HIGH-RESOLUTION SPECTRUM ESTIMATION METHODS FOR SIGNAL ANALYSIS IN POWER SYSTEMS

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Abstract: Modern power frequency converters generate not only a wide spectrum of characteristic harmonics but also non-characteristic harmonics and interharmonics. Interharmonics are considered more damaging than ordinary harmonic components of the distorted signals. Usual tools of the harmonic analysis based on Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long. In this paper a novel approach to the interharmonic analysis is proposed based on the "subspace" methods. The singular value decomposition was also tested for this purpose. Both new high-resolution methods show no disadvantages of the traditional tools and allow exact estimation of the interharmonic frequency.

Keywords: spectrum analysis, frequency conversion, power systems, signal analysis, higher-order statistics, singular value decomposition.

1. INTRODUCTION

Modern frequency power converters generate a wide spectrum of harmonic components, which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converter systems generate not only characteristic harmonics typical for the ideal converter operation, but also a considerable amount of non-characteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply (Carbone, *et al*., 1998). Interharmonics are defined as non-integer harmonics of the main fundamental under consideration.

The estimation of the components is important for control and protection tasks. There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, etc. (Cichocki and Lobos, 1994). Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels.

The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods". MUSIC (Multiple Signal Classification) harmonic retrieval method is an example of eigenstructure-based methods, which yield high resolution and asymptotically exact results. (Mendel, 1992; Therrien, 1991).

The MUSIC method as all of the analyses based on second-order statistics, can also be based on fourth-order statistics. There are several motivations to use higher-order spectra in signal processing. In practical applications, a non-Gaussian signal is received along with additive coloured Gaussian noise of unknown power spectrum.

For Gaussian signals all higher-order spectra of an order greater than two are identical to zero. A transformation to a higher-order cumulant domain eliminates the noise.

In this paper the diagonal slice of the fourth-order cumulant is used to estimate the signal spectrum with the help of the MUSIC method.

The singular value decomposition (SVD) approach is also a very good tool for frequency estimation (Bakamidis, *et al*., 1991; Osowski, 1994; Lobos, *et al*., 1999). The SVD technique is a highly reliable, computationally stable mathematical tool to solve the rectangular overdetermined system of equation. The method can also be applied for frequency estimation of the fundamental component of very distorted periodical signals.

The known non-linear least square methods (James, *et al*., 1994; Pintelon and Schoukens, 1996) are developed and investigated for periodic signals, without interharmonics, and seem to be efficient for the narrow range of harmonics frequencies. In some cases, the proposed method can be used for calculating starting values for the maximum likelihood solution.

To investigate the ability of the methods several experiments were performed. We investigated simulated waveforms as well as real current waveforms at the output of a three-phase frequency converter supplying an induction motor. For comparison, similar experiments were repeated using the FFT with the same number of samples and the same sampling period.

2. SUBSPACE SPECTRUM ESTIMATION METHODS BASED ON CUMULANTS

2.1. Cumulant-based approach

In the case of real harmonic signals the fourth-order cumulant is given by (Mendel, 1991).

$$C_{4,x} = -\tfrac{1}{8}\sum_{k=1}^{p} a_k^4 \left[\cos w_k (t_1 - t_2 - t_3) + \cos w_k (t_2 - t_3 - t_1) + \cos w_k (t_2 - t_3 - t_1)\right] \qquad (1)$$

If *x(n)* is a sum of *p* real-valued sinusoids, then the diagonal slice of the fourth-order cumulant retains all of the pertinent information about the number of harmonics, their amplitudes and frequencies. Set $t_1 = t_2 = t_3 = t$ in (1) to obtain:

$$C_{4,y}(t) = -\tfrac{3}{8}\sum_{k=1}^{p} a_k^4 \cos(w_k t) \qquad (2)$$

which is nearly identical with the autocorrelation of the signal.

It is known that signal parameters can be estimated from the output correlation in the white noise case; they can also be estimated from the fourth-order cumulant, which is useful in the additive coloured noise case. It means that existing high-resolution methods, such as MUSIC, can be applied by replacing correlation quantities with the fourth-order cumulant (Leonowicz, *et al.*, 1998). In this paper the diagonal slice of the fourth-order cumulant is used instead of correlation matrix to estimate the signal spectrum with the help of the MUSIC method.

