Advanced Spectrum Estimation Methods for Applications in Power Systems

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Abstract--**Modern frequency power converters generate a wide spectrum of harmonics components. Large converters systems can also generate noncharacteristic harmonics and interharmonics. The spectrum estimation methods, based on the Fourier transform, suffer from the major problem of resolution. A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. The Prony method as applied for signal analysis was also tested for this purpose. Both the high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonics frequencies. To investigate the methods several experiments were performed using simulated signals, current waveforms at the output of an industrial frequency converter and current waveforms during out-of-step operation of a synchronous generator. For comparison, similar experiments were repeated using the FFT. The comparison proved the superiority of the new methods. However, its computation is much more complex than FFT.**

Index Terms--**Discrete Fourier transforms, frequency conversion, frequency estimation, harmonic analysis, power generation control, power quality, power system harmonics.**

I. INTRODUCTION

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The liberalization of energy markets will strengthen the competition and is expected to drive down the energy prices. This is reason for the requirements concerning the power quality. The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Modern frequency power converters generate a wide spectrum of harmonics components which determine the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of noncharacteristic harmonics and interharmonics which may strongly determine the quality of the power supply voltage [1,2].

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Interharmonics are defined as non-integer harmonics of the main fundamental under consideration. The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

Interharmonics are considered more damaging than characteristic harmonics components of the distorted signals. Their emission is specified lower than those are for the harmonics.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc [3,4,5,6,7]. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels. The linear methods of system spectrum estimation (Blackman-Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in these methods, the estimated spectrum can be a smeared version of the true spectrum [8].

These methods usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long [1]. It is very important to develop better tools of interharmonic estimation to avoid possible damages due to its influence.

In this paper the frequencies of signal components are estimated using the Prony model and min-norm method.

Prony method is a technique for modeling sampled data as a linear combination of exponentials. Although it is not a spectral estimation technique, Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. Prony method seeks to fit a deterministic exponential model to the data in contrast to AR and ARMA methods that seek to fit a random model to the second-order data statistics. The paper [9] presents a new method of real-time measurement of power system frequency based on the Prony model.

The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods" [10]. Its resolution is theoretically independent of the signal-to noise ratio (SNR). The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function. Pisarenko first observed that the zeros of the z-transform of the eigenvector, corresponding to the minimum eigenvalue of the covariance matrix, lie on the unit circle, and their angular positions correspond to the frequencies of the sinusoids. In a later development it was shown that the eigenvectors might be divided into two groups, namely, the eigenvectors spanning the signal space and eigenvectors spanning the orthogonal noise space. The eigenvectors spanning the noise space are the ones whose eigenvalues are the smallest and equal to the noise power. One of the most important techniques, based on the Pisarenko's approach of separating the data into signal and noise subspaces is the min-norm method.

To investigate the ability of the methods several experiments were performed. Simulated signals, current waveforms at the output of an industrial frequency converter as well as current waveforms during out-of-step operation of a synchronous generator were investigated. For comparison, similar experiments were repeated using the FFT.

II. PRONY METHOD

Assuming the N complex data samples $x[1],...,x[N]$ the investigated function can be approximated by M exponential functions:

$$y[n]=\sum_{k=1}^{M} A_k e^{(\alpha_k + j\omega_k)(n-1)T_p + j\psi_k} \tag{1}$$

where

$n = 1,2,..., N$

T_p – sampling period, A_k - amplitude

α_k – damping factor, ω_k – angular velocity

ψ_k – initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{2}$$

where

$$h_k = A_k e^{j\psi_k} \qquad z_k = e^{(\alpha_k + j\omega_k)T_p}$$

The estimation problem bases on the minimization of the squared error over the N data values:

$$\delta = \sum_{n=1}^{N} \left|\varepsilon[n]\right|^2 \tag{3}$$

where

$$\varepsilon[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{M} h_k z_k^{n-1} \tag{4}$$

This turns out to be a difficult nonlinear problem. It can be solved using the Prony method that utilizes linear equation solutions. If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data may be made.

