Spectrum Estimation of Non-stationary Signals in Traction Systems

A. Bracale, G. Carpinelli, Z. Leonowicz, T. Lobos and J. Rezmer

Abstract- **The growing complexity of the AC and DC traction systems in terms of both new technologies and automation requires a careful control of the Power Quality disturbances. Among the disturbances, the waveform distortions are of special concern because of the widespread use of power static converters for both locomotive drives and auxiliary services. Unfortunately, the spectral components of these converters can be highly time-varying both in amplitude and frequency with consequent difficulties in detecting the waveform components and defining proper distortion indices. In this paper the Prony method was tested for this purpose and a novel approach for the analysis of traction non-stationary signals, based on the "subspace" methods, is proposed: the "rootMusic" harmonic retrieval method which is an example of high-resolution eigen structure-based methods. Both methods can allow an accurate and useful estimation of amplitude and frequency of spectral components with their changes in time.**

Index Terms- **Electrical Traction Systems, Waveform Distortion Analysis, Subspace Methods, Prony Method.**

I. INTRODUCTION

As well known, static power converters generate a wide spectrum of components which can deteriorate the quality of the delivered energy and decrease the reliability of the electrical system where they operate. In some cases, their spectra include not only characteristic harmonics, typical for the ideal converter operation, but also considerable amount of non characteristic harmonics and interharmonics which may additionally deteriorate the quality of the power supply voltage. Then, the estimation of the spectral components is very important for control and protection tasks [1]-[3].

In the modern railway systems the static converters are widely used for both locomotive drives and auxiliary services, so that a careful compatibility analysis of the overall electrical traction system is required. Operators specify emission limits, by means of the so called "limit mask"; the limits refer to the locomotive traction current absorbed through the pantograph on a given frequency range. Additional limits exist also for single on-board equipment, like auxiliary converters [4].

In actual traction systems, the waveform spectral components of the converters are highly time-varying in amplitude and in frequency (nonstationary signals); then, the traditional spectral analysis methods based on fixed periodicity intervals can resort to unreliable results.

Representation of non-stationary signals in time-frequency-domain has been of interest in signal processing areas for many years. For studying time-varying power signals the short-time Fourier transform (STFT) can be applied. It is a simple extension of the FT, where the FT is repeatedly evaluated for a windowed version of the time domain signal. The crucial drawback of this method is that the length of the window constraints the frequency resolution.

The STFT has been recently applied in [5], where the problem of measurement and analysis of low-frequency magnetic field emissions (MFEs) on board of rolling stock has been considered. In particular, the nonstationarity of the traction signals under analysis has been investigated and an optimal length of the time window used for STFT has been derived and discussed. Two different procedures for the estimation of the optimal window length have been tested and compared; the first one adopts a constant window length over the entire duration of the signal, the second one is based on adaptive-length time windows.

In this paper the time-varying characteristics of the spectral components due to an auxiliary static converter are estimated using the Prony model and rootMusic methods [6].

Prony method is a technique for modeling sampled data as a linear combination of exponentials. Although it is not a spectral estimation technique, Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation.

Rootmusic method is based on the linear algebraic concepts of subspaces and so it is included in the "subspace methods". It is based on the Pisarenko's approach of separating the data into signal and noise subspaces, modeling the waveforms as a sum of sinusoids in the background of noise.

This paper is organized such that for both methods (Prony and rootMusic) the analytical aspects are presented at first in order to outline how each method can be applied. This is followed by numerical applications of the methods to the case of the non-stationary current waveforms of a 45 kVA static converter for railway application. The aim of the paper is in outlining the characteristics of the signal in study; in fact, the amplitudes and frequencies of the spectral components with their changes in time have to be carefully analysed for an overall evaluation of the power quality problems in the

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electrical system feeding the traction auxiliary services.

II. PRONY METHOD

Prony method is a technique for modeling sampled data by a linear combination of exponentials. It is a parametric method based on the calculation of the exponential parameters minimizing the fitting error.

