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ANALYSIS OF POWER QUALITY DISTURBANCES USINGTHE S-TRANSFORM

Keywords: S-Transform, Power Quality, Disturbances, Time-Frequency Analysis

Introduction

Contemporary power systems, which contain a considerable number of non-linear loads, require advanced methods of spectral analysis for their investigation and control. Fourier-based spectral methods are useful in stationary signal analysis but insufficient in the case of numerous real-life problems when signal contents changes in time. Nonstationary signals are usually analyzed with STFT (Short-Time Fourier Transform). The method presented in this paper employs recently introduced S-transform which is an important development of STFT with improved properties.

Proposed methods allow tracking changes in amplitude and frequency with better precision than STFT and Wigner-Ville transform. Possible applications in diagnosis and power quality problems are targeted.

1. THE S TRANSFORM

The S Transform, introduced by Stockwell [4] is defined by the general equation:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t)\, g(\tau - t, f) e^{-j2\pi ft} dt \qquad (1)$$

where $g(\tau, f)$ is a window function. Window function is a modulated Gaussian function, expressed by:

$$g(\tau, f) = \frac{|f|}{\sqrt{2\pi}} e^{-(t^2/2\sigma^2)} \qquad (2)$$

where σ is defined as:

$$\sigma = \frac{1}{|f|} \qquad (3)$$

Finally, the general formula in (1) takes the form:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{-((\tau-t)^2 f^2/2)} e^{-j2\pi ft} dt \qquad (4)$$

where f represents frequency and t, τ represent time.

The above derivation of the definition of S Transform originates from the STFT idea. However, the S Transform is related to the Wavelet Transform, as well.

Continuous Wavelet Transform $W(\tau, d)$ of the function $h(t)$ is defined by:

$$W(\tau, d) = \int_{-\infty}^{\infty} h(t) w(t-\tau, d) dt \qquad (5)$$

As previously stated, the S – Transform can be also presented as continuous wavelets transform using a specific mother wavelet multiplied by a phase coefficient:

$$S(\tau, f) = e^{j2\pi f\tau} W(\tau, d) dt \qquad (6)$$

Where the *mother wavelet* $\psi(t,d)$ is defined as:

$$\psi(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-(t^2 f^2/2)} e^{-j2\pi ft} \qquad (7)$$

where the coefficient d represents the inverse of the frequency.

Taking into account the above considerations, we can derive the general formula of the S-transform as in (1).

The equation (1) assumes that the width of the window function is inversely proportional to the local frequency. We can also adjust the resolution of the S-transform by introducing a modified coefficient σ:

$$\sigma = \frac{k}{|f|} \qquad (8)$$

Then, the equation (4) takes the form:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{k\sqrt{2\pi}} e^{-((\tau-t)^2 f^2/2k^2)} e^{-j2\pi ft} dt \qquad (9)$$

where the coefficient k controls the resolution in such way that if k>1 the resolution *in frequency* increases and when k<1 the resolution *in time* increases.

The S-transform is a linear transform of the signal $h(t)$. The additive noise can be modeled as follows: $h_{noisy}(t) = h(t) + \eta(t)$. The S-transform of a noisy signal represents a sum of transformed signal and transformed noise, as follows:

$$S\{h_{noisy}(t)\} = S\{h(t)\} + S\{\eta(t)\} \quad (10)$$

The S-transform output is a complex matrix, where the rows correspond to frequencies and the columns to time. Each column thus represents a “local spectrum” for that point in time. Since $S(\tau, f)$ is complex valued, in practice, usually the module $|S(\tau, f)|$ is plotted and this gives time-frequency S spectrum. The S-transform outperforms the STFT in that it has a better resolution in phase space (i.e. a more narrow time window for higher frequencies), giving a fundamentally more sound time frequency representation [4].

2. COMPUTATION OF THE ERROR OF REPRESENTATION (S-TRANSFORM, STFT, WIGNER-VILLE TRANSFORM)

Selected power quality disturbances are investigated using the S-transform (ST) and compared to previously investigated Short-Time Fourier Transform (STFT) and Wigner-Ville transform (WV) [?].

Signal parameters are as follows:

- sampling frequency 1,6 kHz
- signal length equal to 8 periods of main harmonic 50 Hz
- duration of the disturbance equal to 4 periods

Simulated power quality disturbances include:

- voltage drop
- over-voltage (5-50% of the nominal voltage)
- voltage sag (10-100% of the nominal voltage)

Investigations were carried out as follows, independently for each kind of disturbance:

1. Analysis using the S-transform for different values of the *k* coefficient, modifying its resolution in time and in frequency. The range of *k* varied from 0.1 to 1.0 which assures better resolution in time domain of the S-transform. After the transformation a time-frequency of the signal was obtained. An exemplary time-frequency plot is shown in Figure 1. for a 50% voltage sag and *k*=1.0.

