Harmonics and Interharmonics Estimation Using Advanced Signal Processing Methods

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Abstract: Modern frequency power converters generate a wide spectrum of harmonics components. Large converters systems can also generate non-characteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long. A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. Min-norm harmonic retrieval method is an example of high-resolution eigenstructure-based methods. The Prony method as applied for signal analysis was also tested for this purpose. Both the high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonics frequencies. To investigate the methods several experiments were performed using simulated signals, current waveforms at the output of a simulated frequency converter and current waveforms at the output of an industrial frequency converter. For comparison, similar experiments were repeated using the FFT. The comparison proved the superiority of the new methods. However, their computation is much more complex than FFT.

Keywords: Discrete Fourier Transform, frequency conversion, frequency measurement, harmonic analysis, power system, Prony method.

I. INTRODUCTION

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The liberalization of European energy market will strengthen the competition and is expected to drive down the energy prices. This is reason for the requirements concerning the power quality. The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Modern frequency power converters generate a wide spectrum of harmonics components which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of noncharacteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage [1,2]. Interharmonics are defined as non-integer harmonics of the main fundamental under consideration. The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

Interharmonics are considered more damaging than characteristic harmonics components of the distorted signals. Their emission is specified lower than those are for the harmonics.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc [3,4,5,6,7,8]. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels. The linear methods of system spectrum estimation (Blackman-Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in these methods, the estimated spectrum can be a smeared version of the true spectrum [9].

These methods usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long [1]. It is very important to develop better tools of interharmonic estimation to avoid possible damages due to its influence.

In this paper the frequencies of signal components are estimated using the Prony model and min-norm method.

Prony method is a technique for modeling sampled data as a linear combination of exponentials [8]. Although it is not a spectral estimation technique, Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics. The paper [10] presents a new method of real-time measurement of power system frequency based on Prony model.

The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods" [11]. Their resolution is theoretically independent of the SNR. The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function. Pisarenko first observed that the zeros of the z-transform of the eigenvector, corresponding to the minimum eigenvalue of the covariance matrix, lie on the unit circle, and their angular positions correspond to the frequencies of the sinusoids. In a later development it was shown that the eigenvectors might be divided into two groups, namely, the eigenvectors spanning the signal space and eigenvectors spanning the orthogonal noise space. The eigenvectors spanning the noise space are the ones whose eigenvalues are the smallest and equal to the noise power. One of the most important techniques, based on the Pisarenko's approach of separating the data into signal and noise subspaces is the min-norm method.

To investigate the ability of the methods several experiments were performed. Simulated signals, current waveforms at the output of a simulated three-phase frequency converter as well as current waveforms at the output of an industrial frequency converter were investigated. For comparison, similar experiments were repeated using the FFT.

II. PRONY METHOD

Assuming the N complex data samples $x[1],..., x[N]$ the investigated function can be approximated by M exponential functions:

$$y[n] = \sum_{k=1}^{M} A_k e^{(a_k + jw_k)(n-1)T_p + jy_k} \tag{1}$$

where

$n = 1,2,..., N$

T_p – sampling period,

A_k - amplitude

a_k – damping factor,

w_k – angular velocity

y_k – initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n] = \sum_{k=1}^{M} h_k z_k^{n-1} \tag{2}$$

where

$$h_k = A_k e^{jy_k}$$

$$z_k = e^{(a_k + jw_k)T_p}$$

The estimation problem bases on the minimization of the squared error over the N data values:

$$d = \sum_{n=1}^{N} |e[n]|^2 \tag{3}$$

where

$$e[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{M} h_k z_k^{n-1} \tag{4}$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data may be made.

Consider the M-exponent discrete-time function:

$$x[n] = \sum_{k=1}^{M} h_k z_k^{n-1} \tag{5}$$

The p equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_M^0 \\ z_1^1 & z_2^1 & \cdots & z_M^1 \\ \vdots & \vdots & & \vdots \\ z_1^{M-1} & z_2^{M-1} & \cdots & z_M^{M-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[M] \end{bmatrix} \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the z_k exponents as its roots:

$$\begin{aligned} F(z) &= \prod_{k=1}^{M} (z - z_k) = \\ &= (z - z_1)(z - z_2)\ldots(z - z_M) \end{aligned} \tag{7}$$

The polynomial may be represented as the sum:

$$\begin{aligned} F(z) &= \sum_{m=0}^{M} a[m] z^{M-m} = \\ &= a[0]z^M + a[1]z^{M-1} + \ldots + a[M-1]z + a[M] \end{aligned} \tag{8}$$

