Analysis of Traction System Time-Varying Signals using ESPRIT Subspace Spectrum Estimation Method

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Abstract – **Auxiliary static converters in transportation power systems generate non-stationary waveforms, which contain not only characteristic harmonics but also non-characteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform are not suitable for its analysis. Representation of signals in time and frequency domain has been of interest in signal processing areas for many years, especially taking into account time-varying non-stationary signals, which appear in industrial power converters. In this paper, the ESPRIT method is tested for accurate and detail-rich estimation of parameters of currents in the power supply of auxiliary devices in traction systems, also interharmonic frequencies with their changes in time.**

I. INTRODUCTION

Contemporary railway systems require an exact compatibility analysis of the electrical supply system. Operators specify emission limits, that is so called "limit mask" for the locomotive traction current absorbed through the pantograph in a given frequency range. Additional limits exist for single on-board equipment [1].

Traction systems, which consist of power converters, can generate a wide spectrum of harmonic components which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics, typical for the ideal converter operation, but also considerable amount of non-characteristic harmonics and interharmonics which may additionally deteriorate the quality of the power supply voltage. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics can be variable and very long.

There are many different approaches for measuring harmonics [2]. The linear methods of system spectrum estimation (e.g., Blackman-Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation), the estimated spectrum can be a smeared version of the true spectrum. These methods also usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable.

Representation of signals in time and frequency domain has been of interest in signal processing areas for many years, especially for analysis of time-varying non-stationary signals. For studying time-varying signals the short-time Fourier transform (STFT) is widely applied. It is based on the assumption that for a short-time basis a signal can be considered as quasi-stationary.

New approach to analysis of non-stationary electric signals, based on the "subspace" methods, is proposed [3].

ESPRIT harmonic retrieval method is an example of high-resolution eigenstructure-based methods (based on the eigenstructure of the signal autocovariance matrix). The subspace high-resolution methods do not show the disadvantages of the traditional technique and allow exact estimation of the interharmonics frequencies as well as their variation in time with high accuracy [5].

In this paper, the time-varying characteristics of power system signal components are estimated using the STFT (Short-Time Fourier Transform) and ESPRIT (parametric subspace method) for comparison. Due to difficulties in selecting an objective "benchmark" value, it appeared justified to the authors not to show the results of comparison of accuracy of STFT and ESPRIT methods for real signals. Such comparison has been done on simulated waveforms (e.g.,[5]) and clearly confirmed the superiority of ESPRIT method.

II. SHORT-TIME FOURIER TRANSFORM

The *Fourier Transform* (FT) of a general signal $x(t)$, is defined by the equation (1):

$$\mathrm{FT}(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \tag{1}$$

The digital form of the last equation is called DFT:

$$\mathrm{DFT}[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi kn}{N}} \tag{2}$$

where $x(n)$ is the sampled version of the signal and N is the number of samples. It belongs to the group of non-parametric methods.

In practice, the DFT is calculated by using the Fast Fourier Transform (FFT), a class of algorithms very efficient in computation of DFT values, when the number of sample is an integer power of 2.

The *Short Time Fourier Transform* (STFT) gives a time-frequency representation for non-stationary signal analysis. It is an extension of the FT, where the FT is repeatedly evaluated for a windowed version of the time-domain signal. Each FT gives a frequency domain 'slice' associated with the time value at the window centre, so it allows the user to know what frequencies are present in the signal and where they are located in time.

The STFT is defined by:

$$\mathrm{STFT}(f,\tau)=\int_{-\infty}^{\infty}x(t)w(t-\tau)e^{-j2\pi ft}dt \quad (3)$$

The digital form of the last equation:

$$\mathrm{STFT}[k,m]=\sum_{n=0}^{N-1}x[n]\ \ w[n-m]e^{-j\frac{2\pi kn}{N}} \quad (4)$$

where $w[n]$ is the sliding temporal window.

The window length T_w determines the time resolution of the STFT and it fixes also the resolution of the components in frequency according to $\Delta f=1/T_w$.

One of the major drawbacks of STFT is the inherent trade-off between resolution in time and in frequency domains. It also inherits all above-mentioned drawbacks of DFT.

III. ESPRIT METHOD

ESPRIT harmonic retrieval method belongs to the group of parametric methods and is an example of high-resolution eigenstructure-based algorithms [2]. It is based on the eigendecomposition of the signal *autocovariance matrix* $\mathbf{R}_x$.

The subspace high-resolution methods do not show the disadvantages of Fourier techniques and allow exact estimation of the interharmonics frequencies as well as their variation in time with high accuracy [3].

The original ESPRIT algorithm [2] is based on naturally existing shift invariance between the discrete time series, which leads to rotational invariance between the corresponding signal subspaces.

