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ZBIGNIEW **LEONOWICZ**

WROCLAW UNIVERSITY OF TECHNOLOGY, POLAND

PERFORMANCE OF PARAMETRIC SPECTRUM ESTIMATION METHODS

OCENA DOKŁADNOŚCI PARAMETRYCZNYCH METOD ESTYMACJI WIDMA

Streszczenie

W pracy przedstawiono porównanie dokładności wyznaczania częstotliwości i amplitudy składowych sygnałów harmonicznych odkształconych przy pomocy parametrycznych metod estymacji widma: MUSIC i ESPRIT. Miarą dokładności jest błąd średniokwadratowy estymacji określonego parametru dla wielu symulowanych przebiegów. Przebiegi testowe są charakterystyczne dla zagadnień spotykanych w elektroenergetyce, stąd otrzymane wyniki mogą być zastosowane w praktyce, umożliwiając optymalny dobór parametrów tych metod obliczeniowych.

Jako praktyczny przykład zastosowania metod MUSIC i ESPRIT przedstawiono wyznaczanie wskaźników jakości energii. Uzyskano ponad 50% wzrost dokładności po zastąpieniu algorytmu opartego na DFT (dyskretnym przekształceniu Fouriera) parametrycznymi metodami podprzestrzeni.

Introduction

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The proliferation of nonlinear loads connected to power systems has triggered a growing concern with power quality issues. The inherent operation characteristics of these loads deteriorate the quality of the delivered energy, and increase the energy losses as well as decrease the reliability of a power system [4].

The methods of power quality assessment in power systems are almost exclusively based on Fourier Transform. The crucial drawback of the Fourier Transform-based methods is that the length of the window is related to the frequency resolution. Moreover, to ensure the accuracy of Discrete Fourier Transform, the sampling interval of analysis should be an exact integer multiple of the waveform fundamental period [3].

Parametric spectral methods, such as ESPRIT or MUSIC [5] do not suffer from such inherent limitations of resolution or dependence of estimation error on the window length (phase dependence of the estimation error). The resolution of these methods is to high degree independent on signal-to-noise ratio and on the initial phase of the harmonic components.

The author argues that the use of high-resolution spectrum estimation methods instead of Fourier-based techniques can improve the accuracy of measurement of spectral parameters of distorted waveforms encountered in power systems, in particular the estimation of the power quality indices [4].

The paper is composed as follows: After the short description of parametric methods (ESPRIT and MUSIC), the comparison of the frequency and amplitude estimation error, based on numerical simulation is presented. Next part presents basics of selected power quality indices (harmonic sub/groups), followed by comparison of estimation error in the case of application of FFT-based algorithms and parametric methods.

1. PARAMETRIC METHODS

The ESPRIT and the root-Music spectrum estimation methods are based on the linear algebraic concepts of subspaces and so have been called “subspace methods” [5]; the model of the signal in this case is a sum of sinusoids in the background of noise of a known covariance function.

1.1 MUSIC

The MUSIC method assumes the model of the signal as:

$$\mathbf{x} = \sum_{i=1}^{p} A_i \mathbf{s}_i + \eta \; ; \quad A_i = |A_i| e^{j\phi_i} \tag{1}$$

where $\mathbf{s}_i = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$, A_i –amplitudes of the signal components, N – number of signal samples, p – number of the components, η - noise, ϕ_i - components' frequencies. The autocorrelation matrix of the signal is estimated from signal samples as:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathrm{E}\left\{A_i A_i^*\right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} \tag{2}$$

N-p smallest eigenvalues of the correlation matrix (matrix dimension $N>p+1$) correspond to the noise subspace and p largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

The matrix of noise eigenvectors of the above matrix (2) is used

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{p+1} & \mathbf{e}_{p+2} & \cdots & \mathbf{e}_N \end{bmatrix} \quad (3)$$

to compute the projection matrix for the noise subspace:

$$\mathbf{P}_{noise} = \mathbf{E}_{noise}\mathbf{E}_{noise}^{*T} \quad (4)$$

which, by using an auxiliary vector $\mathbf{w} = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$ allows computation of projection of vector $\mathbf{w}$ onto the noise subspace as:

$$\mathbf{w}^{*T}\mathbf{P}_{noise}\mathbf{w} = \mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w} = \\ = \sum_{i=p+1}^{N} E_i\left(e^{j\omega}\right)E_i^*\left(e^{j\omega}\right) \xrightarrow{Z} \sum_{i=p+1}^{N} E_i(z)E_i^*\left(1/z^*\right) \quad (5)$$

The last polynomial in (5) has p double *roots* lying on the unit circle which angular positions correspond to the frequencies of the signal components. This method of finding the frequencies is therefore called *root-MUSIC*.