Consider the M-exponent discrete-time function:

$$x[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{5}$$

The $\boldsymbol{p}$ equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_M^0 \\ z_1^1 & z_2^1 & \cdots & z_M^1 \\ \vdots & \vdots & & \vdots \\ z_1^{M-1} & z_2^{M-1} & \cdots & z_M^{M-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[M] \end{bmatrix} \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes. Prony proposed to define the polynomial that has the z_k exponents as its roots:

$$\begin{aligned} F(z) &= \prod_{k=1}^{M} (z - z_k) = \\ &= (z - z_1)(z - z_2)\ldots(z - z_M) \end{aligned} \tag{7}$$

The polynomial may be represented as the sum:

$$\begin{aligned} F(z) &= \sum_{m=0}^{M} a[m] z^{M-m} = \\ &= a[0]z^M + a[1]z^{M-1} + \ldots + a[M-1]z + a[M] \end{aligned} \tag{8}$$

Shifting the index on (5) from n to n-m and multiplying by the parameter $a[m]$ yield:

$$a[m]x[n-m] = a[m]\sum_{k=1}^{M} h_k z_k^{n-m-1} \tag{9}$$

The (9) can be modified into:

$$\begin{aligned} &\sum_{m=0}^{M} a[m]x[n-m] = \\ &= \sum_{k=1}^{M} h_k z_k^{n-M} \left\{ \sum_{m=0}^{M} a[m] z_k^{M-m-1} \right\} \end{aligned} \tag{10}$$

The right-hand summation in (10) may be recognize as polynomial defined by (8), evaluated at each of its roots z_k yielding the zero result:

$$\sum_{m=0}^{M} a[m]x[n-m] = 0 \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2M$. In the overdetermined data case, the linear equation (11) must be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m] = e[n] \quad (12)$$

The estimation problem bases on the minimization of the total squared error:

$$E = \sum_{n=M+1}^{N} |e[n]|^2 \quad (13)$$

III. MIN-NORM METHOD

The min-norm method involves projection of the signal vector:

$$\boldsymbol{s_i} = \begin{bmatrix} 1 & e^{j\omega_i} & \ldots & e^{j(N-1)\omega_i} \end{bmatrix}^T \quad (14)$$

onto the entire noise subspace.

We consider a random sequence $\boldsymbol{x}$ made up of M independent signals in noise.

$$\boldsymbol{x} = \sum_{i=1}^{M} A_i \boldsymbol{s_i} + \eta; \quad A_i = |A_i| e^{j\phi_i} \quad (15)$$

If the noise is white, the correlation matrix is

$$\mathbf{R_x} = \sum_{i=1}^{M} E\{A_i A_i^*\} \boldsymbol{s_i} \boldsymbol{s_i^T} + \sigma_0^2 \mathbf{I} \quad (16)$$

N-M smallest eigenvalues of the correlation matrix (matrix dimension *N*>*M*+1) correspond to the noise subspace and M largest (all greater than σ_0^2) corresponds to the signal subspace.

We define the matrix of eigenvectors:

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{M+1} & \mathbf{e}_{M+2} & \ldots & \mathbf{e}_N \end{bmatrix} \quad (17)$$

Min-norm method uses one vector $\mathbf{d}$ for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element equal to one. These conditions are expressed by the following equations

$$\begin{aligned} &\mathbf{d} = \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \mathbf{d} \\ &\mathbf{d}^{*T} \ell = 1 \end{aligned} \quad (18)$$

We can express (18) in one equation

$$\mathbf{d}^{*T} \ell = \left(\mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \mathbf{d}\right)^{*T} \ell = \mathbf{d}^{*T} \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \ell = 1 \quad (19)$$

and form the lagrangian

$$\begin{aligned} \mathrm{L} = \mathbf{d}^{*T} \mathbf{d} + \mu\left(1 - \mathbf{d}^{*T} \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \ell\right) + \\ + \mu^* \left(1 - \ell^T \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \mathbf{d}\right) \end{aligned} \quad (20)$$

Gradient of (20) has the form

$$\nabla_{\mathbf{d}^*} \mathrm{L} = \mathbf{d} - \mu \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \ell = \mathbf{0} \quad (21)$$

where μ is chosen in such way that the first element of the vector is equal to one.

For this purpose we present $\mathbf{E}_{noise}$ in the form

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{c}^{*T} \\ \mathbf{E}_{noise}^{'} \end{bmatrix} \quad (22)$$

where $\mathbf{c}^{*T}$ is the upper row of the matrix. Hence $\mathbf{c} = \mathbf{E}_{noise}^{*T} \ell$.

