \sum_{i=1}^{N} w_i x_{(i)} \tag{10}$$

where w_i is a function designed specifically to reduce the influence of certain observations (data points) in form of weighting and $x_{(i)}$ represents the ordered data. For the arithmetic mean it holds $w_i = \frac{1}{N}$.

The arithmetic mean is a standard location estimator used for averaging is not robust. In the case of arithmetic mean, only one outlier may make the estimate infinitely large or small. The breakdown value is: $\lim_{N\to\infty} \frac{1}{N} = 0$ (Fig. 4).

Winsorized mean is one of many robust estimators [5]. The tails of the distribution of the data are not simply ignored (It can lead to the loss of information and should be avoided when the sample size is small) as with trimmed mean [5].

Winsorized mean replaces each observation in each α fraction (p=αN) of the tail of the distribution by the value of the nearest unaffected observation. Weight w_i becomes here (see Fig.4)

$$w_i = \begin{cases} 0,\ i \le p \text{ or } i \ge N - (p-1) \\ \frac{p+1}{N},\ i = p+1 \text{ or } i = N - p \\ \frac{1}{N},\ p+2 \le i \le N-(p+1) \end{cases} \tag{11}$$

Usually, the values in the range $0 \le p \le 0.25N$ are chosen, depending on the heaviness of the tails of the distribution.

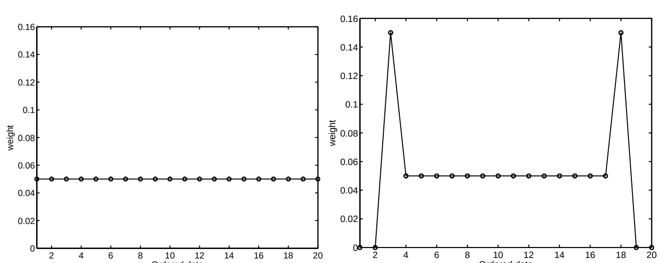

Fig. 4. Weights for different location estimators. Arithmetic mean (left) uses the same weight 1/N for all observations. Winsorized mean (right) shifts the weights of the ignored extreme observations to the last accepted data points.

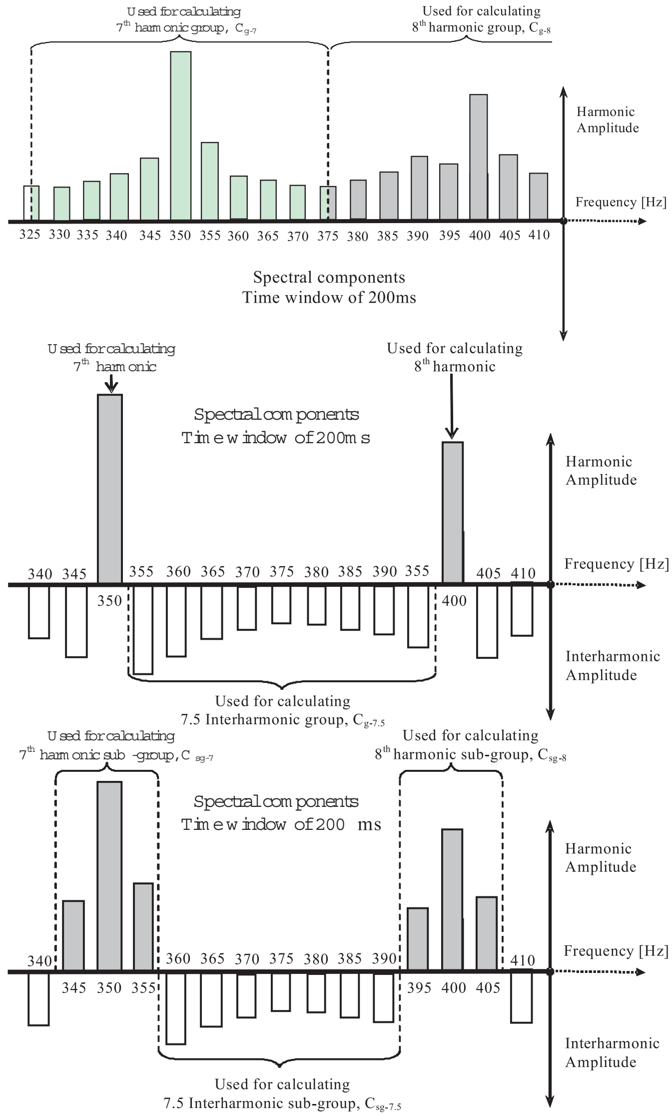

Fig. 5. Examples of harmonic (↑) and interharmonic (↓) (sub)groups according to IEC Standard drafts 61000-4-7 and 61000-4-30.

