HARMONICS AND INTERHARMONICS ESTIMATION USING ADVANCED SIGNAL PROCESSING METHODS

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1. Introduction

- Modern frequency power converters generate a wide spectrum of harmonic components

- In some cases, large converter systems generate not only characteristic harmonics typical for the ideal converter operation, but also a considerable amount of non-characteristic harmonics and interharmonics

  - this may strongly deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of the power system.

The estimation of the signal parameters is important for control and protection tasks.

Most of commonly used approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, operate adequately only in the narrow range of frequencies and at moderate noise levels.

Spectrum estimation of discretely sampled processes is usually based on procedures employing the FFT. This approach is computationally efficient and produces reasonable results for a large class of signal processes. However, there are several performance limitations of the FFT.

The most prominent limitation:

- frequency resolution, i.e. the ability to distinguish the spectral responses of two or more signals;

The second limitation:

- due to windowing of the data.

Fourier algorithms are accurate only when the sampling interval is equal to one period or more periods of the main component. In the presence of interharmonics the period can be very long and change with time.

Windowing manifests as leakage in the spectral domain:

energy in the main lobe of a spectral response “leaks” into the sidelobes obscuring and distorting other spectral responses.

The limitations are troublesome when analyzing short data records.

Short data records occur frequently in practice because many processes:

- are brief in duration,
- have slowly time-varying spectra, that may be considered constant only for short record lengths.

The time-varying spectra of a nonstationary time series commonly used are the spectrogram from the short-time Fourier transform (STFT) and the scalogram obtained from the wavelet transform.

Another type of time-frequency distribution is the Wigner-Ville (WV) distribution.

Wigner-Ville spectrum of signals with time-limited windows shows better frequency concentration and less phase dependence than Fourier spectra.

Many alternative spectral estimation procedures have been proposed within the last four decades.

In many cases an improved spectral fidelity can be achieved.

However, the computational requirements of that method may be significantly higher than FFT processing.

Windowing of data makes the implicit assumption that the unobserved data outside the window are zero.

A smeared spectral estimate is a consequence.

If one:

- has more knowledge about the process,
- is able to make a more reasonable assumption,

it is usually possible to obtain a better estimate.

The spectrum analysis becomes a three step procedures:

- to select a time series model,
- to estimate the parameters of the assumed model,
- to obtain the spectral estimate.

Three main time series models:

- autoregressive (AR),
- moving average (MA),
- autoregressive-moving average (ARMA).

Many deterministic and stochastic discrete-time processes are well aproximated by a rational transfer function model:

$$x_n = \sum_{l=0}^{q} b_l n_{n-l} - \sum_{k=1}^{p} a_k x_{n-k} \qquad (1)$$

where:

$\{n_n\}$ - input driving sequence,

$\{x_n\}$ - output sequence

The most general linear model is termed an ARMA model.

The system function:

$$H(z) = \frac{B(z)}{A(z)} \qquad (2)$$

where: $A(z)$ – Z-transform of AR branch = $\sum_{m=0}^{p} a_m z^{-m}$

$B(z)$ – Z-transform of MA branch = $\sum_{m=0}^{q} b_m z^{-m}$

Often the driving process is assumed to be a white-noise sequence of zero mean and variance s^2.

Specification of the parameters $\{a_k\}$, $\{b_k\}$ and s^2 is equivalent to specifying the spectrum of the process $\{x_n\}$.

The estimation of parameters for an AR model results in linear equations.

An overdetermined system of equations can be solved using the SVD (Singular Value Decomposition) approach.

Two kinds of AR parameter estimation methods:

- -autocorrelation method,
- -covariance method.

The only difference between the covariance method and the autocorrelation method is the range of summation in the prediction error power estimate.

The covariance method is identical to the modern version of the **Prony** method.

Subspace Methods

We assume that the data consists of p complex sinusoids in complex white Gaussian noise.

$$x[n]=\sum_{i=1}^{M} A_i \exp(j2pf_i n+\Phi_i)+h$$

$$\mathbf{x} = \sum_{i=1}^{M} A_i \mathbf{s}_i + \eta \qquad (3)$$

for n = 0,1,...,N-1

η is complex white Gaussian noise with zero-mean and variance s_0^2.