From (21) and (22) results that the first element of the vector $\mathbf{d}$ is equal to $\mu \mathbf{c}^{*T} \mathbf{c}$.

Finally, vector $\mathbf{d}$ is equal to

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T} \mathbf{c}} \mathbf{E}_{noise} \mathbf{c} = \begin{bmatrix} 1 \\ \left(\mathbf{E}_{noise}^{'} \mathbf{c}\right) / \left(\mathbf{c}^{*T} \mathbf{c}\right) \end{bmatrix} \quad (23)$$

Pseudospectrum defined with the help of $\mathbf{d}$ is defined as.

$$\hat{P}\left(e^{j\omega}\right) = \frac{1}{\left|\mathbf{w}^{*T} \mathbf{d}\right|^2} = \frac{1}{\mathbf{w}^{*T} \mathbf{d} \mathbf{d}^{*T} \mathbf{w}} \quad (24)$$

where $\mathbf{w}$ is defined as in (14).

IV. EXPERIMENTS WITH SIMULATED WAVEFORMS

Several experiments were performed with the signal waveform described in [2]. The investigated signal is characteristic for DC arc furnace installations without compensation. It consists of basic harmonic (50 Hz) one higher harmonic (125 Hz), one interharmonic (25 Hz) and is additionally distorted by 5% random noise. The sampling interval was 0.5 ms.

The signal was investigated using the Prony and min-norm methods. Both the methods enable us to detect all the signal components already using 100 sample (Fig. 1). For detection the 25 Hz component using the Fourier algorithm much more samples were needed.

V. INDUSTRIAL FREQUENCY CONVERTER

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter composed of a single-phase half-controlled bridge rectifier and a voltage source converter. The waveforms of the converter output current under normal conditions (Fig.2) were investigated using the Prony, min-norm and FFT methods. The main frequency of the waveform was 40 Hz.

Using the Prony and min-norm methods the following harmonics have been detected: 5th, 7th, 17th, 19th, 25th, 35th, and 41th. It is also possible to estimate the frequency of the fundamental component. Estimation of the main component frequency enables to choose an appropriate sampling window for the FFT.

VI. OUT-OF-STEP OPERATION OF A SYNCHRONOUS GENERATOR

If the machine running in parallel with others is disturbed from its synchronous-state conditions, the rotor winding and the stator winding fluxes rotate with different velocities. The stator winding flux generates an electromotive force (e.m.f.) in the rotor winding which angular velocity depends on the rotor slip. The current in the rotor winding, caused by the e.m.f., produces a pulsating magnetomotive force (m.m.f.), which can be resolved into two "rotating" m.m.f.s of constant and equal amplitude revolving in opposite directions. These m.m.f.s are assumed to set up corresponding gap fluxes. The angular velocities of the fluxes are equal to the angular frequencies of the alternating components of the rotor winding current.

$$\omega_f = s \cdot \omega_s ; \omega_b = -s \cdot \omega_s \qquad (25)$$

where:

ω_f - angular velocity of the forward field component,

ω_b - angular velocity of the backward field component,

ω_s - angular velocity of the rotating stator field,

s - rotor slip.

The angular velocity of the rotor ω_r is described as:

$$\omega_r = (1-s)\omega_s \qquad (26)$$

The field components cut the stator conductors at velocities depending on the velocities of the components and velocity of the rotor. Hence, in the stator windings are induced corresponding e.m.f.s. causing currents components to flow. The angular frequencies of the components are: ω_s and $(1-2s)\omega_s$. Direct current in rotor windings produces m.m.f.s, which set up corresponding gap fluxes. The fluxes rotate with the angular velocity ω_r, cut the stator conductors at sleep speed, induce corresponding e.m.f.s and cause an other component of the stator currents.

A. Simulation of the Out-of step Operation

In the recent years, simulation programs for complex electrical circuits and control systems have been improved essentially. The EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program) as a FORTRAN based and to MS-DOS/WINDOWS adapted program serves for modelling complex 1- or 3-phase networks occurring in drive, control and energy systems.

In the paper we show investigation results of a fault operation of a synchronous generator powered by hydraulic turbine combined to a PID governor system and excitation system. Excitation system implements IEEE type 1 synchronous machine voltage regulator combined to an exciter.

Generator data: salient-pole synchronous generator: Nominal power 200MVA, nominal voltage 13800 V, nominal frequency 50 Hz.