2. Analysis using the STFT for analysis window length equal to ¼ of the signal length.

3. Analysis using the WV transform with smoothing.

From each time-frequency representation the main harmonic 50 Hz was filtered out and its amplitude was compared to the reference waveform. Additionally, for the S-transform, the investigations included the evaluation of the influence of the *k* coefficient on the error of representation.

Reference signal composed of one main harmonic of 50 Hz was generated independently for each analysis method with randomly variable phase.

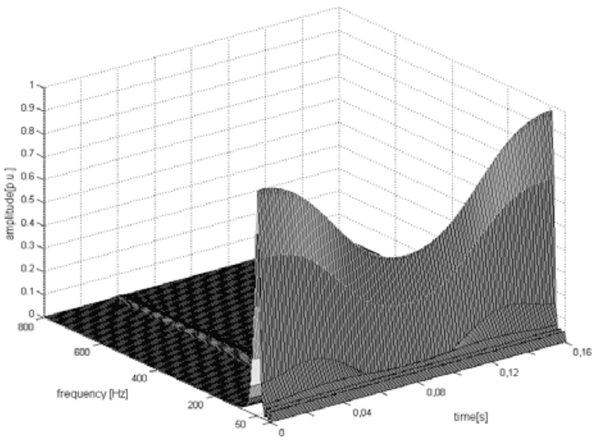

Fig.1. Time-frequency representation of a 50% voltage sag and *k*=1.0 using the S-transform.

Figure 2 shows the representation of the reference signal using the S-transform, the Short-Time Fourier Transform and the Wigner-Ville Transform for a 50% voltage sag. For each group of waveforms (as in Figure 2), the error of representation was calculated, according to the formula:

$$MSE = \frac{1}{256}\sum_{n=1}^{256}\sqrt{\left(A_{50Hz\mathrm{Repr}}(n) - A_{50Hz\mathrm{Ref}}(n)\right)^2} \quad (11)$$

where n- number of samples , $A_{50Hz\mathrm{Rep}}(n)$ - representation of the amplitude of the 50 Hz main harmonic for the time instant corresponding to the sample number *n*, $A_{50Hz\mathrm{Ref}}(n)$ - reference amplitude of the 50 Hz main harmonic for the time instant corresponding to the sample number *n* .

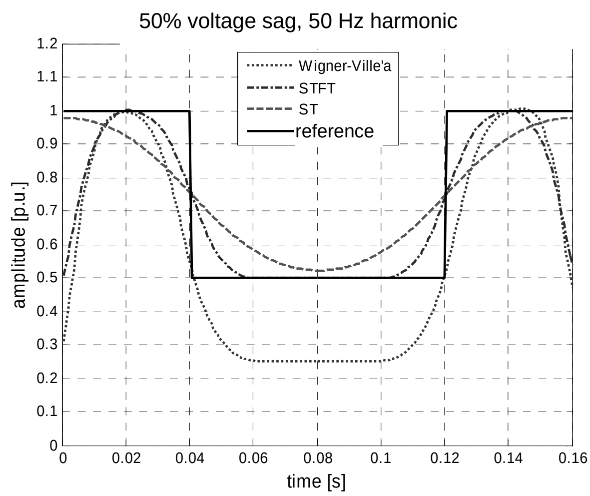

Fig. 2. Representation of the reference signal using the S-transform (ST) for K=1, the Short-Time Fourier Transform (STFT) – window length = 0.04 s and the Wigner-Ville Transform with smoothing window length of 0.04s – of the test waveform: 50% voltage sag.

2.1. Comparison of representation error.

The representation error computed as in (11) for exemplary power disturbances is shown in Figures 3 and 4. In Figure 3 voltage sag of variable amplitude from 10% to 100% is investigated and in Figure 4 a case of over-voltage with variable amplitude from 5% to 50% is shown. The S-transform shows overall smallest representation error, especially for the case when k=0.1 (small value of k means higher resolution in time domain).

Best performance of the S-transform can be explained by the *multi-resolution* capabilities of this representation – see Figure 5. The S-transform has better time resolution for higher harmonics (narrow analysis window) and better frequency resolution for low harmonics (wide analysis window).