After the calculation of the frequencies, the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relations:

$$\mathbf{e}_i^{*T}\mathbf{R}_x\mathbf{e}_i = \lambda_i \quad \text{and} \quad \mathbf{R}_x = \sum_{i=1}^{p} P_i s_i s_i^{*T} + \sigma_0^2\mathbf{I} \quad (6)$$

and solving for P_i – components' powers.

1.2 ESPRIT

The original ESPRIT algorithm [5] is based on naturally existing shift invariance between the discrete time series, which leads to rotational invariance between the corresponding signal subspaces.

The assumed signal model is as in (1). The eigenvectors $\mathbf{E}$ of the autocorrelation matrix of the signal define two subspaces (signal and noise subspaces) by using two selector matrices $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$.

$$\mathbf{S}_1 = \mathbf{\Gamma}_1\mathbf{E} \text{ and } \mathbf{S}_2 = \mathbf{\Gamma}_2\mathbf{E} \quad (7)$$

The rotational invariance between both subspaces leads to the equation:

$$\mathbf{S}_1 = \mathbf{\Phi}\mathbf{S}_2 \quad (8)$$

where:

$$\mathbf{\Phi} = \begin{bmatrix} e^{j\omega_1} & 0 & \cdots & 0 \\ 0 & e^{j\omega_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\omega_p} \end{bmatrix} \quad (9)$$

The matrix $\mathbf{\Phi}$ contains all information about p components' frequencies. Additionally, the TLS (total least-squares) approach assumes that both estimated matrices $\mathbf{S}$ can contain errors and finds the matrix $\mathbf{\Phi}$ as minimization of the Frobenius norm of the error matrix. Amplitudes of the components can be found in similar way as with MUSIC method using (6).

2. ACCURACY OF PARAMETRIC METHODS

Comparison of mean square error of the frequency and amplitude estimation is useful for practical assessment of accuracy of both methods: root-MUSIC and ESPRIT. Both methods are similar in the sense that they are both eigendecomposition-based methods which rely on decomposition of the estimated correlation matrix into two subspaces: noise and signal subspace. On the other hand, MUSIC uses the *noise subspace* to estimate the signal components while ESPRIT uses the *signal subspace*. Also the approach is in many points different. Numerous publications were dedicated to the analysis of the performance of the aforementioned methods. Unfortunately, due to many assumed simplifications, and the complexity of the problem, published results are often contradictory and sometimes misleading.

Several experiments with simulated, stochastic signals were performed, in order to compare performance aspects of both parametric methods MUSIC and ESPRIT. Testing signal is designed to belong to a class of waveforms often present in power systems [3][1]. Each run of spectrum and power estimation is repeated many times (Monte Carlo approach) and the mean--square error (MSE) is computed.

Parameters of test signals:

- one 50 Hz main harmonic with unit amplitude.
- random number of higher odd harmonic components with random amplitude (lower than 0.5) and random initial phase (from 0 to 8 higher harmonics).
- sampling frequency 5000 Hz.
- each signal generation repeated 1000 times with re-initialization of random number generator.
- SNR=40 dB if not otherwise specified.
- size of the correlation matrix = 50 if not otherwise specified.
- signal length 200 samples if not otherwise specified.

In the Fig. 1 it can be seen that the performance of both methods is similar for frequency estimation, only MUSIC performs better for SNR higher than 60 dB and lower than 20 dB. The error of power estimation is significantly lower for ESPRIT algorithm in the whole SNR range.

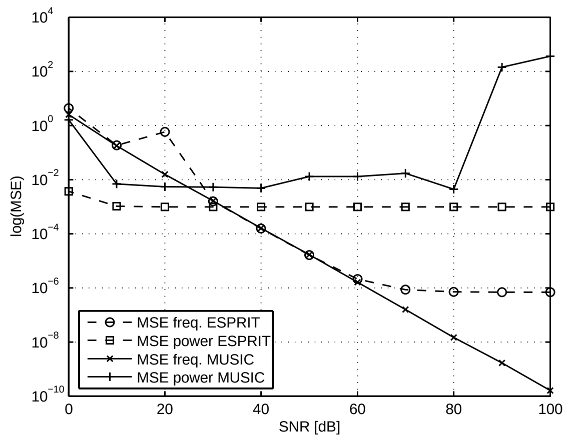

Fig.1. MSE of frequency and power estimation depending on SNR.