The NxN autocorrelation matrix for N > M+1

$$\mathbf{R_x} = \sum_{i=1}^{M} E\{A_i A_i^*\}\mathbf{s_i s_i}^T + s_0^2\mathbf{I} = \mathbf{R_{ss}} + \mathbf{R_{zz}} \qquad (4)$$

$\mathbf{R_x}$ is the sum of a signal autocorrelation matrix $\mathbf{R_{ss}}$ and a noise autocorrelation matrix $\mathbf{R_{zz}}$.

The frequency information is contained in the matrix $\mathbf{R_{ss}}$.

The decomposition of the matrix $\mathbf{R_{xx}}$ bases on the eigenvectors and eigenvalues.

The eigenvectors corresponding to the M largest eigenvalues contain information about signal parameters.

To extract the information we can use the orthogonality of the eigenvectors.

It is said that eigenvectors containing the information about signal span the signal subspace and the remaining span the noise subspace.

The signal vectors are orthogonal to all vectors in the noise subspace.

The earliest application of the property is the Pisarenko Harmonic Decomposition (PHD).

Other subspace methods:

- MUSIC (Multiple Signal Classification)
- Min-Norm

We have investigated the following metods:

- Prony method,
- subspace methods,

  and especially MIN-NORM

2. Prony method

Assuming the N complex data samples $x[1],\dots,x[N]$ the investigated function can be approximated by M exponential functions:

$$y[n]=\sum_{k=1}^{M} A_k e^{(a_k+jw_k)(n-1)T_p+jy_k} \qquad (1)$$

where

$n=1,2,\dots,N$

T_p – sampling period,

A_k - amplitude
a_k – damping factor,
w_k – angular velocity
y_k – initial phase.

The discrete-time function may be expressed as:

$$y[n] = \sum_{k=1}^{M} h_k z_k^{n-1} \quad (2)$$

where $h_k = A_k e^{j y_k}$

$z_k = e^{(a_k + j w_k) T_p}$

The estimation problem - minimization of the squared error over the N data values:

$$d = \sum_{n=1}^{N} |e[n]|^2 \quad (3)$$

where

$$e[n] = x[n] - y[n] = x[n] - \sum_{k=1}^{M} h_k z_k^{n-1} \quad (4)$$

The nonlinear problem can be solved using the Prony method.

First case

The number of data samples is equal to the number of exponential parameters,.
The M-exponent discrete-time function:

$$x[n] = \sum_{k=1}^{M} h_k z_k^{n-1} \quad (5)$$

The p equations of (5) may be expressed in matrix form.

The set of linear equations can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the z_k exponents as its roots:

$$F(z) = \prod_{k=1}^{M} (z - z_k) = \\ = (z - z_1)(z - z_2)\ldots(z - z_M) \tag{7}$$

The polynomial may be represented as the sum:

$$F(z) = \sum_{m=0}^{M} a[m] z^{M-m} = \\ = a[0]z^M + a[1]z^{M-1} + \ldots + a[M-1]z + a[M] \tag{8}$$

The right-hand summation in (8) may be recognize as polynomial, evaluated at each of its roots z_k yielding the zero result:

$$\sum_{m=0}^{M} a[m]x[n-m] = 0 \tag{11}$$

The equation can be solved for the polynomial coefficients.

In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

Second case

The number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2M$.

In the overdetermined data case, the linear equation (11) must be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m] = e[n] \tag{12}$$

The estimation problem bases on the minimization of the total squared error:

$$E = \sum_{n=M+1}^{N} \left|e[n]\right|^2 \tag{13}$$

3. Min-Norm method

Min-Norm method uses one vector **d** for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element is equal to one.

4. Experiments with simulated waveforms

The investigated signal:

- basic harmonic (*50 Hz*),
- one higher harmonic (*125 Hz*),

- one interharmonic (*25 Hz*)
- distorted by *5%* random noise.

The sampling interval - *0.5 ms*.

The Prony and Min-Norm methods enable to detect all the signal components already using *100* sample.

The Fourier algorithm indicates the frequencies ca. *21,5; 51; 127Hz*.