Reactances: $X_d = 1.305$, $X_d^{'} = 0.296$, $X_d^{''} = 0.252$, $X_q = 0.474$, $X_q^{''} = 0.243$ (p.u.)

Block transformer: 210 MVA, 13.8 kV/230 kV, dY, $R_1 = 0.0027$, $L_1 = 0.08$ (p.u.)

System: 10 GVA, 230 kV and sampling frequency: 200 Hz

Under normal steady state conditions a three-phase to ground fault at the transformer output was simulated. The fault was switched on at t=0. After the fault was cleared out-of-step operation conditions occurred. In Fig. 3 the current waveform at the generator output for the short-circuit duration of 300 ms and Fig 4. the rotor speed during the fault are shown. Time-frequency distribution of the waveform has been calculated applying the Min-Norm method and the window of 40 samples (0.2 s).

On this way a three-dimensional spectrum has been obtained. In the paper, only some cross sections of the spectrum are shown to demonstrate the high resolution of the method.

At the beginning of the asynchronous running three current frequency components have been detected (Fig. 5). Afterwards the component with the smallest frequency disappeared (Fig. 6). The difference between the frequencies of the two other current components are for t=4 seconds smaller than estimated for t=2 seconds. Decrease of the frequency differences of the signal components over the time confirms the trend towards the retrieval of the generator from the out-of-step state.

VII. CONCLUSIONS

It has been shown that a high-resolution spectrum estimation method, such as min-norm, could be effectively used for parameter estimation of distorted signals. The Prony method could also be applied for estimation of the frequencies of signal component. The accuracy of the estimation depends on the signal distortion, the sampling window and on number of samples taken into the estimation process.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics.

For identification of the asynchronous operation of a synchronous generator, the frequencies of the current components are estimated. The appearance of additional current frequency components can be used as indicator of out-of-step operation of a synchronous machine. Decrease of the frequency differences of the detected current components over the time indicates that the generator is leaving the out-of-step state.