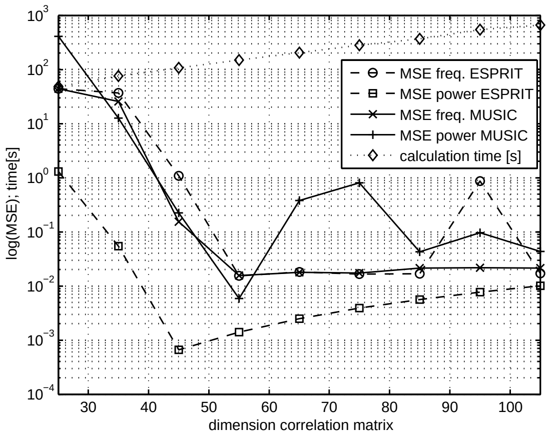

Fig.2. MSE of frequency and power estimation depending on the size of the correlation matrix.

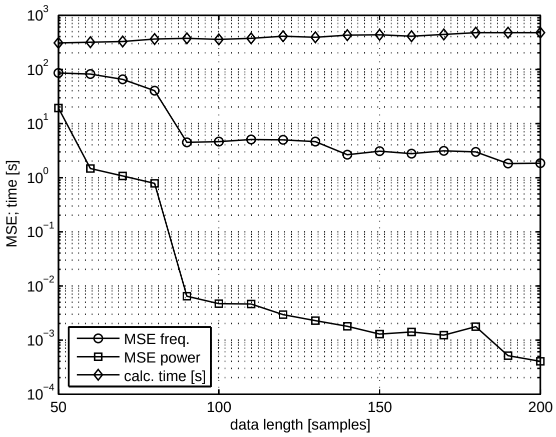

Fig.3. MSE of frequency and power estimation (ESPRIT) depending on the data window length.

From the Fig. 2 appears that there exists an optimal size of the correlation matrix which assures the lowest possible estimation error (tradeoff between accuracy of estimation of the correlation matrix and increase of numerical errors with the size of the correlation matrix). In Fig. 3 appears a sharp decrease of the estimation error for a specific length of the data sequence (for ESPRIT method, MUSIC results are similar). ESPRIT method performs better for both problems shown in Figs. 2 and 3.

3. POWER QUALITY INDICES

A number of power system applications require an accurate knowledge of the spectral components of non-stationary current and voltage waveforms.

The main application of spectral components in the field of Power Quality refers to the calculation of waveform distortion indices [2].

Several indices are in common use for the characterization of waveform distortions. However, they generally refer to periodic signals which allow an „exact" definition of harmonic components and deliver only one numerical value to characterize them.

When the spectral components are time-varying in amplitude and/or in frequency (as in case of non-stationary signals), a wrong use of the term harmonic can arise and several numerical values are needed to characterize the time-varying nature of each spectral component of the signal.

I this paper the IEC harmonic and interharmonic subgroups calculation as introduced by the IEC Standard drafts (IEC Std 61000-4-7, 61000-4-30), will be compared.

Cited IEC Standard drafts - with reference to DFT with 5 Hz resolution in frequency (200 ms of window length for 50 Hz fundamental frequency) - introduce the concept of harmonic and interharmonic groupings and characterize the waveform distortions with the amplitudes of these groupings, as shown in Fig. 4.