5. Simulation of a frequency converter

3 kVA – PWM – converter;
modulation frequency - *1 kHz*;
2-pole, *1 kW* asynchronous motor (*U = 380 V, I = 2.8 A*);
output frequency - *0.1 ÷ 150 Hz*.

Fig. 2 - current waveform at the converter output for the frequency *100 Hz*; sampling interval - *0.2 ms*.

The Prony and Min-Norm methods enable to estimate the frequencies of all the signal components:*100, 800, 1000, 1200 Hz* using *100* samples.

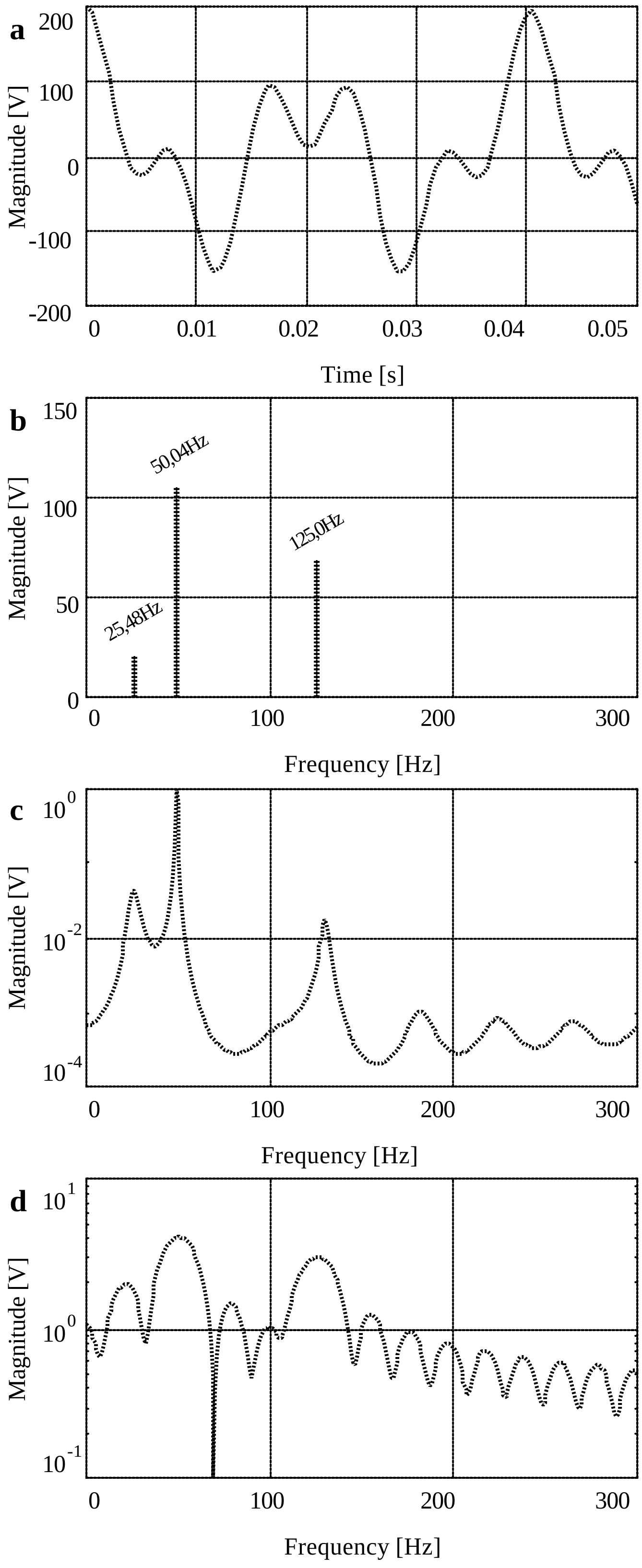
a
200
100
0
-100
-200
Magnitude [V]
0
0.01
0.02
0.03
0.04
0.05
Time [s]
b
150
100
50
0
Magnitude [V]
50,04Hz
125,0Hz
25,48Hz
0
100
200
300
Frequency [Hz]
c
10^0
10^{-2}
10^{-4}
Magnitude [V]
0
100
200
300
Frequency [Hz]
d
10^1
10^0
10^{-1}
Magnitude [V]
0
100
200
300
Frequency [Hz]

Fig. 1. Voltage waveform at the output of a simulated DC arc furnace power supply installation (a); investigation results: Prony, M=30 (b); min-norm (c); FFT (d); f_p=2000Hz, N=100.