Let us consider the M data samples $\begin{bmatrix} x_1 & x_2 & \cdots & x_M \end{bmatrix}$; the investigated function can be approximated by p exponential functions:

$$y[n] = \sum_{k=1}^{p} A_k e^{(\alpha_k + j\omega_k)(n-1)T_s + j\psi_k} \tag{1}$$

where $n = 1, 2, ..., M$,

T_s – sampling period, A_k – amplitude,

α_k – damping factor, ω_k – angular velocity,

ψ_k – initial phase.

The following vectors can be defined:

$$\mathbf{x}_{w0} = \begin{bmatrix} x_p & x_{p-1} & \cdots & x_1 \end{bmatrix} \tag{2}$$

$$\mathbf{x}_{k0} = \begin{bmatrix} x_p & x_{p+1} & \cdots & x_{M-1} \end{bmatrix}^T \tag{3}$$

$$\mathbf{x}_{k1} = \begin{bmatrix} -x_{p+1} & -x_{p+2} & \cdots & -x_M \end{bmatrix}^T \tag{4}$$

$$\mathbf{x}_{k2} = \begin{bmatrix} x_1 & x_2 & \cdots & x_p \end{bmatrix}^T \tag{5}$$

where p is the assumed number of exponential components, which can approximate the investigated signal $(2p \le M)$.

The Toeplitz matrix:

$$\mathbf{t} = \begin{bmatrix} x_p & x_{p-1} & \cdots & x_1 \\ x_{p+1} & x_p & \cdots & x_2 \\ \vdots & \vdots & & \vdots \\ x_{M-1} & x_{M-2} & \cdots & x_{M-p} \end{bmatrix} \tag{6}$$

created from (2) and (3) makes it possible to determine the vector of coefficients **a** of the characteristic polynomial:

$$\mathbf{t} \cdot \mathbf{a} = \mathbf{x}_{k1}, \tag{7}$$

where:

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_p \end{bmatrix}^T \tag{8}$$

The roots of the characteristic polynomial:

$$z^p + a_1 z^{p-1} + \ldots a_{p-1} z + a_p = 0 \tag{9}$$

written in a vector form:

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 & \mathbf{z}_2 & \cdots & \mathbf{z}_p \end{bmatrix}^T \tag{10}$$

define the Vandermonde matrix:

$$\mathbf{v} = \begin{bmatrix} \mathbf{z}_1^0 & \cdots & \mathbf{z}_{p-1}^0 & \mathbf{z}_p^0 \\ \mathbf{z}_1^1 & \cdots & \mathbf{z}_{p-1}^1 & \mathbf{z}_p^1 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{z}_1^{p-1} & \cdots & \mathbf{z}_{p-1}^{p-1} & \mathbf{z}_p^{p-1} \end{bmatrix}. \tag{11}$$

Vector of complex values **h** can be calculated from (5) and (11) solving the system of equations:

$$\mathbf{v} \cdot \mathbf{h} = \mathbf{x}_{k2} \tag{12}$$

where:

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \cdots & \mathbf{h}_p \end{bmatrix}^T. \tag{13}$$

Using the equations (10) and (13), parameters of exponential components for *k=1, 2 ,... ,p* can be calculated from:

$A_k = |\mathbf{h}_k|$ amplitude,

$\phi_k = \arg(\mathbf{h}_k)$ initial phase,

$\alpha_k = f_p \cdot \ln|\mathbf{z}_k|$ damping factor,

$\omega_k = f_p \cdot \arg(\mathbf{z}_k)$ angular velocity.

III. MUSIC (MULTIPLE SIGNAL CLASSIFICATION) METHOD

A general model of the signal which is considered for estimation of the sinusoidal components within the signal is:

$$\mathbf{x} = \sum_{i=1}^{p} A_i \mathbf{s}_i + \eta \; ; \quad A_i = |A_i| e^{j\phi_i} \;, \tag{14}$$

where $\mathbf{s}_i = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(M-1)\omega_i} \end{bmatrix}^T$; M - number of samples; p number of exponential components, η -noise.

