Spectrum Estimation Methods for Signal Analysis in the Supply System of a DC Arc Furnace

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Abstract **– In this paper, Prony and root-MUSIC methods are applied for analysis of waveforms from the DC arc furnace supply. Several experiments were performed using non-stationary voltage and current signals. The experiments proved the effectiveness of the applied methods for time-varying spectral analysis.**

I. INTRODUCTION

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Frequency power converters and arc furnaces generate a wide spectrum of harmonic components which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of non characteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels. The linear methods of system spectrum estimation (Blackman-Tukey), based on the Fourier transform, suffer from the major problem of resolution. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in these methods, the estimated spectrum can be a smeared version of the true spectrum.

These methods usually assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long.

In this paper the parameters of signal components are estimated using the Prony model and root-MUSIC method. Prony method is a technique for modeling sampled data as a linear combination of exponentials. Although it is not a spectral estimation technique, Prony method has a close relationship to the least squares linear prediction algorithms used for AR and ARMA parameter estimation. The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods". The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function.

One of the most important techniques, based on the approach of separating the data into signal and noise subspaces is the root-MUSIC method.

The standard method for study time-varying signals is short-time Fourier transform (STFT) that is based on the assumption that for a short-time the signal can be considered as stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the center of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the non stationarity occurring during this interval will be smeared in time and frequency. This inherent relationship between time and frequency resolution becomes more important when dealing with signals whose frequency content is changing. In the paper the time frequency characteristics of signals have been calculated applying sliding sampling windows. To investigate the methods several experiments were performed using non-stationary signals in a supply system of a dc arc furnace.

II. PRONY METHOD

The Prony signal model [4] assumes that the investigated waveform $\begin{bmatrix} x_1 & x_2 & \cdots & x_N \end{bmatrix}$ can be approximated by M exponential functions, as follows:

$$y[n] = \sum_{k=1}^{M} A_k e^{(\alpha_k + j\omega_k)(n-1)T_s + j\psi_k} \qquad (3)$$

where T_s – sampling period, A_k – amplitude, α_k –

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damping factor, ω_k – angular velocity, ϕ_k – initial phase.

The Toeplitz matrix created from samples makes it possible to determine the vector of coefficients **a** of the characteristic polynomial:

$$z^M + a_1 z^{M-1} + \ldots a_{M-1} z + a_M = 0 \qquad (4)$$

The roots of the characteristic polynomial define a Vandermonde matrix:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{z}_1^0 & \cdots & \mathbf{z}_{M-1}^0 & \mathbf{z}_M^0 \\ \mathbf{z}_1^1 & \cdots & \mathbf{z}_{M-1}^1 & \mathbf{z}_M^1 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{z}_1^{M-1} & \cdots & \mathbf{z}_{M-1}^{M-1} & \mathbf{z}_M^{M-1} \end{bmatrix} \qquad (5)$$

Vector of complex values **H** can be calculated from:

$$\mathbf{Z} \cdot \mathbf{H} = \mathbf{X} \qquad (6)$$

where

$$\mathbf{X} = \left[x_1 \quad x_2 \quad \cdots \quad x_M \right].$$

Parameters of exponential components for k=1, 2,…,M can be calculated from:

$$A_k = \left|\mathbf{h}_k\right| \text{ - amplitude} \qquad (7)$$

$$\alpha_k = f_s \cdot \ln\left|\mathbf{z}_k\right| \text{-damping factor} \qquad (8)$$

$$\omega_k = f_s \cdot \arg\left(\mathbf{z}_k\right) \text{ - angular velocity} \qquad (9)$$

$$\phi_k = \arg\left(\mathbf{h}_k\right) \text{ - initial phase} \qquad (10)$$

III. MUSIC (MULTIPLE SIGNAL CLASSIFICATION) METHOD

A general model of the signal which is considered for estimation of the sinusoidal components within the signal is [5]:

$$\mathbf{x} = \sum_{i=1}^{p} A_i \mathbf{s}_i + \eta \;; \quad A_i = \left|A_i\right| e^{j\phi_i} \qquad (11)$$

where $\mathbf{s}_i = \left[1 \quad e^{j\omega_i} \quad \cdots \quad e^{j(M-1)\omega_i}\right]^T$; M - number of samples; p - number of exponential components, η - noise.