Shifting the index on (5) from n to n-m and multiplying by the parameter $a[m]$ yield:

$$a[m]x[n-m] = a[m]\sum_{k=1}^{M} h_k z_k^{n-m-1} \tag{9}$$

The (9) can be modified into:

$$\begin{aligned} &\sum_{m=0}^{M} a[m]x[n-m] = \\ &= \sum_{k=1}^{M} h_k z_k^{n-M} \left\{ \sum_{m=0}^{M} a[m] z_k^{M-m-1} \right\} \end{aligned} \tag{10}$$

The right-hand summation in (10) may be recognize as polynomial defined by (8), evaluated at each of its roots z_k yielding the zero result:

$$\sum_{m=0}^{M} a[m]x[n-m] = 0 \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2M$, In the overdetermined data case, the linear equation (11) must be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m] = e[n] \tag{12}$$

The estimation problem bases on the minimization of the total squared error:

$$E = \sum_{n=M+1}^{N} \left|e[n]\right|^2 \tag{13}$$

III. MIN-NORM METHOD

The min-norm method involves projection of the signal vector:

$$\boldsymbol{s_i} = \begin{bmatrix} 1 & e^{jw_i} & \dots & e^{j(N-1)w_i} \end{bmatrix}^T \tag{14}$$

onto the entire noise subspace.

We consider a random sequence $\boldsymbol{x}$ made up of M independent signals in noise.

$$\boldsymbol{x} = \sum_{i=1}^{M} A_i \boldsymbol{s_i} + h ; \quad A_i = |A_i| e^{jf_i} \tag{15}$$

If the noise is white, the correlation matrix is

$$\mathbf{R_x} = \sum_{i=1}^{M} E\left\{A_i A_i^*\right\} \boldsymbol{s_i} \boldsymbol{s_i^T} + s_0^2 \mathbf{I} \tag{16}$$

N-M smallest eigenvalues of the correlation matrix (matrix dimension $N>M+1$) correspond to the noise subspace and M largest (all greater than s_0^2) corresponds to the signal subspace.

We define the matrix of eigenvectors:

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{M+1} & \mathbf{e}_{M+2} & \dots & \mathbf{e}_N \end{bmatrix} \tag{17}$$

Min-norm method uses one vector **d** for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element equal to one. These conditions are expressed by the following equations

$$\begin{aligned} \mathbf{d} &= \mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{d} \\ \mathbf{d}^{*T}\ell &= 1 \end{aligned} \tag{18}$$

We can express (18) in one equation

$$\mathbf{d}^{*T}\ell = \left(\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{d}\right)^{*T}\ell = \mathbf{d}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\ell = 1 \tag{19}$$

and form the lagrangian

$$\begin{aligned} \mathrm{L} = \mathbf{d}^{*T}\mathbf{d} + m\left(1 - \mathbf{d}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\ell\right) + \\ + m^*\left(1 - \ell^T\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{d}\right) \end{aligned} \tag{20}$$

Gradient of (20) has the form

$$\nabla_{\mathbf{d}^*}\mathrm{L} = \mathbf{d} - m\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\ell = \mathbf{0} \tag{21}$$

where m is chosen in such way that the first element of the vector is equal to one.

For this purpose we present $\mathbf{E}_{noise}$ in the form

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{c}^{*T} \\ \mathbf{E}'_{noise} \end{bmatrix} \tag{22}$$

where $\mathbf{c}^{*T}$ is the upper row of the matrix. Hence $\mathbf{c} = \mathbf{E}_{noise}^{*T}\ell$.

From (21) and (22) results that the first element of the vector **d** is equal to $m\mathbf{c}^{*T}\mathbf{c}$.

Finally, vector **d** is equal to

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T}\mathbf{c}}\mathbf{E}_{noise}\mathbf{c} = \begin{bmatrix} 1 \\ \left(\mathbf{E}'_{noise}\mathbf{c}\right)/\left(\mathbf{c}^{*T}\mathbf{c}\right) \end{bmatrix} \tag{23}$$

Pseudospectrum defined with the help of **d** is defined as.

$$\hat{P}\left(e^{jw}\right) = \frac{1}{\left|\mathbf{w}^{*T}\mathbf{d}\right|^2} = \frac{1}{\mathbf{w}^{*T}\mathbf{d}\mathbf{d}^{*T}\mathbf{w}} \tag{24}$$

where **w** is defined as in (14).