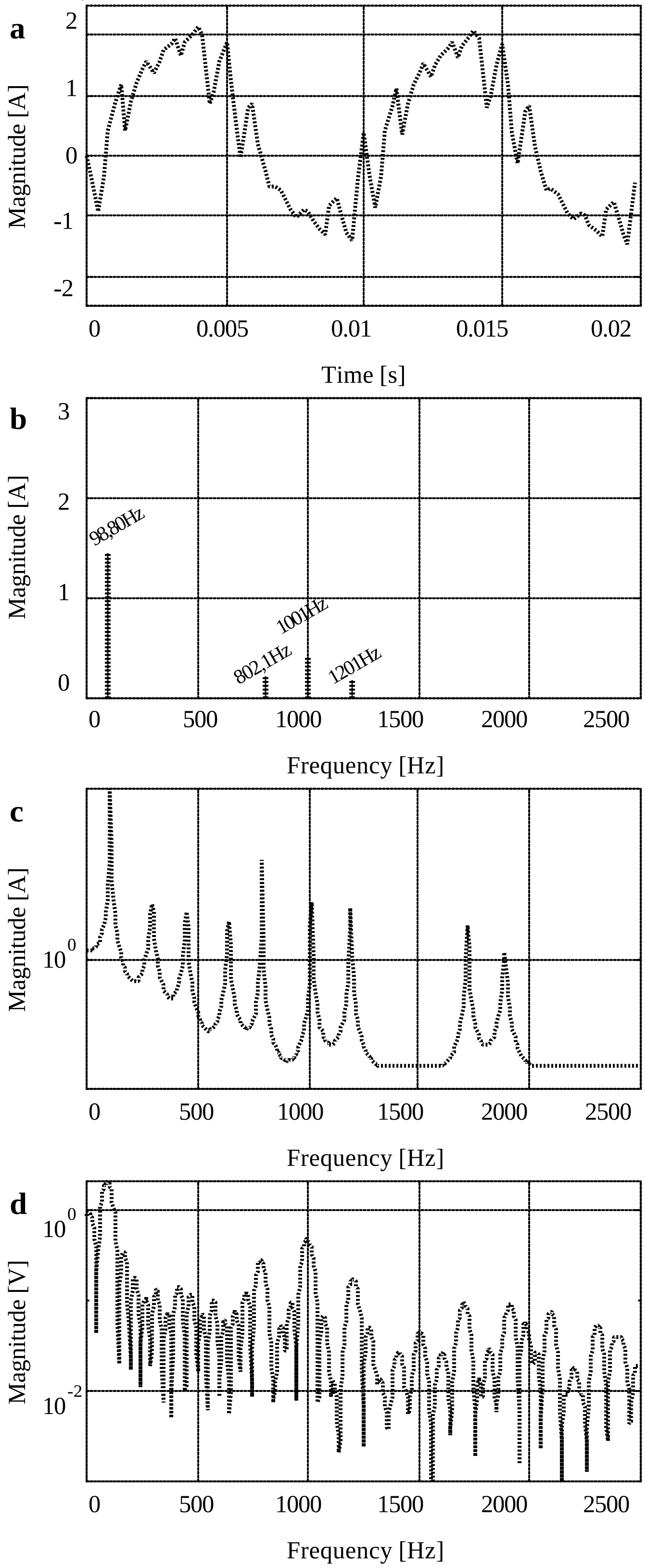

Fig. 2. Current waveform at the output of a simulated frequency converter (a); investigation results: Prony N=80,M=40 (b); min-norm N=100 (c); FFT N=80 (d); f_p=5000Hz.

6. Industrial frequency converter

Investigated industrial drive:

- a three-phase asynchronous motor,
- a power converter, composed of a single-phase half-controlled bridge rectifier and a voltage source converter.

The current waveform at the converter output (Fig. 3) , under normal conditions, was investigated using the Prony, Min-Norm and FFT methods.