VI. Experimental Setup and Results

The waveforms obtained from a power supply of a typical for dc arc furnace plant are analyzed, (as shown in Fig. 6), which consists of a dc arc connected to a medium voltage ac busbar with two parallel thyristor rectifiers that are fed by transformer secondary winding [1]. The IEC groups and subgroups were estimated by using DFT and the results are compared to those obtained using subspace methods: the ESPRIT and the root-MUSIC.

In order to compare the different processing techniques, we used a *reference technique*. We assumed as reference the technique proposed in [1], named as “Ideal IEC”, where the respective harmonic groupings are computed on the whole interval of 3 s. In Fig. 7-11 the progressive average of the (inter-) harmonic groups of the current and voltage are shown; the value of “Ideal IEC” is shown as constant unit value, whereas other results are scaled with respect to it.

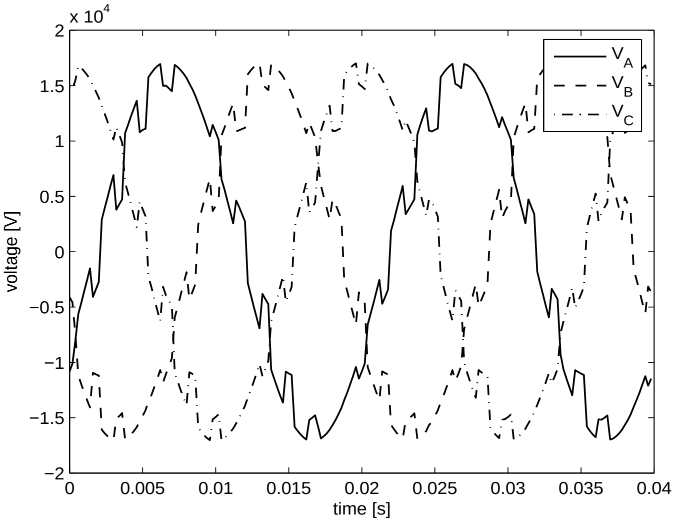

Fig. 6. Voltage waveform at the power supply of the dc arc furnace.

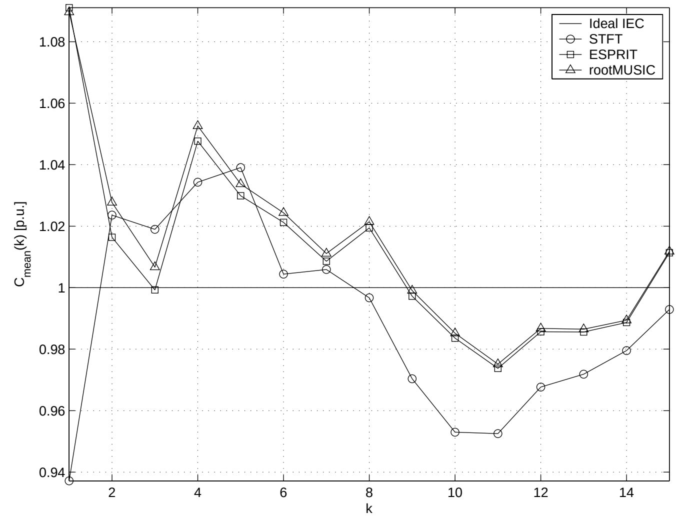

Fig. 9. Progressive average of the 13th harmonic component of the current.