The assumed signal model is the following:

$$y[n]=\sum_{k=1}^{M}A_k e^{(j\omega_k n)}+w[n] \quad (4)$$

where $w[n]$ represents additive noise. The eigenvectors $\mathbf{U}$ of the autocorrelation matrix of the signal define two subspaces (signal and noise subspaces) by using two selector matrices $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$.

$$\mathbf{S}_1=\mathbf{\Gamma}_1\mathbf{U} \quad \mathbf{S}_2=\mathbf{\Gamma}_2\mathbf{U} \quad (5)$$

The rotational invariance between both subspaces leads to the equation:

$$\mathbf{S}_1=\mathbf{\Phi}\mathbf{S}_2 \quad (6)$$

where:

$$\mathbf{\Phi}=\begin{bmatrix} e^{j\omega_1} & 0 & \cdots & 0 \\ 0 & e^{j\omega_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\omega_M} \end{bmatrix} \quad (7)$$

The matrix $\mathbf{\Phi}$ contains all information about *M* components' frequencies. Additionally, the TLS (total least-squares) approach (applied for solving of (6) for $\mathbf{\Phi}$) assumes that both estimated matrices **S** can contain errors and finds the matrix $\mathbf{\Phi}$ as minimization of the Frobenius norm of the error matrix.

After calculation of the components' frequencies, the amplitudes of the components can be found in the next step, from eigenvalues and eigenvectors of the correlation matrix, using the relation:

$$\mathbf{e}_i^{*T}\mathbf{R}_x\mathbf{e}_i=\lambda_i \quad (8)$$

by substituting:

$$\mathbf{R}_x=\sum_{i=1}^{p}\mathrm{E}\left\{A_iA_i^*\right\}\mathbf{y}_i\mathbf{y}_i^T+\sigma_0^2\mathbf{I}=\sum_{i=1}^{M}P_i\mathbf{y}_i\mathbf{y}_i^{*T}+\sigma_0^2\mathbf{I} \quad (9)$$

where: P_i -power (squared amplitude)of the component.

The resulting equations can be solved for P_i [2].

The analysis of non-stationary signals requires a similar approach as in short time Fourier transform (STFT). The time varying signal is broken up into minor segments with the help of the temporal window function and each segment (with overlapping) is analysed [5].

IV. INVESTIGATIONS

The current in the power supply of auxiliary traction converters has been analysed for two cases, in the network with a capacitor suppressing the harmonics and without this filtering. The 45 kVA converter with the output frequency of 56.5 Hz was supplied by a 2300 V dc.

The investigations have been carried out using the STFT and ESPRIT method. The signals were sampled with the frequency of 5000 Hz. For parameter estimation of signal components two filters have been applied. The bandpass Butterworth IIR filter of the 3rd order (40-70Hz) for the main component. The second bandpass Butterworth of the 3rd order (15-40Hz) focuses on subharmonic components. Other bandpass filters were also applied for higher harmonic bands.

The time-frequency characteristics have been visualized using the STFT and ESPRIT methods. The STFT window length was 2500 samples with 20 samples overlapping. In the case of ESPRIT method the sampling window was 200 samples and the overlapping 20 samples. Fig 1 shows the 3-D STFT representation of the current in the supply system without filter.

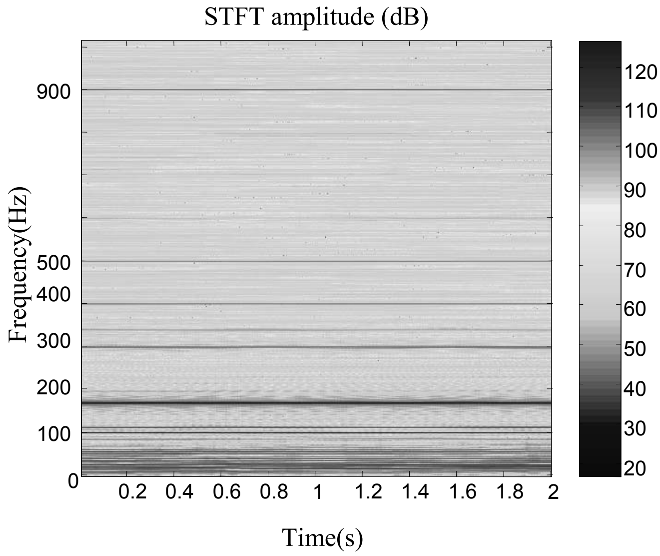

Fig. 1. Short-Time Fourier Transform of the supply current without filter.