The MUSIC method [8] involves projection of this vector (signal vector) onto the entire noise subspace.

If the noise is white, the correlation matrix of **x** can be expressed as:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathcal{E}\left\{A_i A_i^*\right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} \qquad (12)$$

After the calculation of eigenvector and eigenvalues of the correlation matrix above, it can be observed that M-p smallest eigenvalues of the correlation matrix (while the matrix dimension is $M>p+1$) correspond to the *noise subspace* and p largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

MUSIC method uses only the noise subspace for the estimation of frequencies of sinusoidal components. $\mathbf{E}_{noise}$ can be used to form the projection matrix $\boldsymbol{P_X}$ for the noise subspace:

$$\mathbf{P}_{noise} = \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} = \mathbf{I} - \mathbf{P}_{signal} \qquad (13)$$

The MUSIC *pseudospectrum* is defined as:

$$\widehat{P}(e^{j\omega}) = \left. \frac{1}{\sum_{i=p+1}^{M} E_i(z) E_i^*(z)} \right|_{z = e^{j\omega}} \qquad (14)$$

and it exhibits sharp peaks at the signal frequencies where $z = e^{j\omega}$.

The denominator polynomial from (14) has p double roots lying on the unit circle. These roots correspond also to the frequencies of the signal components. This method of finding the frequencies is therefore called *root-MUSIC*.

After the calculation of the frequencies, the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relation:

$$\mathbf{e}_i^{*T} \mathbf{R}_x \mathbf{e}_i = \lambda_i \qquad (15)$$

by substituting:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathcal{E}\left\{A_i A_i^*\right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} = \sum_{i=1}^{M} P_i s_i s_i^{*T} + \sigma_0^2 \mathbf{I} \qquad (16)$$

where P_i -power of the i-th component.

The resulting equations can be solved for P_i.

IV. DC ARC FURNACE

A typical dc arc furnace plant is shown in Fig. 1. It consists of a dc arc connected to a medium voltage ac busbar with two parallel thyristor rectifiers that are fed by transformer secondary winding with Δ and Y connection, respectively.

The medium voltage busbar is connected to the high voltage busbar with a HV/MV transformer whose windings are Y-Δ connected. The power of the furnace is 80 MW. The other parameters are: Transformer T_1 - 80 MVA, 220kV/21kV; Transformer T_2 - 87 MVA, 21kV/0.638kV/0.638kV.

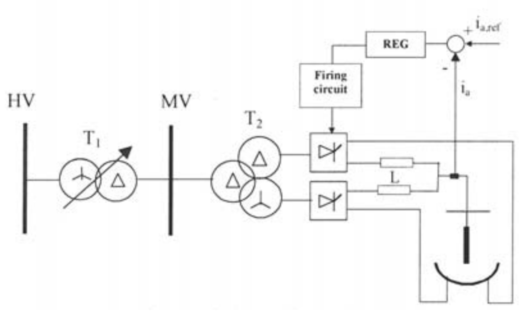

Fig. 1. Modeled DC arc furnace plant

The DC arc furnace is characterized by the presence of the ac/dc static converter and the random motion of the electric arc, of the non linear and time-varying nature, which are responsible for strong perturbations, in particular waveform distortions and voltage fluctuations. These perturbations are time-varying.

The voltage and current waveforms recorded at the MV busbar of Fig. 1 have been sampled with the frequency of 5000 Hz. The Fig. 2 shows a MV current waveform and its spectrum. The amplitudes of different values are relative to the maximum value throughout in the paper.

The analysis of Fig. 2 shows the presence of the ac/dc converter characteristic harmonics ($h = 12p \pm 1$, $p = 1, 2, ...$). There are also interharmonics around the characteristic harmonics and around the fundamental component due to the arc fluctuations and, additionally, non-characteristic harmonics appear.