The MUSIC method uses the signal correlation matrix to separate the signal space in two parts: a signal subspace and a noise subspace. The frequency and amplitude estimation of exponential components is based on the projection of the signal vector onto the entire noise subspace.

If the noise is white, the correlation matrix of **x** can be expressed as:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathsf{E}\left\{ A_i A_i^* \right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I}. \tag{15}$$

After the calculation of eigenvector and eigenvalues of the correlation matrix, it can be observed that the *M-p* smallest eigenvalues of the correlation matrix (matrix dimension *M>p+1*) correspond to the noise subspace and the p largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

The matrices of eigenvectors can be divided into signal and noise matrices:

$$\mathbf{E}_{signal} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_p \end{bmatrix} \tag{16}$$

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{p+1} & \mathbf{e}_{p+2} & \cdots & \mathbf{e}_M \end{bmatrix} \tag{17}$$

and, similarly, two matrices of eigenvalues $\Lambda_{signal}, \Lambda_{noise}$ can be built.

It is possible then to write $\mathbf{R}_x$ as:

$$\mathbf{R}_x = \mathbf{E}_{signal} \Lambda_{signal} \mathbf{E}^{*T}_{signal} + \mathbf{E}_{noise} \Lambda_{noise} \mathbf{E}^{*T}_{noise} \ . \quad (18)$$

MUSIC method uses only the noise subspace for the estimation of frequencies of sinusoidal components. $\mathbf{E}_{noise}$ can be used to form the projection matrix $\mathbf{P}_X$ for the noise subspace:

$$\mathbf{P}_{noise} = \mathbf{E}_{noise} \mathbf{E}^{*T}_{noise} = \mathbf{I} - \mathbf{P}_{signal} \quad (19)$$

The squared magnitude of the projection of $\mathbf{w}$ (an auxiliary vector defined similarly to $\mathbf{s}$ - $\mathbf{w} = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(M-1)\omega_i} \end{bmatrix}^T$) onto the noise subspace is given by:

$$\mathbf{w}^{*T} \mathbf{P}_{noise} \mathbf{w} = \mathbf{w}^{*T} \mathbf{E}_{noise} \mathbf{E}^{*T}_{noise} \mathbf{w} = = \sum_{i=p+1}^{M} E_i(e^{j\omega}) E_i^*(e^{j\omega})$$

(20)

The MUSIC pseudospectrum is defined as:

$$\widehat{P}(e^{j\omega}) = \left. \frac{1}{\sum_{i=p+1}^{M} E_i(z) E_i^*(z)} \right|_{z = e^{j\omega}} \quad (21)$$

and it exhibits sharp peaks at the signal frequencies where $z = e^{j\omega}$.

The denominator polynomial:

$$\widehat{P}^{-1}(z) = \sum_{i=p+1}^{M} E_i(z) E_i^*(1/z) \quad (22)$$

has p double roots lying on the unit circle. These roots correspond also to the frequencies of the signal components.

This method of finding the frequencies is therefore called ***root-Music***.

After the calculation of the frequencies, the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relation:

$$\mathbf{e}_i^{*T} \mathbf{R}_x \mathbf{e}_i = \lambda_i \quad (23)$$

by substituting:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathrm{E}\left\{ A_i A_i^* \right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} = \sum_{i=1}^{M} P_i s_i s_i^{*T} + \sigma_0^2 \mathbf{I} \ ,$$

P_i -power of the component.

The resulting equations can be solved for P_i [6].

IV. Investigations

The current waveforms in the supply system of an auxiliary static converter have been analysed in two different conditions, with a filtering capacitor suppressing the high order harmonics and without it [4]. The 45 kVA converter with the output frequency of 56.5 Hz was supplied by 2300 V dc.