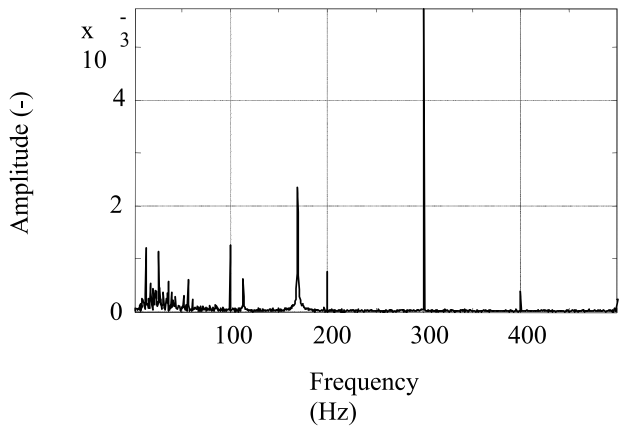

Fig. 2. Power spectrum of the current in the supply system with a filter.

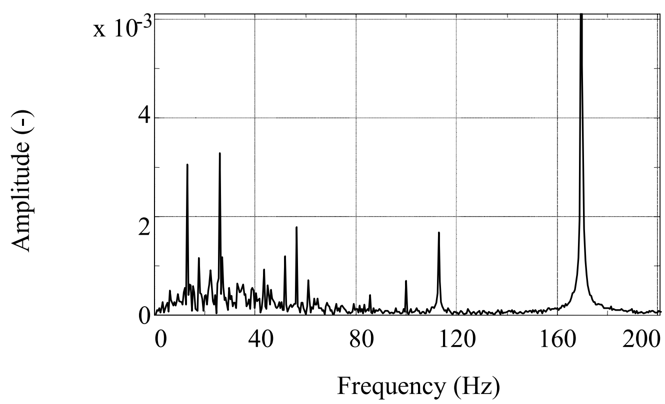

Fig. 3. Power spectrum of the current in the supply system without filter.

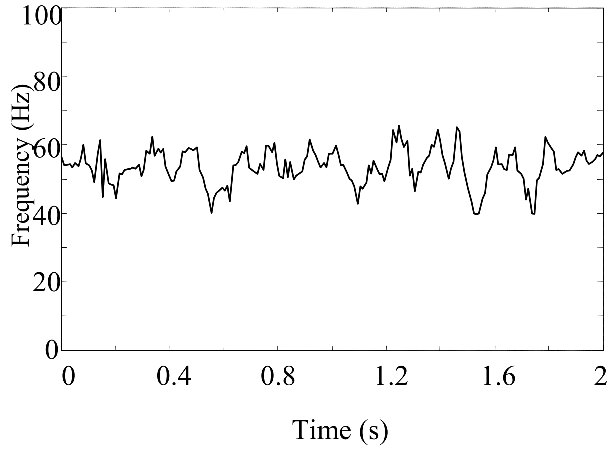

Fig.4. Time-frequency representation of the main component estimated by ESPRIT method (current without filter).

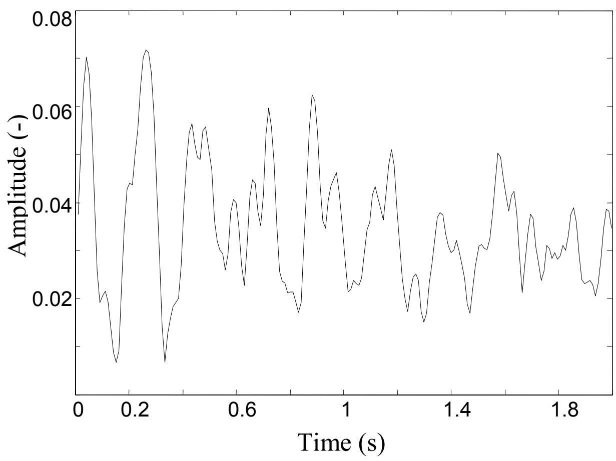

Fig.5. Time-amplitude representation of the main component estimated by ESPRIT method (current with a filter).

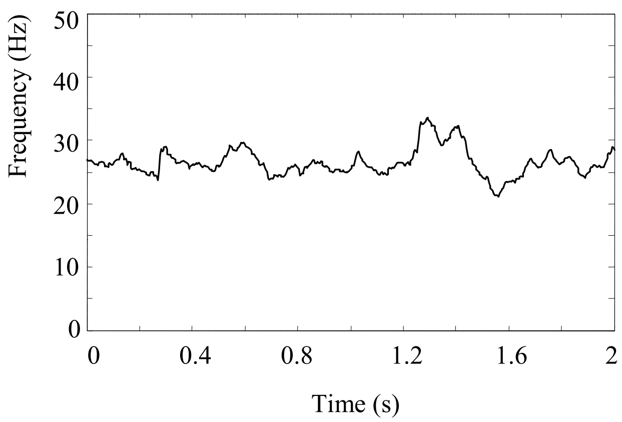

Fig. 6. Time-frequency representation of the subharmonic in the system with a filter, estimated by ESPRIT method.