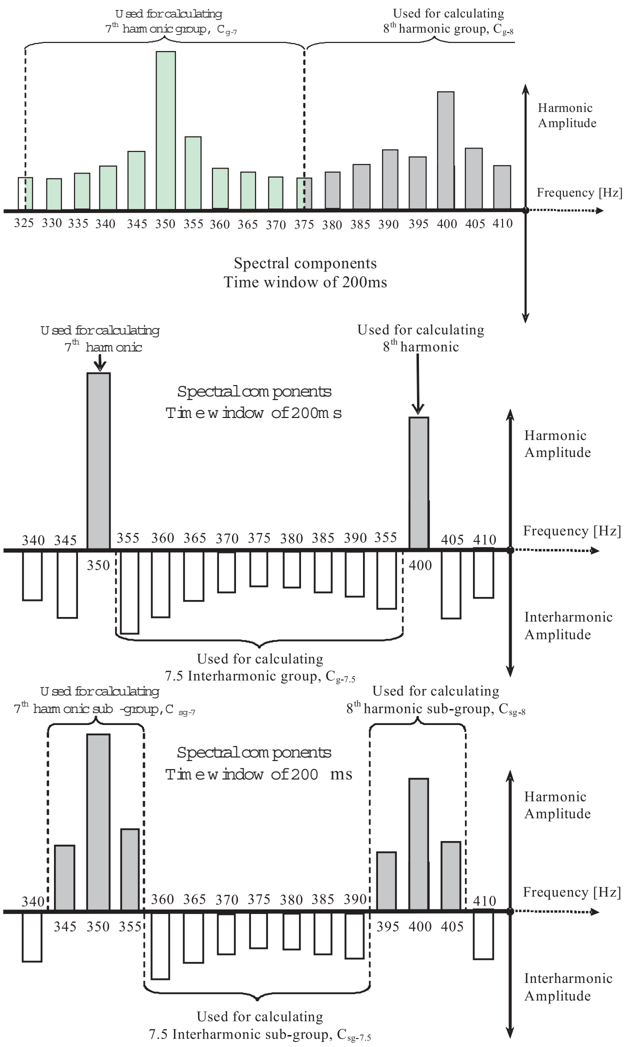

Fig. 4. Examples of harmonic (↑) and interharmonic (↓) (sub)groups according to IEC Standard drafts 61000-4-7 and 61000-4-30.

3.1. Experimental setup and results

The waveforms obtained from a power supply of a typical for dc arc furnace plant are analyzed [1]. The IEC groups and subgroups are estimated by using DFT and the results are compared to those obtained with subspace methods: the ESPRIT and the root-MUSIC.

In order to compare the different processing techniques, a *reference technique* is adopted. We assumed as reference the technique proposed in [1], named as "Ideal IEC", where the respective harmonic groupings are computed on the whole interval of 3s. In Fig. 5 the progressive average of the harmonic groups of the current is shown, as an example; the value of "Ideal IEC" is shown as constant unit value, whereas other results are scaled with respect to it. In Fig. 6 the progressive average of the second interharmonic subgroup is shown.

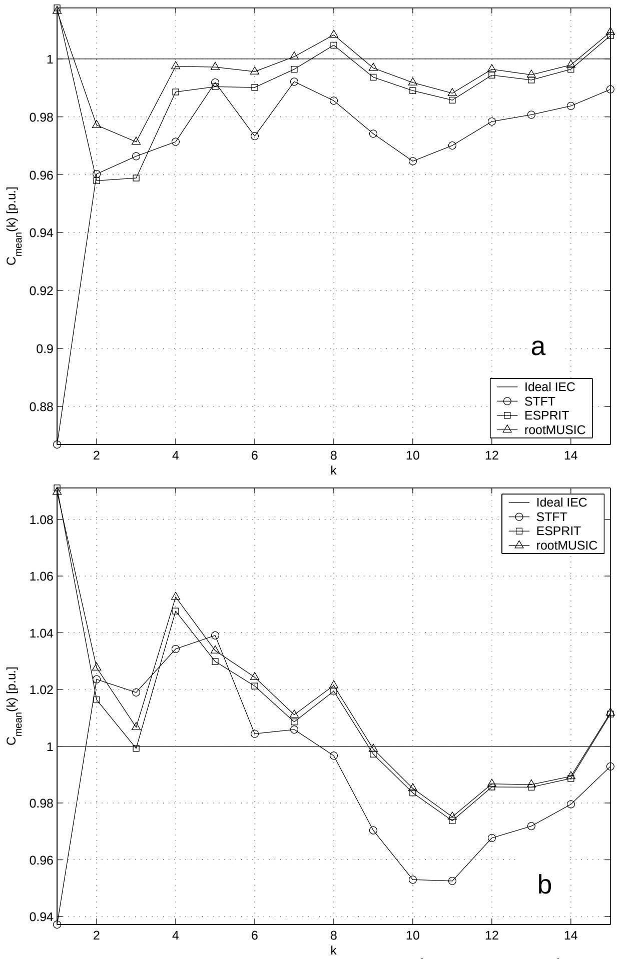

Fig.5. Progressive average of the 11th (a) and 13th (b) harmonic component of the current.