IV. EXPERIMENTS WITH SIMULATED WAVEFORMS

Several experiments were performed with the signal waveform described in [2]. The investigated signal is characteristic for DC arc furnace installations without compensation. It consists of basic harmonic (50 Hz) one higher harmonic (125 Hz), one interharmonic (25 Hz) and is additionally distorted by 5% random noise. The sampling interval was 0.5 ms. The signal was investigated using the Prony and min-norm methods. Both the methods enable us to detect all the signal components already using 100 samples (Fig. 1). For detection the 25 Hz component using the Fourier algorithm much more samples were needed. When using the same number of samples (100) the Fourier algorithm indicates the frequencies ca. 21.5, 51,and 127 Hz.

V. SIMULATION OF A FREQUENCY CONVERTER

In recent years, simulation programs for complex electrical circuits and control systems have been essentially improved. The simulation of characteristic transient phenomena concerning the electrical quantities becomes feasible without any arrangement of hardware. Among many available simulation programs, the EMTP – ATP as a Fortran based program adapted to DOS/Windows serves for modeling complex 1- and 3-phase networks occurring in drive, control and power systems.

In the paper we show investigation results of a 3 kVA – PWM – converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U = 380 V, I = 2.8 A). The simulated converter can change the output frequency within a range 0.1 ÷ 150 Hz.

Fig. 2 shows the current waveform at the converter output for the frequency 100 Hz. The sampling interval was 0.2 ms.

The Prony and min-norm methods enable to estimate the frequencies of all the signal components using 100 samples. The following frequencies have been detected: ca. 100, 800, 1000, 1200 Hz. The estimation accuracy is a little better than when using the Fourier algorithm.

VI. INDUSTRIAL FREQUENCY CONVERTER

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter composed of a single-phase half-controlled bridge rectifier and a voltage source converter. The waveforms of the converter output current under normal conditions (Fig.3) were investigated using the Prony, min-norm and FFT methods. The main frequency of the waveform was 40 Hz.

Using the Prony and min-norm methods the following harmonics have been detected: 7th, 17th, 19th, 25th, 35th, and 41th. The min-norm method has additionally detected the 5th harmonic. It is also possible to estimate the frequency of the fundamental component. Estimation of the main component frequency enables to choose an appropriate sampling window for the FFT.