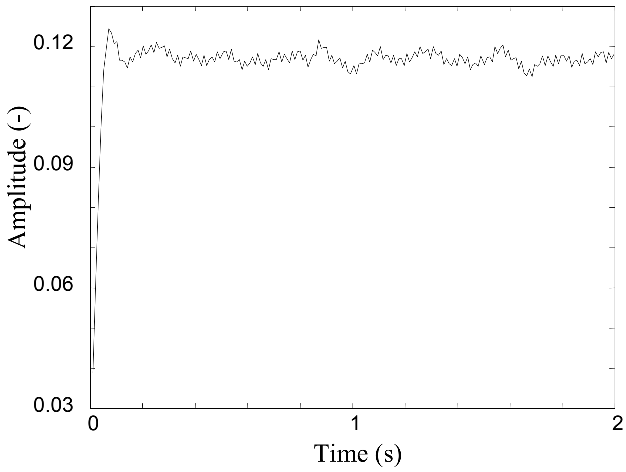

Fig. 7. Time-amplitude representation of the ~170 Hz component in the system with a filter, estimated by ESPRIT method.

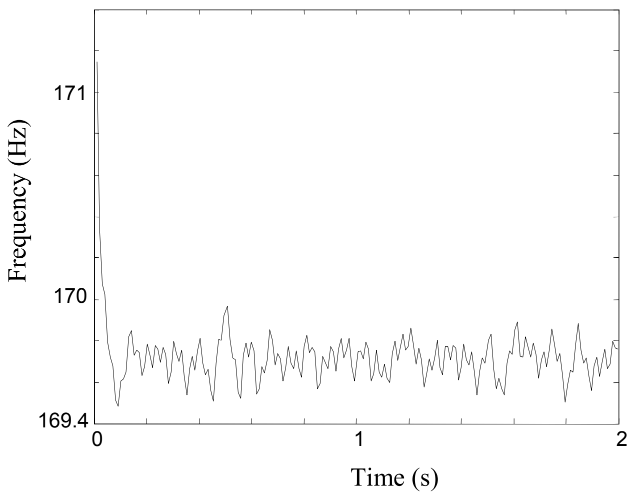

Fig. 8. Time-frequency representation of the ~ 170 Hz component in the system with a filter, estimated by ESPRIT method.

From the STFT in Fig. 1 it is visible, that most important higher harmonics are located in the main frequency band, the subharmonic band, in the ~170 Hz band and in the ~300 Hz band. The relatively strong 300 Hz component is probably caused by 6-pulse rectifiers in the power substations. In the case with the compensation capacitor the 300 Hz component could not be identified.

All components have strongly time-varying behaviour, especially visible for amplitude characteristics. Frequency characteristics show obviously less variability. It can be stressed that ESPRIT method allows unprecedented accuracy and very detailed representation of spectral components which can not be obtained using classical Fourier-based techniques [5]. Application of bandpass filters allows selection of only one component for time-frequency analysis, makes straightforward the selection of the number of components *M* (4) for subsequent parameter estimation via ESPRIT method, improves also the SNR and resolution [4].

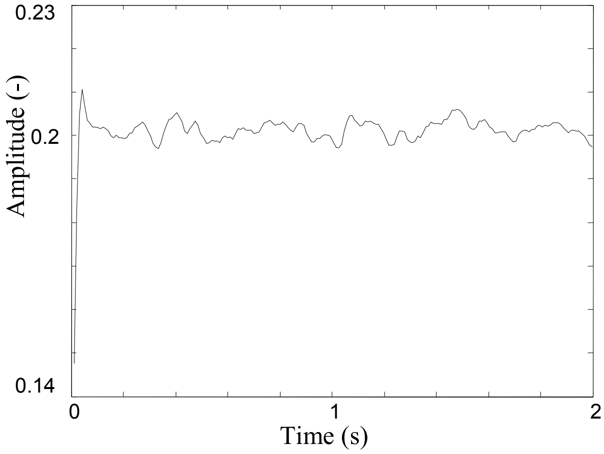

Fig. 9. Time-amplitude representation of the ~300 Hz component in the system without filter, estimated by ESPRIT method.

V. CONCLUSIONS

It has been shown that high-resolution spectrum estimation parametric method, such as ESPRIT could be effectively used for parameter estimation of distorted, non-stationary signals. The estimation accuracy is much better than for the Fourier algorithm [3]. Application of proposed advanced method makes it possible the estimation the changes in time the parameters of signal components. High accuracy of parameter estimation, inherent to parametric methods, can bring advantages eg., in fault evaluation, optimization of converter control, evaluation of power quality and others.

VI. ACKNOWLEDGMENT

This work is supported by the Ministry of Education and Science (Poland) under grant No. 3T10A04030.

VII. REFERENCES

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