Figs. 1 and 2 show the spectra of the converter currents without and with capacitor, respectively. From the analysis of these spectra it clearly appears that without (with) capacitor the ~300 (~170) Hz component has the highest amplitude. Moreover, particularly interesting are the spectral components in the range around the 55 Hz frequency since in this range stringent limits can be imposed [7][1].

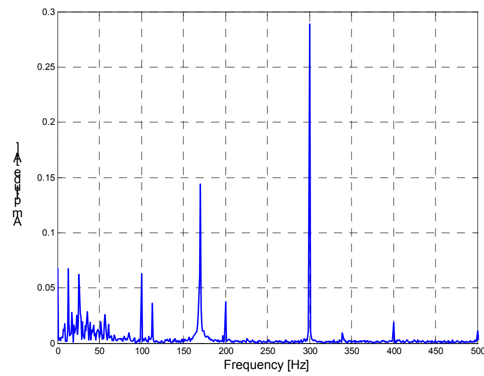

Fig.1. Spectrum of the current in the supply system without capacitor.

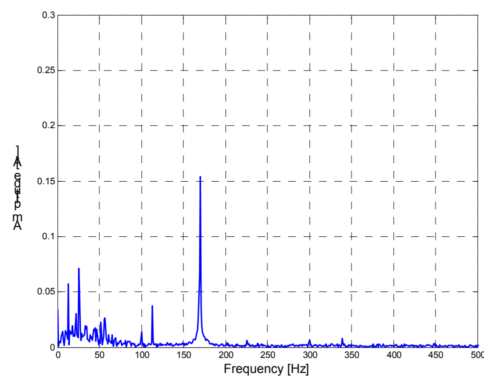

Fig.2. Spectrum of the current in the supply system with capacitor.

Several spectral component analyses have been carried out using Prony and root-Music methods. As an example, in the next the following cases will be presented:

- *Case 1*: time-frequency characteristics of the *main component* with and without the capacitor;
- *Case 2*: time-frequency and time-amplitude characteristics of the most significant component around 170 Hz without capacitor;
- *Case 3*: time-frequency and time-amplitude characteristics of the most significant component around 300 Hz component without capacitor;
- *Case 4*: time-frequency characteristics of a sub harmonic, with capacitor.

For parameter estimation of the above signal spectral components, the following filters have been applied to pre-processing data:

[1] In the following the most critical spectral component in the range around 55 Hz will be named "*main component*"; the spectral components at lower (higher) frequencies will be named subharmonics (high order harmonics).

- a bandpass Butterworth IIR of the 3rd order (40-70Hz) passes the main component;
- a bandpass Butterworth of the 3rd order (15-40Hz) passes a subharmonic component;
- a bandpass Butterworth of the 3rd order (160-180Hz and 290-310Hz) passes two high order harmonic components.

In the case of the Prony method, the sampling window with 32 samples and a median filter with 20 samples at the output have been applied. The sliding step was 20 samples. In the case of the rootMusic method the sampling window was 200 samples and the sliding step 20 samples too.

A. Case 1

The Figs. 3 and 4 show the time-frequency characteristics of the main component obtained using Prony method while Figs 5 and 6 show the corresponding plots obtained with rootMusic method.

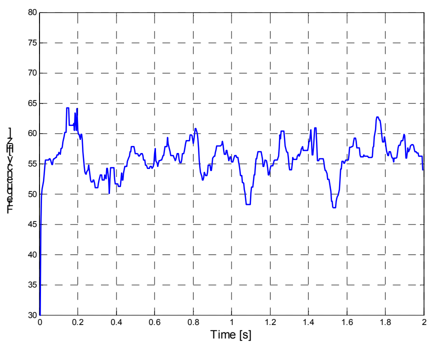

Fig.3. - Time-frequency characteristic of the main component in the system without capacitor, estimated with Prony method.