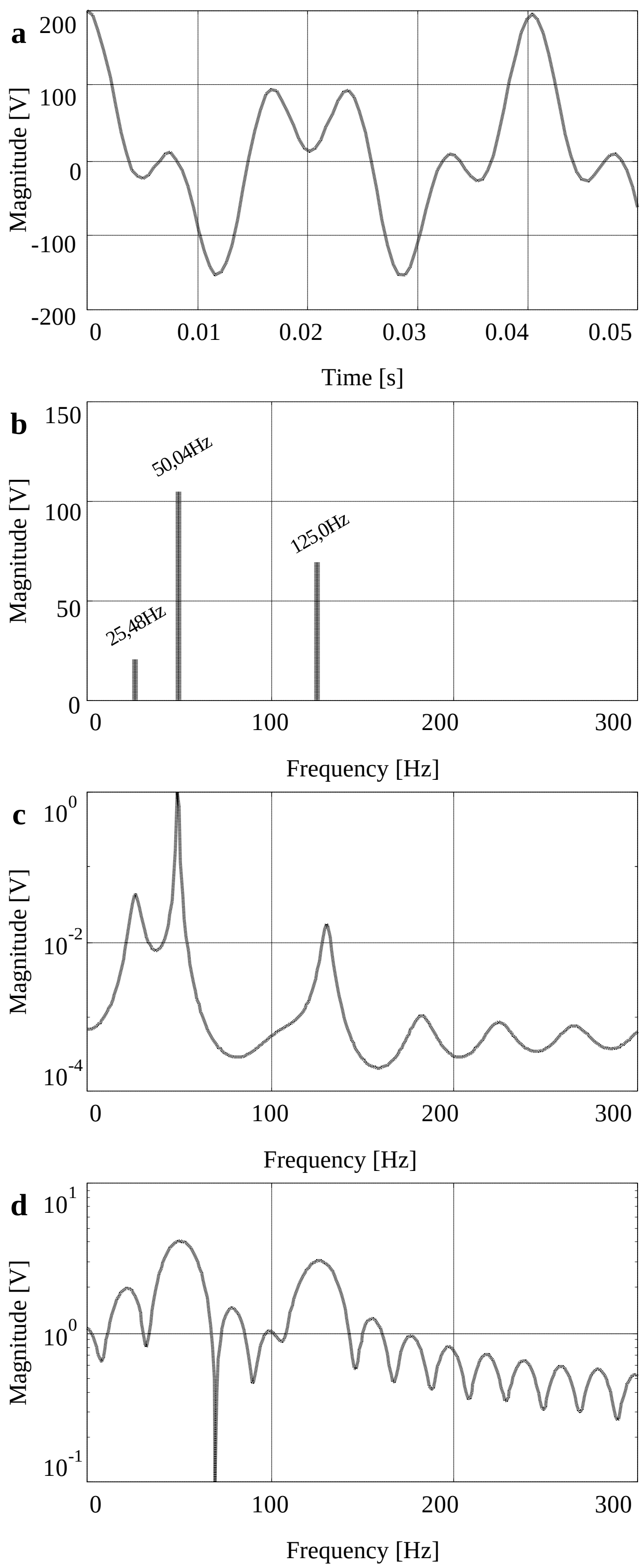

Fig. 1. Voltage waveform at the output of a simulated DC arc furnace power supply installation (a); investigation results: Prony, M=30 (b); min-norm (c); FFT (d); f_p=2000Hz, N=100.

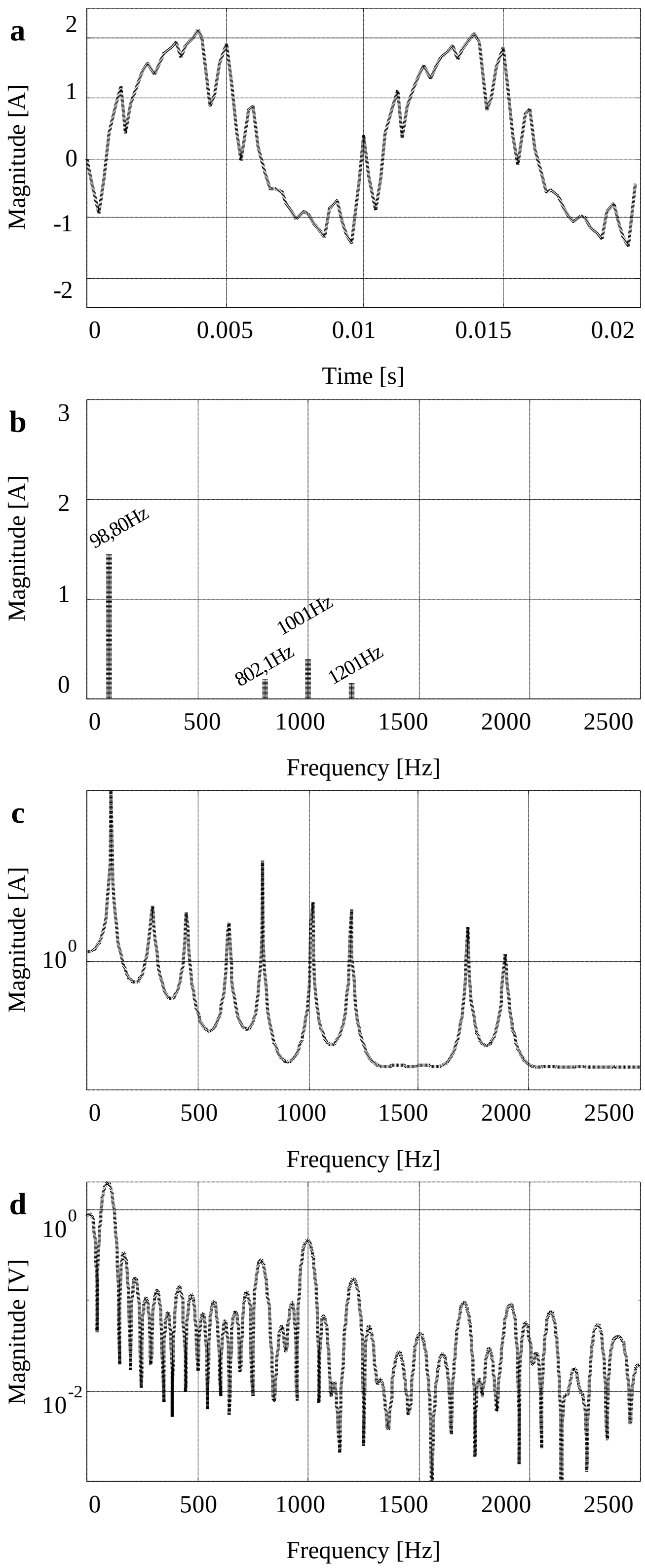

Fig. 2. Current waveform at the output of a simulated frequency converter (a); investigation results: Prony N=80,M=40 (b); min-norm N=100 (c); FFT N=80 (d); f_p=5000Hz.