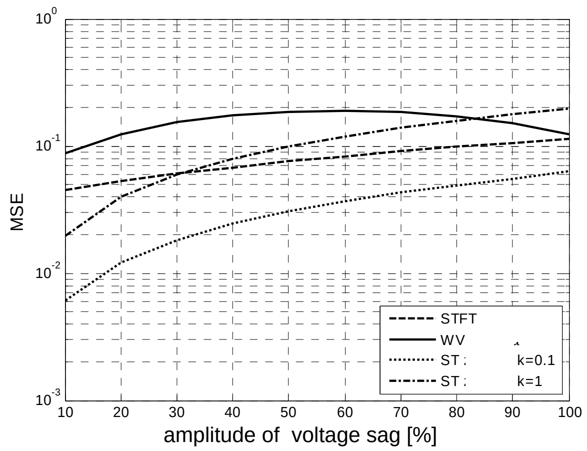

Fig. 3. Representation error for varying amplitude of voltage sag.

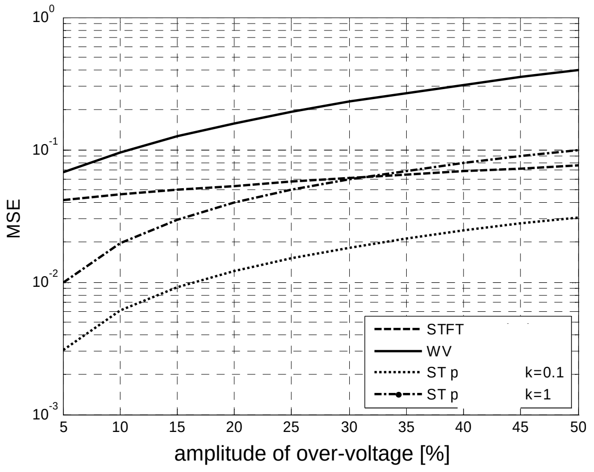

Fig. 4. Representation error for varying amplitude of over-voltage.

As seen from Figures 2-4, the S-transform in comparison to smoothed Wigner-Ville Transform and Short-Time Fourier Transform allows more accurate representation of dynamic changes of signal spectral components. The S-transform, providing the optimal setting of the parameter k, has the smallest representation error.

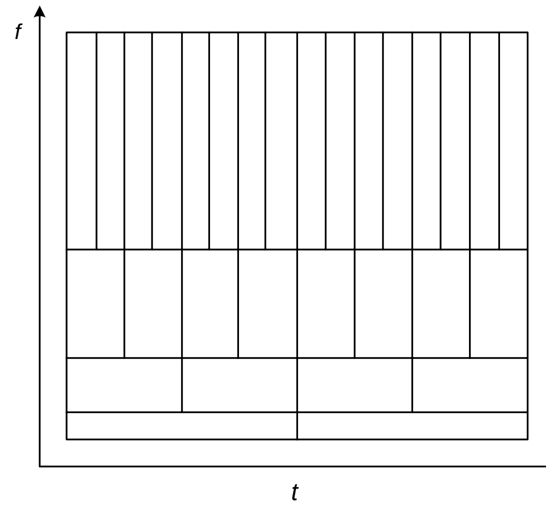

Fig. 5. Frequency-dependent resolution of the S-transform.

2.2. Analysis of switching of capacitor banks

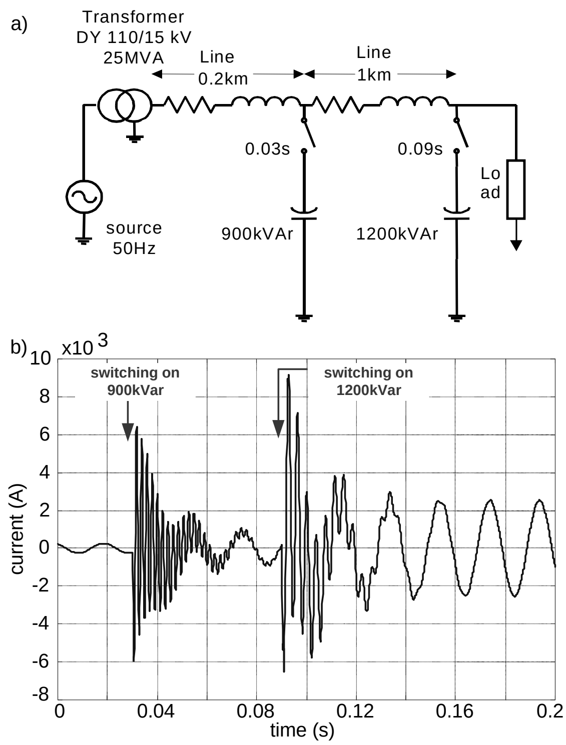

Fig. 6. a) One-phase diagram of the simulated distribution system. b) Voltage waveform at the beginning of the feeder.