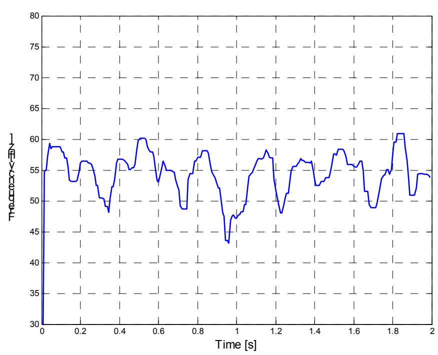

Fig.4. Time-frequency characteristic of the main component in the system with capacitor, estimated with Prony method.

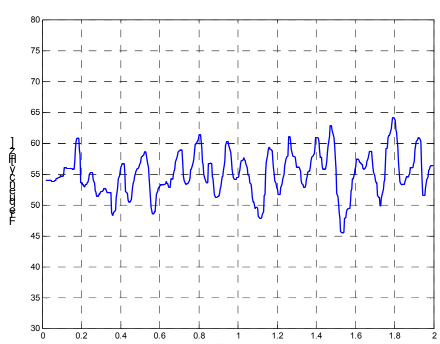

Fig.5. as in Fig. 3, but estimated with root Music method.

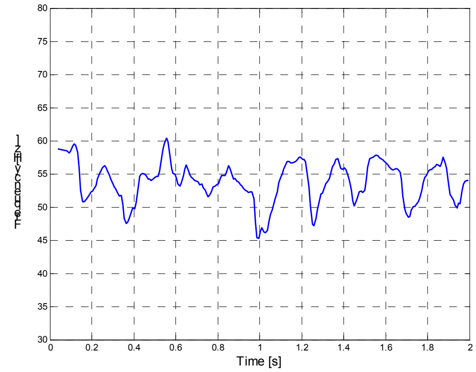

Fig.6. as in Fig. 4, by estimated with root Music method

From the analysis of the Figs from 3 to 6 it clearly appears a considerable time-variation of the main component frequency; moreover, as foreseeable, the Prony results are characterized by faster fluctuations than rootMusic results because of a smaller sampling window.

It should be noted that a significant time-variation has been detected also for the main component amplitude (not shown in the figures).

B. Case 2

The Figs. from 7 to 10 show the time-frequency and time-amplitude characteristics of the ~170 Hz component obtained using both methods.

From the analysis of the Figs from 7 to 10 it clearly appears a very slow time-variation of the frequency; instead, a not negligible time-variation has been detected for the amplitude.

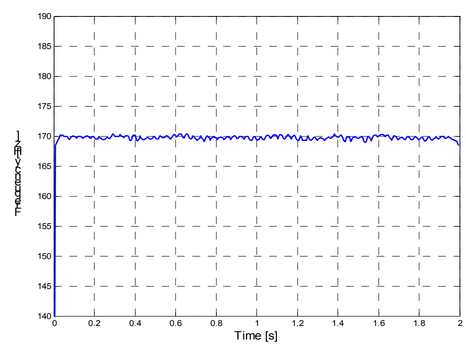

Fig.7. Time-frequency characteristic of the ~170 Hz component in the system without capacitor, estimated with Prony method

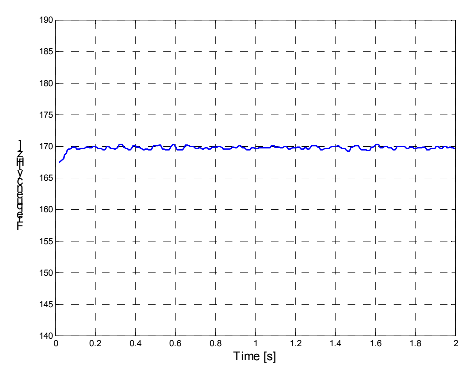

Fig. 8. Time-frequency characteristic of the ~170 Hz component in the system without capacitor, estimated with rootMusic method