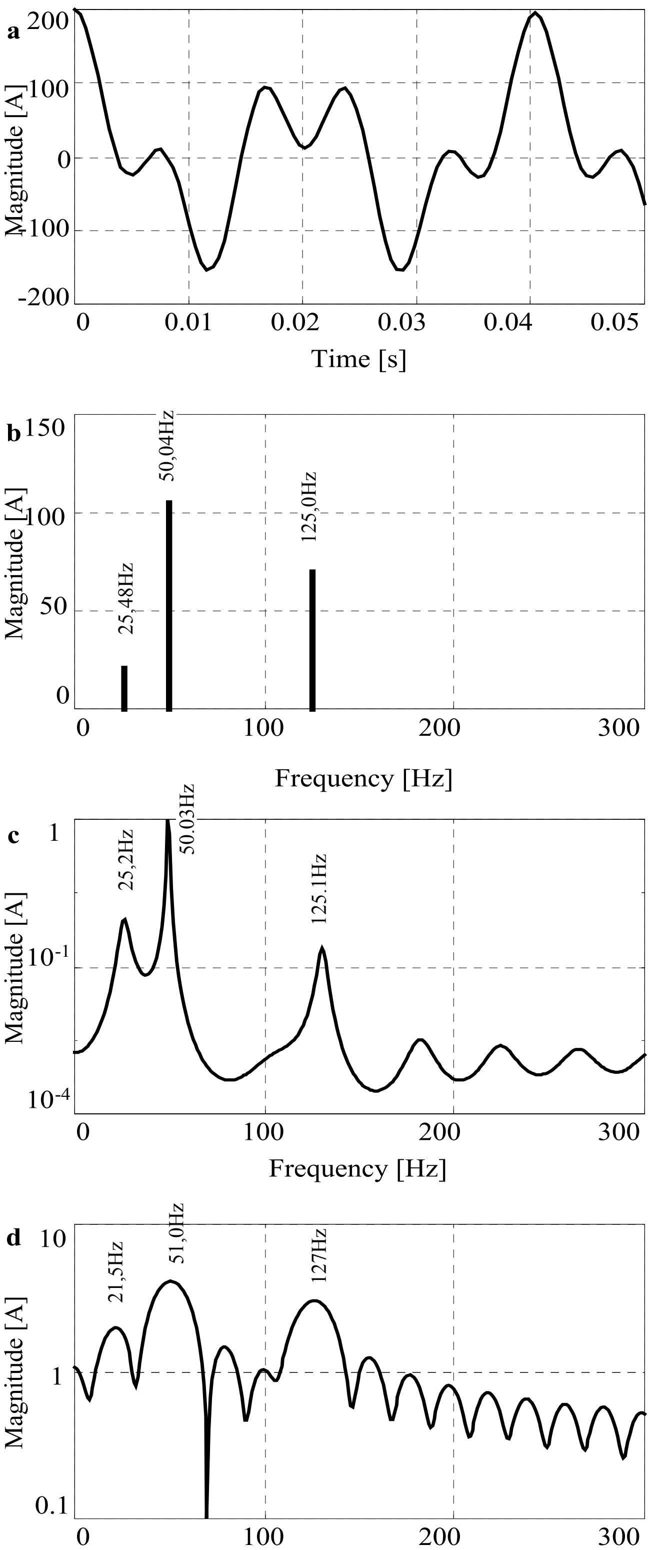

Fig. 1. Voltage waveform at the output of a simulated DC arc furnace power supply installation (a); investigation results: Prony, M=30 (b); min-norm (c); FFT (d); f_p=2000Hz, N=100.

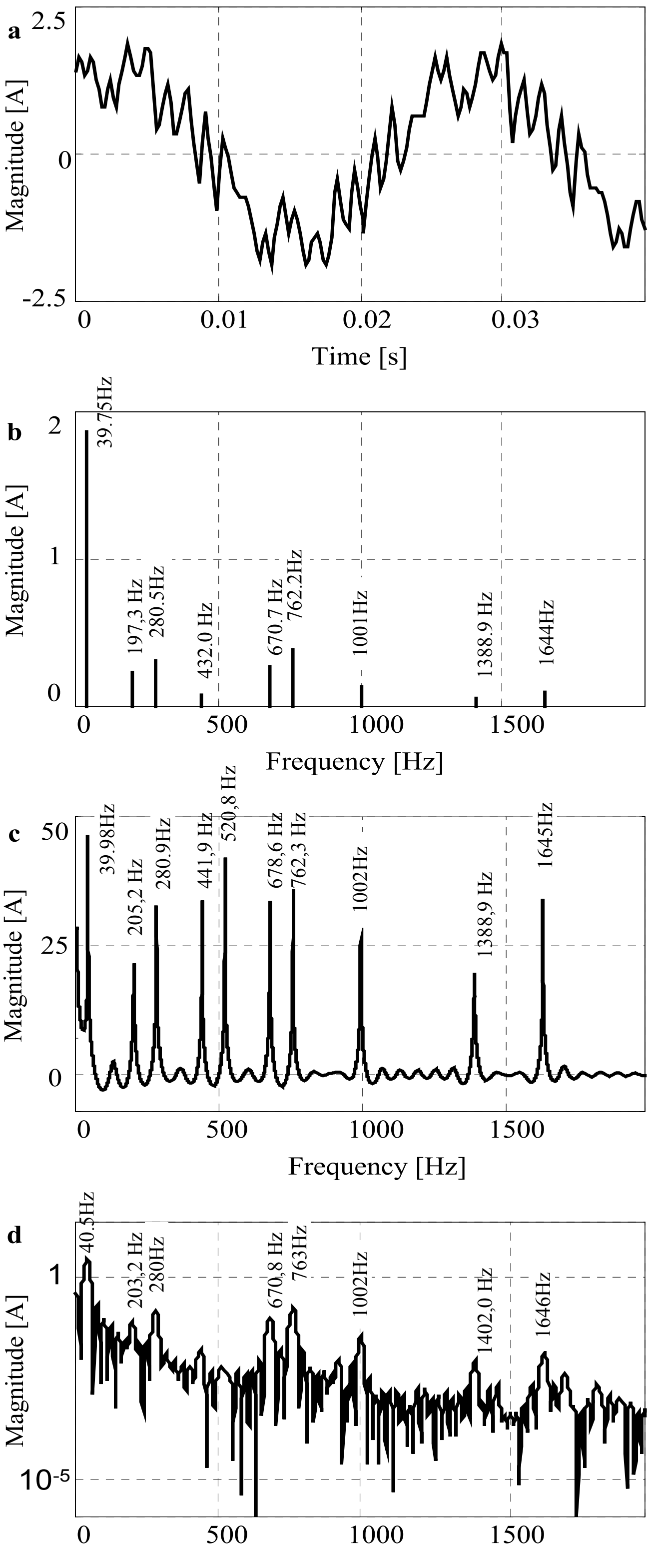

Fig. 2. Current waveform at the output of a real frequency converter (a); investigation results: Prony N=200,M=80 (b); min-norm N=100 (c); FFT N=200 (d), f_p=5000Hz