The investigation results in a distribution system as in Figure 6a are shown in this section. Two capacitor banks (CB) were installed along the feeder. Several cases were simulated and both currents and voltages were recorded. Figure 6b shows the voltage waveform at the beginning of the feeder for the case that the first CB 900 (kVAr) was switched on at 0.03 s and the second CB 1200 (kVAr) at 0.09 s. Figure 7 shows three-dimensional time frequency representations obtained using a) S-transform b) STFT - of the complex waveform.

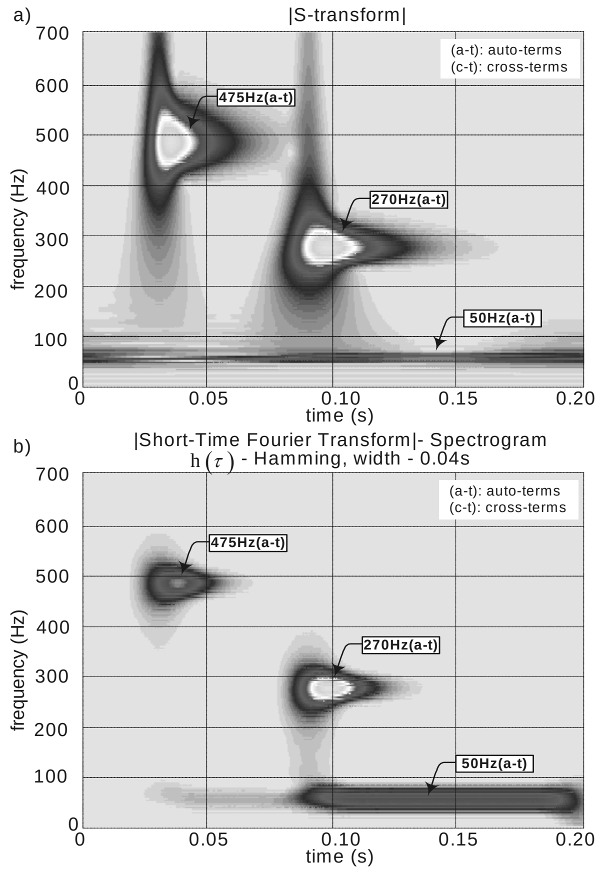

Fig. 7. Comparison of S-transform representation and STFT spectrum of the waveform.

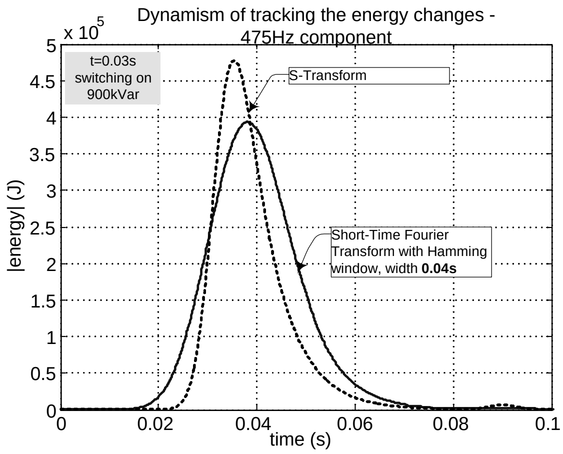

Fig. 8. Comparison of tracking capabilities of dynamic amplitude changes of the 475 Hz components.

The S-transform shows again better energy concentration and ,therefore, lower error when estimating the amplitude of time-varying amplitude of one spectral component in a signal composed of multiple spectral components and noise (Figure 8).

3. CONCLUSIONS

In all performed experiments (in the paper only limited number of them is reported) the S-transform showed the lowest error when comparing the error of representation to STFT and Wigner-Ville transform, especially when analyzing dynamically changing spectral components. Sometimes it was necessary to adjust the *k* coefficient to obtain the optimal representation for a given problem and adjust in this way the multi-resolution property of the S-transform. The same advantageous tracking capability showed the S-transform when analyzing the time-varying multi-component signal.

Performed experiments allow concluding that the S-transform is a reliable and accurate tool for the analysis non-stationary waveforms in power systems and its properties can be used for diagnostic and power quality applications.

5. REFERENCES

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6. SHORT ABSTRACT

The S-transform outperforms the STFT in that it has a better resolution in phase space giving a fundamentally more sound time frequency representation. Investigations of the representation error show that optimally adjusted S-transform can also outperform the Wigner-Ville transform when dealing with time-frequency representations of the signal. The S- transform is also tested on nonstationary electric signals where it shows excellent tracking capability.

These properties show that S-transform can be effectively used for analysis of electric signals, especially when dealing with multi-component time-varying waveforms.

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