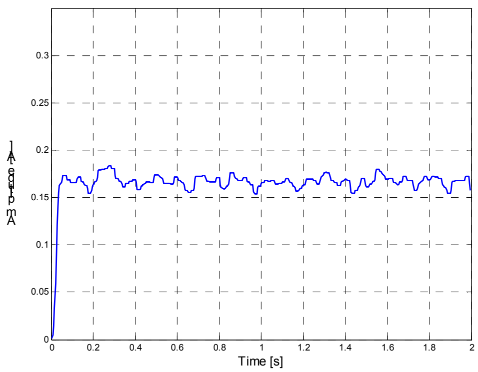

Fig.9. Time-amplitude characteristic of the ~170Hz component in the system without capacitor, estimated with Prony method

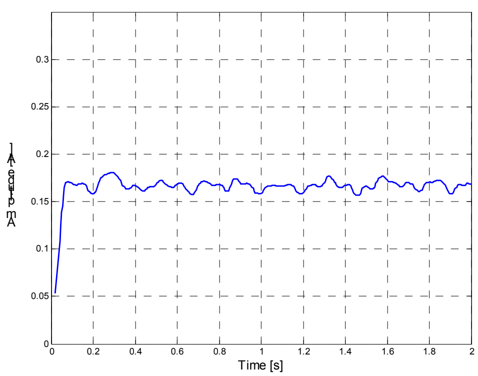

Fig.10. Time-amplitude characteristic of the ~170Hz component in the system without capacitor, estimated with rootMusic method

C. Case 3

The Figs. from 11 to 14 show the time-frequency and time-amplitude characteristics of the ~300 Hz component obtained using Prony and root-Music methods.

From the analysis of the Figs from 11 to 14 it clearly appears the same behavior as for the ~170 Hz component.

D. Case 4

In the Figs. 15 and 16 the time-frequency characteristics of a subharmonic component are shown.

In this case the significant frequency time-change makes more visible the averaging effect due to the larger sampling window used with rootMusic method. It should be noted that a significant time-variation has been detected also for the component amplitude (not shown in the figures).

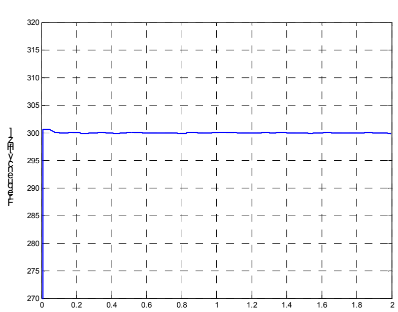

Fig. 11. Time-frequency characteristic of the ~300 Hz component in the system without capacitor, estimated with Prony method.

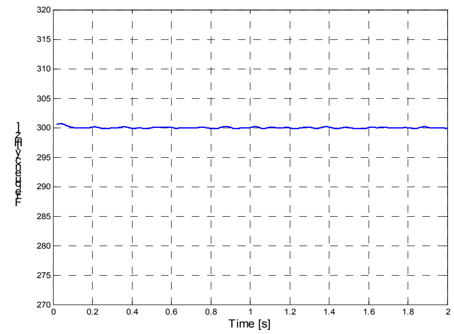

Fig.12. Time-frequency characteristic of the ~300 Hz component in the system without capacitor, estimated with rootMusic method.

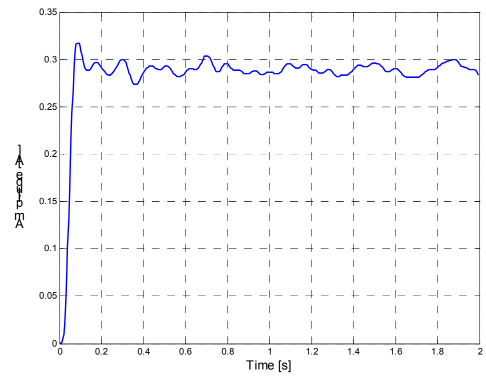

Fig.13. Time-amplitude characteristic of the ~300 Hz component in the system without capacitor, estimated with Prony method.