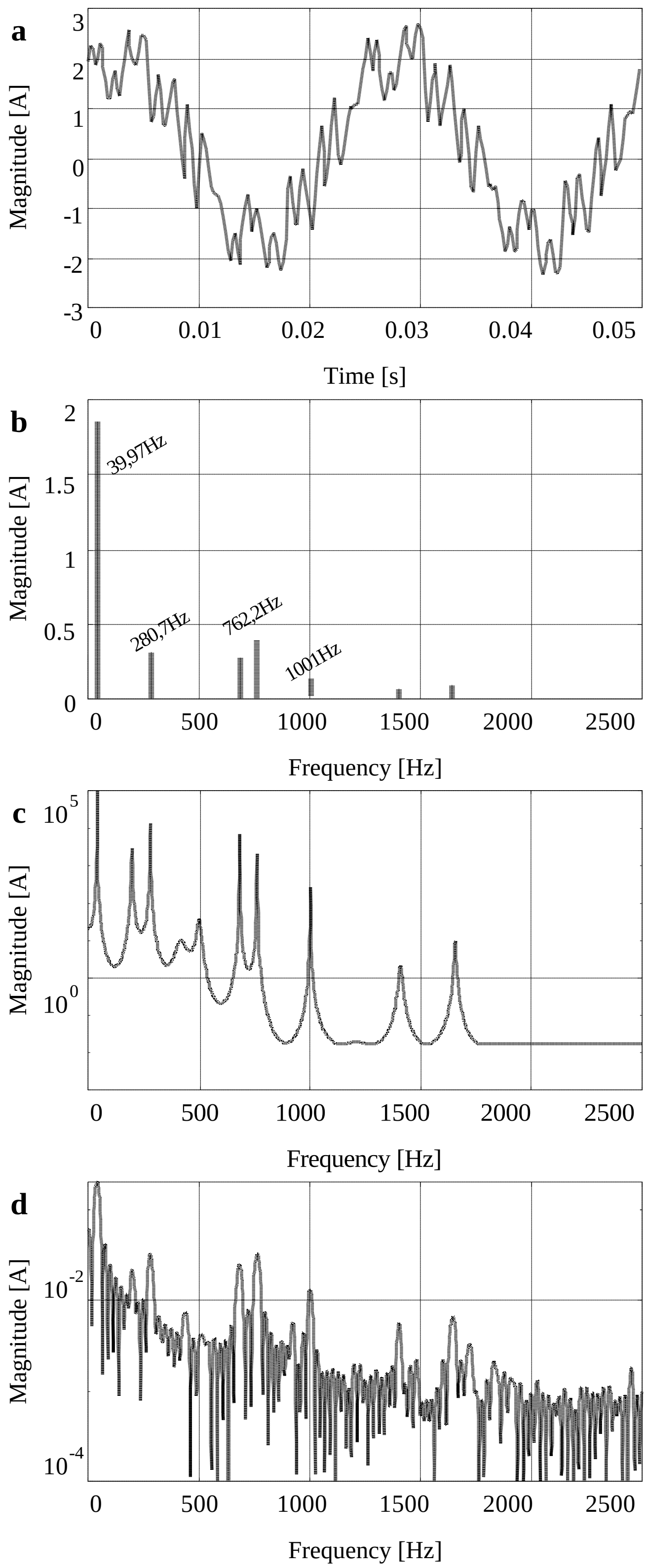

Fig. 3. Current waveform at the output of a real frequency converter (a); investigation results: Prony N=200,M=80 (b); min-norm N=100 (c); FFT N=200 (d), f_p=5000Hz

VII. CONCLUSIONS

It has been shown that that a high-resolution spectrum estimation method, such as min-norm could be effectively used for parameter estimation of distorted signals. The Prony method could also be applied for estimation the frequencies of signal components. The accuracy of the estimation depends on the signal distortion, the sampling window and the number of samples taken into the estimation process.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics.

When using both the high-resolution methods: Prony and min-norm the estimation accuracy in most cases is better than when using the Fourier algorithm. Application of the proposed advanced methods makes it possible the estimation of the components, which frequencies differ insignificantly. However, their computation is much more complex than FFT.

VIII. ACKNOWLEDGMENTS

The authors would like to thank the State Committee for Scientific Research KBN (Poland) for its financial support (grant 8T10A02316).

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