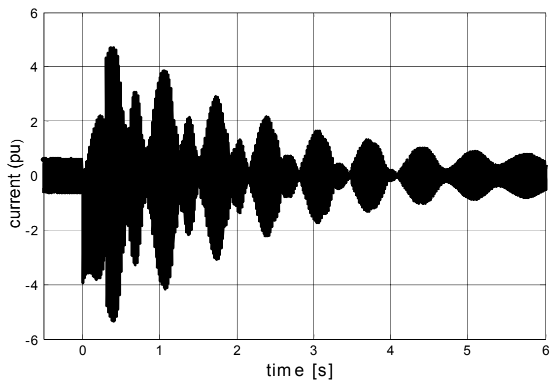

Fig. 3 Current waveform at the generator output.
Duration of the fault 300 ms.

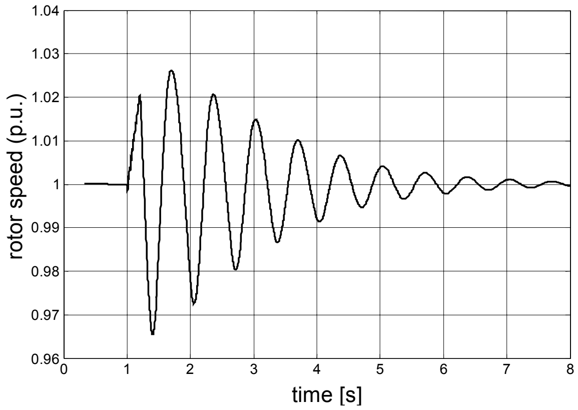

Fig. 4 Rotor speed of the generator during out-of-step operation (p.u.)

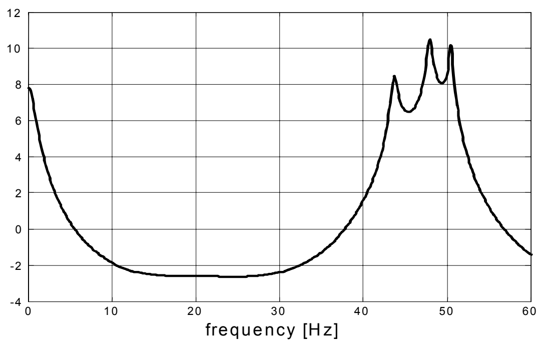

Fig. 5. Stator current spectrum of the signal in Fig. 3, for t=2 s after fault incipience. Detected signal components with frequencies: 43.75, 47.95 and 50.49 Hz.

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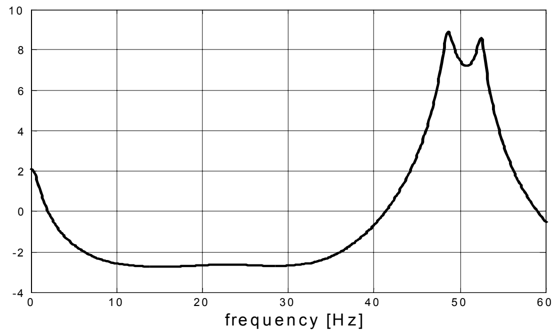

Fig. 6. Stator current spectrum of the signal in Fig. 3 for t=4 s after fault incipience. Detected signal components with frequencies: 48.63 and 52.54 Hz.

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Peter Schegner (M' 1999) studied Electrical Power Engineering at the Darmstadt University of Technology (Germany) where he received the M.Sc. degree in 1982. After that he worked as a system engineer in the field of power system control and became a member of the scientific staff at the Saarland University (Germany), receiving the PH.D. degree in 1989 with a thesis on the earth-fault distance protection. From 1989 until 1995 he worked as head of the development department of protection systems at the AEG, Frankfurt a.M./Germany. In 1995 he became a Full Professor and Head of the Institute of Electrical Power Systems and High Voltage Engineering at the Dresden University of Technology (Germany).