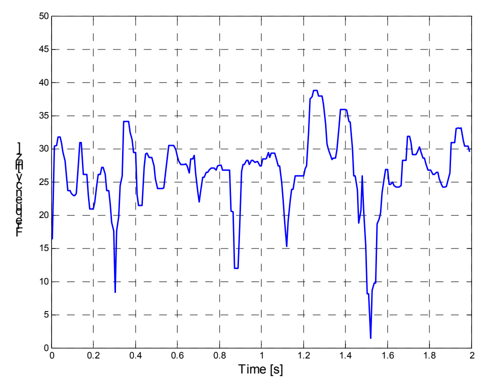

Fig.15. Time-frequency characteristic of a subharmonic component in the system with capacitor, estimated with Prony method

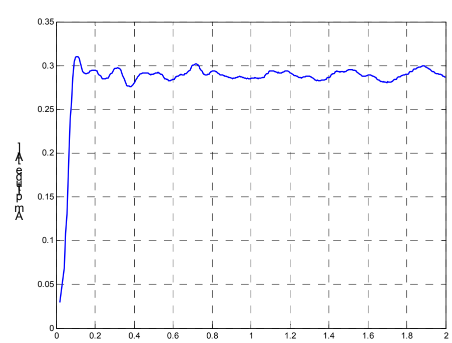

Fig.14. Time-amplitude characteristic of the ~300 Hz component in the system without capacitor, estimated with rootMusic method.

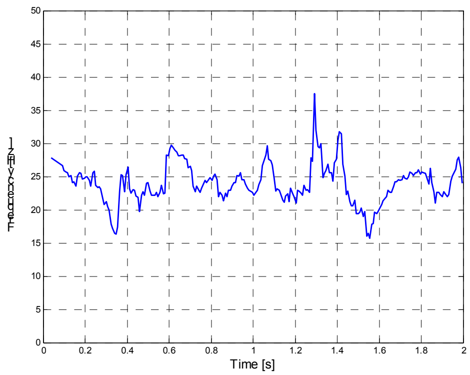

Fig.16. as in Fig. 15, estimated with root-Music method

VI. References

[1] T. Lobos, Z. Leonowicz, and J. Rezmer, "Harmonics and interharmonics estimation using advanced signal processing methods", 9th Int. Conf. on Harmonics and Quality of Power, Orlando (USA), 2000, vol. 1, pp. 335-340.

[2] Z. Leonowicz, T. Lobos, J. Rezmer, "Advanced spectrum estimation methods for signal analysis in power electronics", *IEEE Trans. on Industrial Electronics, June 2003, vol. 50, no. 3*, pp. 514-519.

[3] A. Bracale, G. Carpinelli, D. Lauria, Z. Leonowicz,, T. Łobos and J. Rezmer J: "On Some Spectrum Estimation Methods for Analysis of Non-stationary Signals in Power Systems – Part I Theoretical Aspects - Part II: Numerical Applications", 11th International Conference ICHQP 2004, Lake Placid, N.Y. (USA), September 2004

[4] A. Mariscotti, G. Armanino, " In-house test of low frequency conducted emissions of static converters for railway application", XVII IMEKO World Congress, Dubrovnik (Croatia) 2003.

[5] D. Bellan, A. Gaggelli, F. Maradei, A. Mariscotti, S. Pignari,, "Time-domain Measurements and Spectral Analysis of Nonstationary Low-frequency Magnetic Field Emissions on Board of Rolling Stock", *IEEE Trans. on Electromagnetic Compatibility, Vol. 46, No. 1, February 2004,* pp. 12-23.

[6] C. W. Therrien: *"Discrete Random Signals and Statistical Signal Processing",* Prentice-Hall, Englewood Cliffs, New Jersey, 1992, pp. 614-655.

[7] Italian Traction Recommendation FS N. 370582 - 1.0: Limit Mask, FS, 1996 (In Italian)