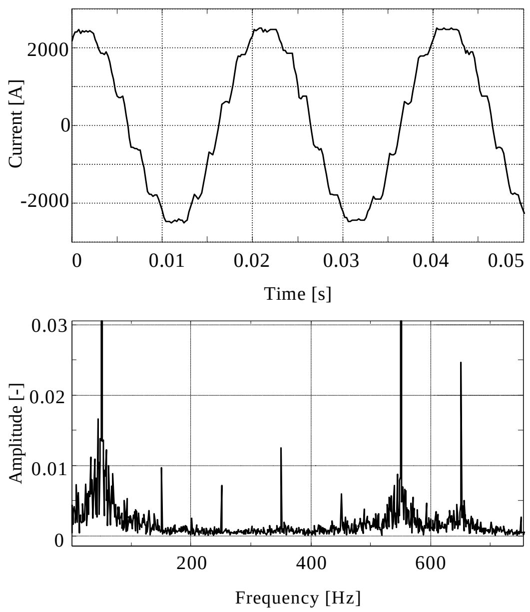

Fig. 2. Current waveform at MV busbar of the dc arc furnace from Fig. 1 (upper fig.) and its power spectrum (lower fig.).

V. INVESTIGATIONS

The range 300-500 Hz has been analysed of the supply currents (Fig.2). Figs. 3 show the time-frequency characteristics calculated using the Prony and the root-MUSIC methods. In the case of the Prony method, the sampling window with 22 samples and a median filter with 50 samples at the output have been applied. The sliding step was 10 samples. In the case of the root-MUSIC methods the sampling window was 500 samples and the sliding step 20 samples.

In the case of voltage waveforms the frequency range of 100-300 Hz was chosen. Figs. 4 show the time-frequency characteristics calculated using the Prony and the root-MUSIC methods.

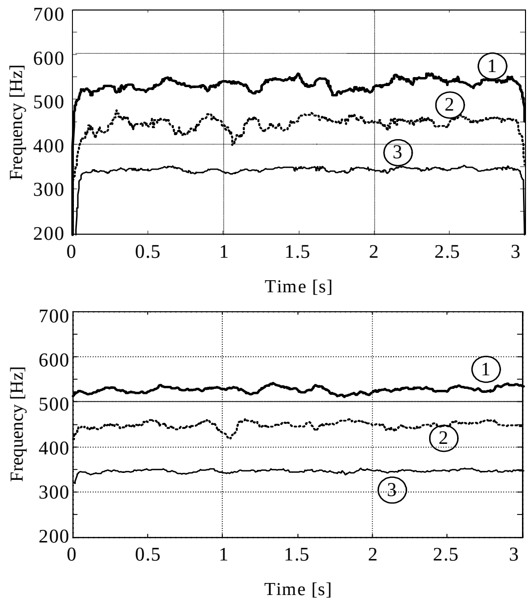

Fig. 3. Time frequency characteristics of the 3 current components in the range 300-500 Hz: Prony method (upper fig.), root-MUSIC method (lower fig.)

Finally, the subharmonics (frequencies under 50Hz) have been analysed.

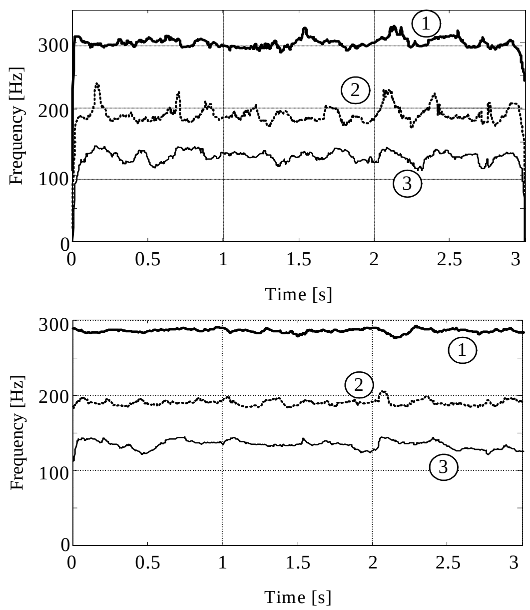

Fig. 4. Time frequency characteristics of the 3 voltage components in the range 100-300 Hz, Prony method (upper fig.), root-MUSIC method (lower fig.)

The investigations show fluctuations of frequencies. For the Prony method the fluctuations are larger because of a smaller sampling window.