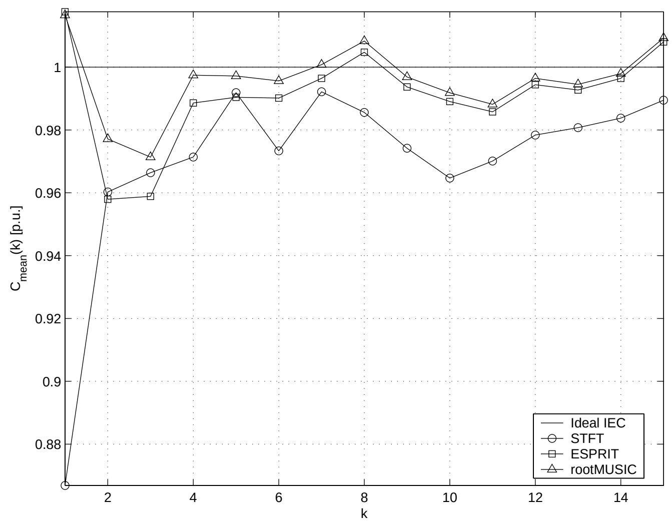

Fig. 7. Progressive average of the 11th harmonic component of the current.

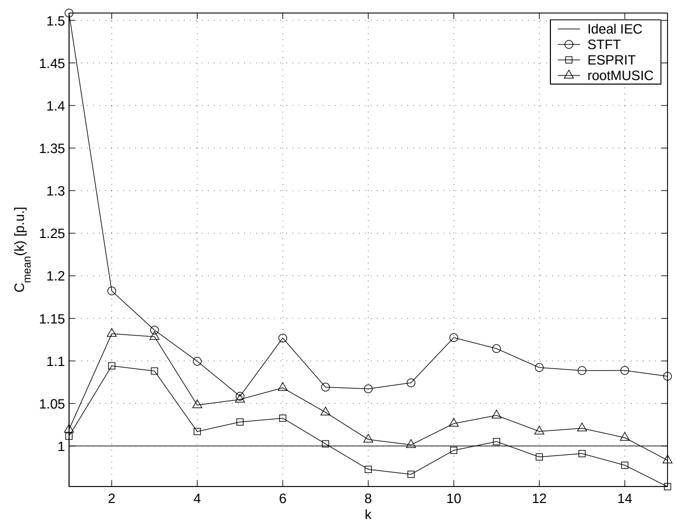

Fig. 10. Progressive average of the 2nd interharmonic component of the current.

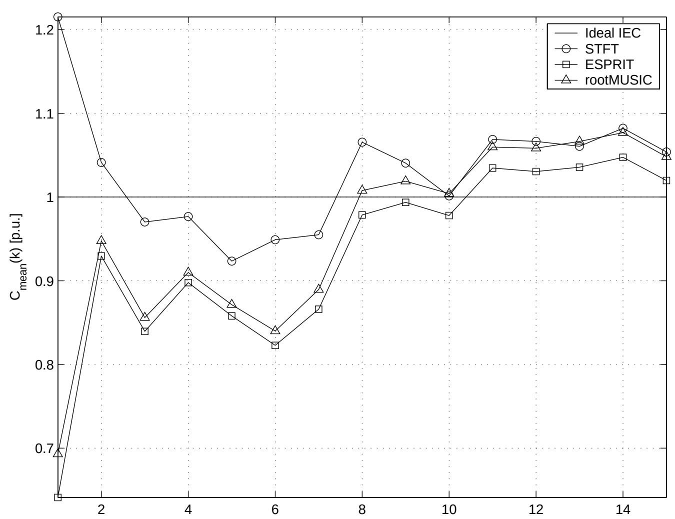

Fig. 8. Progressive average of the 7th harmonic component of the voltage.

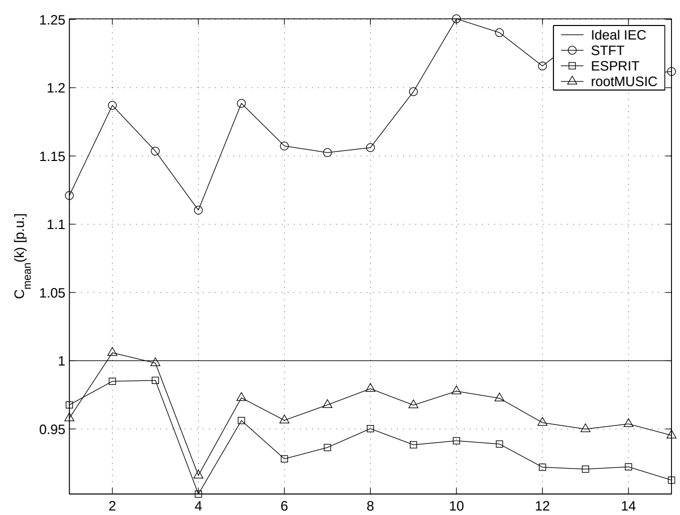

Fig. 11. Progressive average of the 2nd interharmonic component of the voltage.