2.2. MUSIC (MUltiple SIgnal Classification)

The MUSIC method (Therrien, 1991) involves projection of the signal vector:

$$\boldsymbol{s_i} = \left[1 \quad e^{jw_i} \quad \cdots \quad e^{j(N-1)w_i}\right]^T \qquad (3)$$

onto the entire noise subspace, where N is the number of the signal vector components.
If the noise is white, the correlation matrix is:

$$\mathbf{R}_x = \sum_{i=1}^{M} \mathrm{E}\left\{A_i A_i^*\right\} \boldsymbol{s_i} \boldsymbol{s_i^T} + s_0^2 \mathbf{I} \qquad (4)$$

where A_i are complex amplitudes. N-M smallest eigenvalues of the correlation matrix (matrix dimension $N>M+1$) correspond to the noise subspace and M largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

The matrix of eigenvectors is defined by:

$$\mathbf{E}_{noise} = \left[\mathbf{e}_{M+1} \quad \mathbf{e}_{M+2} \quad \cdots \quad \mathbf{e}_N\right] \qquad (5)$$

$\mathbf{E}_{noise}$ can be used to form a projection matrix $\mathbf{P}_{\boldsymbol{X}}$ for the noise subspace

$$\mathbf{P}_{noise} = \mathbf{E}_{noise}\mathbf{E}_{noise}^{*T} \qquad (6)$$

The squared magnitude of the projection of an auxiliary vector **w** (defined as in (3)) onto the noise subspace is given by:

$$\mathbf{w}^{*T}\mathbf{P}_{noise}\mathbf{w} = \mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w} \qquad (7)$$

Since each of the elements of the signal vector is orthogonal to the noise subspace, the quantity (7) goes to zero for the frequencies where $\mathbf{w}=\boldsymbol{s}_i$.

The MUSIC pseudospectrum is defined as:

$$\hat{P}\left(e^{jw}\right) = \left[\mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w}\right]^{-1} \qquad (8)$$

It exhibits sharp peaks at the signal frequencies where $\mathbf{w}=\boldsymbol{s}_i$.

3. PRINCIPLES OF THE SVD APPROACH

The waveform of the voltage or current is assumed as the sum of harmonics of unknown magnitudes and phases.

$$x(t) = \sum_{k=1}^{N} X_k \cos(w_k t + j_k) + k_s e(t) \qquad (9)$$

in which X_k, w_k and j_k are the unknown amplitude, angular frequency and phase of the *k-th* harmonic and *N* is the number of these harmonics. The variable *e(t)* represents the additive Gaussian noise with unity variance and k_s - the gain factor. Further, the set of *n* measured samples x_1, x_2, ... x_n of the waveform is considered. The number of measurements is usually higher than the number of harmonics. Estimation of harmonics is then equivalent to solving the overdetermined system of algebraic equation (James, *et al.*, 1994).

$$\mathbf{Ah} = \mathbf{b} \qquad (10)$$

where the matrix **A** and vectors **h** and **b** are given as follows

$$\mathbf{A} = \begin{bmatrix} x_l & x_{l-1} & \dots & x_1 \\ x_{l+1} & x_l & \dots & x_2 \\ \dots & \dots & \dots & \dots \\ x_{n-1} & x_{n-2} & \dots & x_{n-l} \\ x_2 & x_3 & \dots & x_{1+1} \\ x_3 & x_4 & \dots & x_{1+2} \\ \dots & \dots & \dots & \dots \\ x_{n-l+1} & x_{n-l+2} & \dots & x_n \end{bmatrix} \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ h_l \end{bmatrix} \mathbf{b} = \begin{bmatrix} x_{l+1} \\ x_{l+2} \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ x_{n-l} \end{bmatrix} \qquad (11)$$

l is the order of predicted **AR** model of the data ($N \le l \le n\text{-}N/2$). The vector **h** is composed of the coefficients of the impulse response of this model

$$H(z) = 1 - \sum_{i=1}^{l} h_i \ z^{-i} \qquad (12)$$

The solution for vector **h** of (10) is possible in the least square (LS) sense, i.e. by minimising the summed squared error between the left and right hand sides of the equation. The objective function to be minimised may be expressed in the norm - 2 vector notation form as

$$E = \frac{1}{2} \|\mathbf{Ah} - \mathbf{b}\|_2^2 \qquad (13)$$

To solve (10) the most suitable method seems to be the application of singular value decomposition. In this approach the rectangular matrix **A** is represented as the product of three matrices.