The main frequency of the waveform - *40 Hz*.

The following harmonics have been detected at the converter output: *7th*, *17th*, *19th*, *25th*, *35th*, and *41th*.

The Min-Norm method has additionally detected the 5th harmonic.

It is also possible to estimate the frequency of the fundamental component.

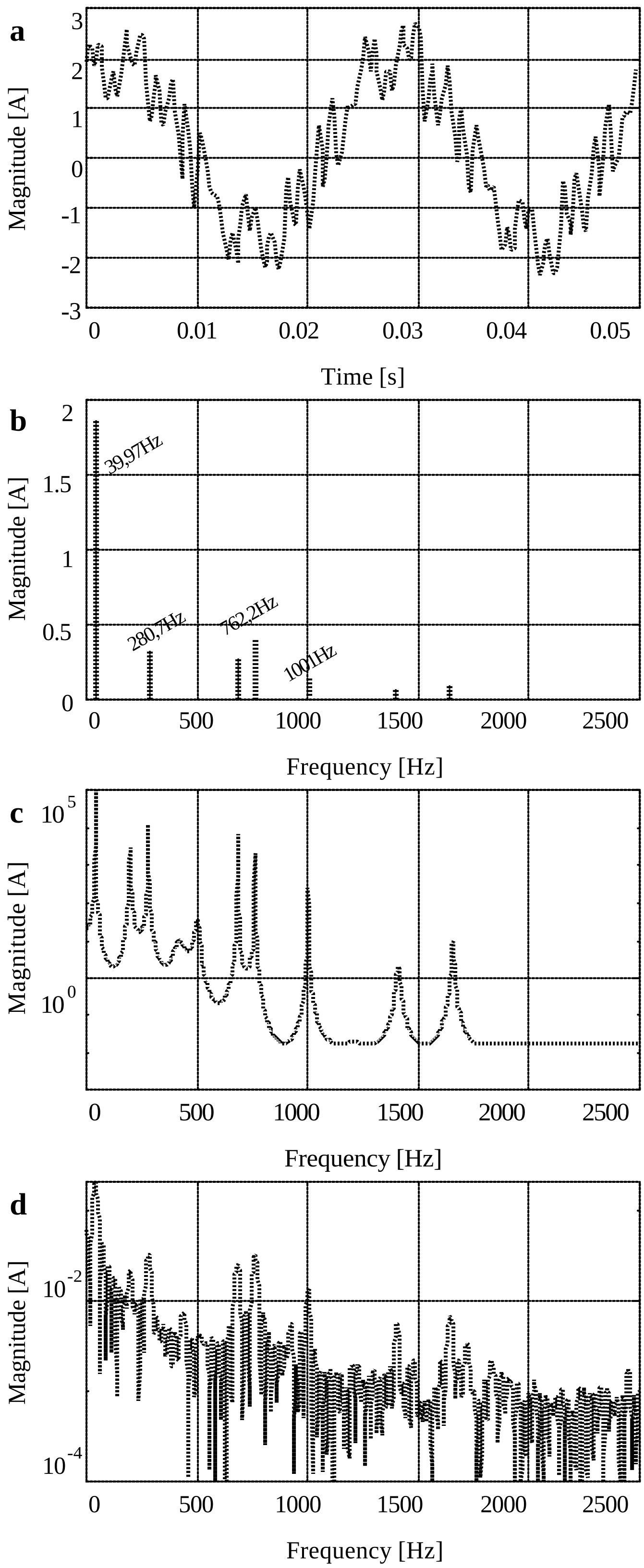

Fig. 3. Current waveform at the output of a real frequency converter (a); investigation results: Prony N=200,M=80 (b); min-norm N=100 (c); FFT N=200 (d), f_p=5000Hz

7. Conclusions

High-resolution spectrum estimation method, such as min-norm could be effectively used for parameter estimation of distorted signals.
The Prony method could also be applied for estimation the frequencies of signal components.
The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics.
When using both the high-resolution methods: Prony and min-norm the estimation accuracy in most cases is better than when using the Fourier algorithm. Application of the proposed advanced methods makes it possible the estimation of the components, which frequencies differ insignificantly.
Their computation is much more complex than FFT.