From Fig. 7-11 it can be seen that MUSIC and ESPRIT methods usually show lower estimation error.

In some cases, especially for harmonic components (Fig.8), MSE can be higher than the error obtained using FFT. In the case of interharmonic groupings, the MSE is always significantly lower, when using parametric methods (Fig. 10, 11).

In Table I the results of harmonics and interharmonics subgroups estimation of the current are summarized. The mean-square error is calculated as a value relative to the value of respective quantities. From over 1000 independent calculations we may conclude that the ESPRIT method offers an average reduction of the error of harmonic groups and subgroups estimation by 55% and MUSIC method by 51%, comparing to DFT-based method.

When comparing values of power quality indices obtained from different parts of the same recorded waveform, a high variability of results appears. To alleviate this problem, authors used winsorized mean to compute averages from spectral data. Winsorized mean was chosen because of relatively small number of data, taken into averaging procedure.

When using the value of α=0.2 (11) which means that 20% of ordered data points were discarded and replaced by nearest unaffected data. In such way the outliers were removed and replaced by data, which are assumed to belong to "true" spectral content of investigated waveform. The use of winsorized mean instead of usual arithmetic mean allowed reducing the variance of results by nearly 35%.

TABLE I
RELATIVE MSE OF THE PROGRESSIVE AVERAGE OF HARMONIC AND INTERHARMONIC SUBGROUPS ESTIMATION

Method	MSE of harmonics estimation	MSE of interharmonics estimation
DFT	0.059	0.791
ESPRIT	0.021	0.169
MUSIC	0.027	0.201

VII. CONCLUSIONS

In the paper we compared the performance of parametric spectrum estimation methods (MUSIC and ESPRIT). The performance was estimated as accuracy of estimation of frequencies and amplitudes of harmonic multi-component signals. The results show slightly better performance of ESPRIT over MUSIC method for applications where the analyzed waveforms consists of multiple harmonics with variable amplitudes and random initial phases (waveforms often encountered in power system analysis). Based on these results, optimal parameters were chosen for the following calculations of selected power quality indices.

As a practical application we choose the calculation of harmonic and interharmonic subgroups (IEC Std 61000-4-7, 61000-4-30). Both parametric methods were used and the results compared to those obtained with commonly used DFT-based algorithms. Results show that the highest improvement of accuracy can be obtained by using the ESPRIT method (especially for interharmonics estimation), closely followed by MUSIC method, which outperform classical DFT approach by over 50%. Partially stochastic nature of investigated arc furnace waveforms caused high variability of calculated power quality indices. The use of robust averaging (winsorized mean) helped to reduce this unwanted variability.

VIII. REFERENCES

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IX. BIOGRAPHIES

Tadeusz Lobos received the M.Sc., Ph.D. and Habilitate Doctorate (Dr Sc.) degrees, all in electrical engineering, from the Wroclaw University of Technology., Poland in 1960, 1967, and 1975, respectively. He has been with the Department of Electrical Engineering, Wroclaw University of Technology, since 1960, where he became a Full Professor in 1989. From 1982 to 1986, he worked at the University of Erlangen-Nuremberg, Germany. His current research interests are in the areas of transients in power systems, control and protection, and especially application of neural networks and signal processing methods in power systems. The Alexander von Humboldt Foundation, Germany awarded Dr. Lobos a Research Fellowship in 1976 and he spent this fellowship at the Technical University of Darmstadt. He was awarded the Humboldt Research Award, Germany in 1998.

Zbigniew Leonowicz (M' 2003) received the M.Sc. and PhD degrees in electrical engineering from the Wroclaw University of Technology, Poland in 1997 and 2001, respectively. The NATO Organization awarded Dr. Leonowicz an Advanced Fellowship in 2001 - Technical University of Dresden (Germany). From 2002 to 2004 he worked as research scientist at the Brain Science Institute, RIKEN (Japan). He has been with the Department of Electrical Engineering at Wroclaw University of Technology (Poland), since 1997. His current research interests include modern digital signal processing methods applied to power system analysis.