$$\mathbf{A} = \mathbf{U}\ \mathbf{S}\ \mathbf{V}^{\mathbf{t}} \qquad (14)$$

where **U** and **V** are orthogonal matrices of the dimension *n´n* and *l´l* respectively, while **S** is the quasidiagonal *n´l* matrix of singular values s_1, s_2, ... ,s_p ordered in a descending way, i.e. $s_1 \ge s_2 \ge ... \ge s_p \ge 0$. The essential information of the system is contained in the first non-zero singular values and first *p* singular vectors, forming the orthogonal matrices **U** and **V**. By cutting the appropriate matrices to this size and denoting them by $\mathbf{U_r}$, $\mathbf{S_r}$, and $\mathbf{V_r}$, respectively, the solution of (10) is obtained in the form.

$$\mathbf{h} = \mathbf{V_r}\ \mathbf{S_r^{-1}}\ \mathbf{U_r^t}\ \mathbf{b} \qquad (15)$$

where

$$\mathbf{S}^{-1} = \text{diag} \left[\frac{1}{s_1}, \frac{1}{s_2}, \dots, \frac{1}{s_p} \right]. \qquad (16)$$

On the basis of the determined coefficients **h** the zeros of polynomial (11) can be found. The phases of the roots closest to the unit circle denote the angular frequencies of the sinusoids forming the waveform (9). These frequencies can also be determined on the basis of the frequency characteristics of the model (5). They correspond to the frequencies in the range $-0.5 \le f \le 0.5$ for which the magnitude response $½H(e^{j2\pi f})½$ is equal or closest to zero.

4. EXPERIMENTS WITH SIMULATED WAVEFORMS

To investigate the efficiency of the proposed approaches several experiments were performed with the signal waveform described in (Mattavelli, 1998). The investigated signal is characteristic for DC arc furnace installations without compensation. It consists of basic harmonic (50 Hz), one higher harmonic (125 Hz), one interharmonic (25 Hz) and is additionally distorted by 5% random noise. The sampling interval was 0.5 ms.

The signal was investigated using the MUSIC and SVD methods. The MUSIC method enables to detect all the signal components using already 40 samples. The more samples taking into the calculation the more exact are the results. Figure 1 shows the results for n=80. For comparison similar experiments were repeated using Fourier algorithm with the same number of samples and the same sampling interval. For detection the 25 Hz component using the Fourier algorithm much more samples were needed. When using the same number of samples (80) the Fourier algorithm indicates the frequencies ca. 20, 55 and 130 Hz.

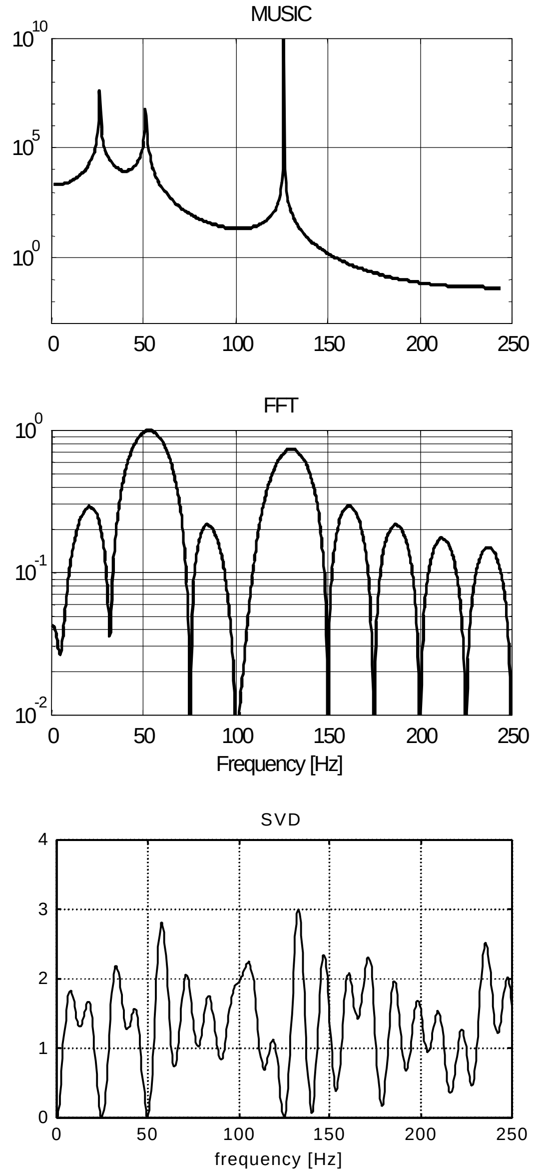


Fig. 1. Magnitude characteristics of the signal calculated by the MUSIC, FFT algorithms (n=80) and SVD (l=80, n=90).