Each method delivers a certain middle value over a sampling window. The non-stationarity of the subharmonic components is especially visible on the Figs. 4 and 5. The fundamental component (50 Hz) has a constant frequency and the frequency of the subharmonic fluctuates, because of the non-stationarity of the arc (Fig. 6). The results obtained with the Prony and the root-MUSIC methods show a significant similarity.

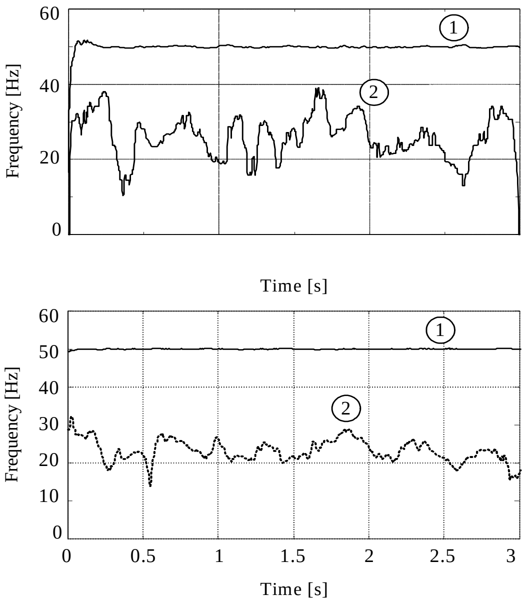

Fig. 5. Time frequency characteristics of the 2 current components in the range 0-50 Hz, Prony method (upper fig.), root-MUSIC method (lower fig.).

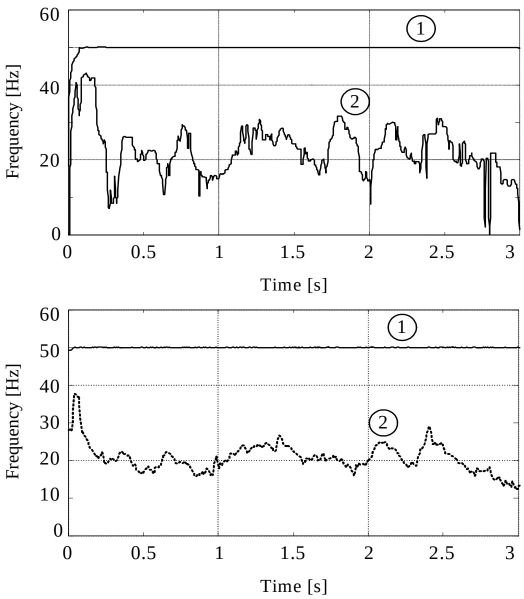

Fig. 6. Time frequency characteristics of the 2 voltage components in the range 0-50 Hz Hz, Prony method (upper fig.), root-MUSIC method (lower fig.).

VI. CONCLUSIONS

The proposed high-resolution methods were investigated under different conditions and found to be valuable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics and the calculation of the time-frequency characteristics of signal components [2].

Another important application, as shown in [3], is the accurate real-time determination of power system frequency when the voltage waveforms are distorted due the presence of nonlinear loads. This is a very important problem in power system applications because some power system protection and control equipment require accurate and fast estimation of frequency.

An accurate knowledge of the system frequency is useful also for static compensator control systems.

An important application of advanced spectrum estimation methods refers to the calculation of the frequencies of the signal components, when they are characterized by a time-varying behavior. As pointed out in [1], there exist cases (e.g.. the adjustable speed drives) where the knowledge of interharmonic frequencies is very useful; allowing to evaluate in advance the true Fourier fundamental frequency defined as the maximum common divisor of all the spectral component frequencies that are present; other reasons are related to interharmonic effects, such as light flicker, asynchronous motor aging, power system resonance excitation, and so on.

In particular, the application of the proposed advanced methods makes possible to estimate the changing in time the signal component parameters, even in situations when the frequencies of the components differ insignificantly. In the time-frequency characteristics of signal components fluctuations of frequency characteristics is visible, mainly when comparing the frequency of the main component and the subharmonic. However, the computation of the methods is usually more difficult than STFT.

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