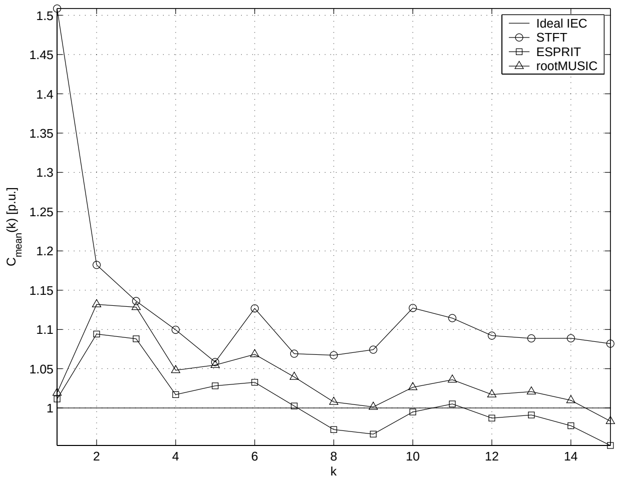

Fig.6. Progressive average of the 2nd interharmonic component of the current.

In Table 1 the results of harmonics and interharmonics subgroups estimation of the current are summarized. The mean-square error is calculated relative to the value of respective quantities. From over 500 independent calculations we may conclude that the ESPRIT method offers an average reduction of the error of harmonic groups and subgroups estimation by 54% and MUSIC method by 50%, comparing to DFT-based method.

Table 1

Relative MSE of the progressive average of harmonic and interharmonic subgroups estimation.

Method	MSE of harmonics estimation	MSE of interharmonics estimation
DFT	0.057	0.771
ESPRIT	0.029	0.180
MUSIC	0.027	0.231

4. CONCLUSIONS

In the paper the performance of parametric spectrum estimation methods (MUSIC and ESPRIT) were compared. The performance was estimated as accuracy of estimation of the frequency and amplitude of harmonic multi-component signals. The results show better performance of ESPRIT over MUSIC method for applications where the analyzed waveforms consists of many higher harmonics with variable amplitude and random initial phase (waveforms often encountered in power system analysis). Based on these results, optimal parameters were chosen for the following calculations.

As a practical application, the calculation of harmonic and interharmonic subgroups was chosen. Both parametric methods were used and the results compared to those obtained with commonly used DFT-based algorithms. Results show that the highest improvement of accuracy can be obtained by using the ESPRIT method (especially for interharmonics estimation), closely followed by MUSIC method, which outperform classical approach by over 50%.

5. ACKNOWLEDGEMENT

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6. REFERENCES

1. Bracale A., Carpinelli G., Lauria D., Leonowicz Z., Łobos T., Rezmer J: *On Some Spectrum Estimation Methods for Analysis of Non-stationary Signals in Power Systems – Part I: Theoretical aspects, Part II: Numerical Applications,* 11th International Conference on Harmonics and Quality of Power. ICHQP Lake Placid, New York, September 12-15, 2004.
2. Iaramillo S.H., Heydt G.T., Neill Carrello E.O.: *Power Quality Indices for Aperiodic Voltages and Currents*. IEEE Trans. on Power Delivery, Vol. 15, April 2000, pp. 784-790.
3. Leonowicz Z., Łobos T., Rezmer J., *Advanced Spectrum Estimation Methods for Signal Analysis in Power Electronics*: IEEE Trans. on Industrial Electronics, , vol. 50, no. 3, June 2003, pp. 514-519.
4. Łobos T., Leonowicz Z., Rezmer J., Schegner P.: *High-Resolution Spectrum-Estimation Methods for Signal Analysis in Power Systems*. IEEE Trans. on Instr. and Meas., Vol. 50, No. 1, Feb. 2006.
5. Therrien C.W.: *Discrete Random Signals and Statistical Signal Processing*. Prentice-Hall, Englewood Cliffs, New Jersey, 1992.

Dr inż. Zbigniew Leonowicz
Politechnika Wrocławska
Wydział Elektryczny
Instytut Podstaw Elektrotechniki i Elektrotechnologii
Wyb. Wyspiańskiego 27, 50-370 Wrocław
E-mail: zbigniew.leonowicz@pwr.wroc.pl