When applying the SVD method, taking into account only the dominant singular values, we can approximately assess the number of components existing in the measured waveform. For detection all the components at least n=90 samples were needed. Figure 1 presents the obtained magnitude frequency characteristic of the system. The zeros of the characteristic or the points closest to zero determine the exact values of the frequencies. The superiority of the MUSIC and SVD methods over Fourier algorithms is visible.

5. SIMULATION OF A FREQUENCY CONVERTER

In recent years, simulation programs for complex electrical circuits and control systems have been essentially improved. The simulation of characteristic transient phenomena concerning the electrical quantities becomes feasible without any arrangement of hardware. Among many available simulation programs, the EMTP – ATP as a Fortran based program adapted to DOS/Windows serves for modelling complex 1- and 3-phase networks occurring in drive, control and energy systems.

In the paper we show investigation results of a 3 kVA – PWM – converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U=380V, I=2.8A). The simulated converter can change the output frequency within a range 0.1 ÷ 150 Hz.

Figure 2 shows the current waveform at the converter output for the frequency 100 Hz. The sampling interval was 0,05 ms. The MUSIC method enables us to estimate the frequencies of all the signal components already using 100 samples. The number of samples corresponds to a half period of the main component.. When applying the SVD method, for detection all the components n=260 samples were needed. Using the MUSIC and SVD methods with l=254 and n=260 we can detect the following frequencies: 100, 800, 1000 and 1200 Hz.

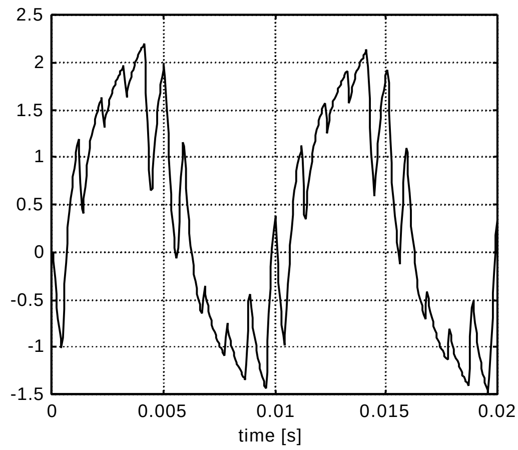
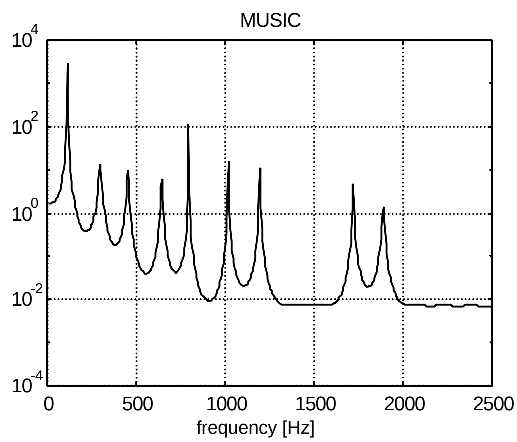


Fig. 2. Current waveform at the output of a simulated frequency converter, f = 100 Hz.



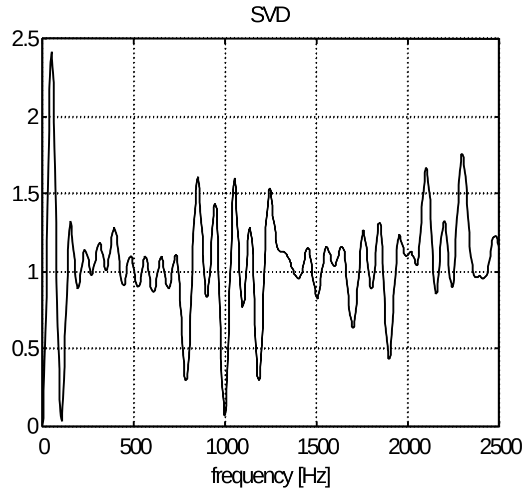


Fig. 3. Magnitude characteristics of the signal in Figure 2 calculated by the MUSIC (n=100) and SVD (l= 254, n= 260) method.

6. REAL INDUSTRIAL FREQUENCY CONVERTER

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter composed of a single-phase half-controlled bridge rectifier and a voltage source inverter. The waveforms of the inverter output current under normal conditions (Fig.4) were investigated using the SVD and MUSIC methods. The main frequency of the waveform was 40 Hz. Using the MUSIC method and SVD method (Fig.5) with l= 70, n= 100 we can detect the following harmonics 5^{th}, 7^{th}, 19^{th}, 25^{th}, 35^{th} and 41^{st}. It is also possible to estimate the frequency of the fundamental component. Estimation of the main component frequency enables to choose an appropriate sampling window for the FFT.

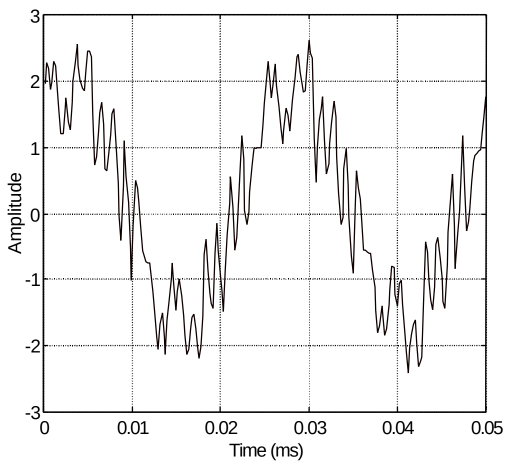


Fig. 4. Current waveform at the output of frequency converter.

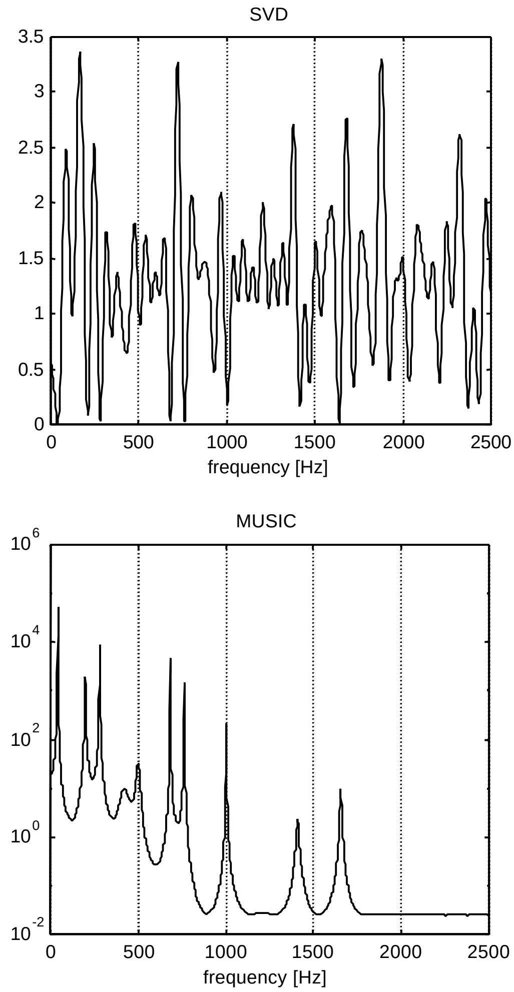


Fig. 5. Magnitude characteristic SVD method and by the MUSIC method of the signal in Fig. 4, l = 70, n=100.

7. CONCLUSIONS

It has been shown that a high-resolution spectrum estimation method, such as MUSIC combined with the use of higher–order statistics could be effectively used for parameter estimation of distorted signals. First of all, the method has been applied for frequency estimation of harmonic and interharmonic signal components. The frequency estimation of the main harmonic makes it possible to calculate the stationary space-phasor for system visualisation and to determine an appropriate window for a DFT-analysis.

The linear least square method for harmonics and interharmonics detection in a power system has also been investigated in this paper using the singular value decomposition (SVD).

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection and location of all higher harmonics existing in the system. The accuracy of the estimation method depends on the signal distortion, the sampling window and on the number of samples taken into the estimation process. The comparison to the standard FFT technique has proved the superiority of the approach for signals buried in noise, especially when estimating the parameters of interharmonics.

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