Time-Frequency Analysis of Three Phase Signals Using Wigner Distribution

Zbigniew Leonowicz, Tadeusz Łobos, Tomasz Sikorski

***Abstract* - New method of observation and diagnosis of inverter-fed induction motor drives is developed and tested. Unsymmetrical conditions concerning the machine impedances or valves operation are reflected in the spectrum of the current space-phasor. The spectrum of the space-phasor was estimated with the help of the Wigner-Ville distribution (WVD) and its time-frequency representation with excellent time and frequency resolution has been obtained. The proposed method was also tested with non-stationary multiple-component signals in the supply installation of compensated AC arc furnace.**

I. INTRODUCTION

Reliability of power electronic drive systems is important in many industrial applications. The analysis of fault mode behaviour can be utilised for development of monitoring and diagnostic systems. In this paper we present some results of simulation investigations of a converter-fed induction motor drive. PWM converters supplying asynchronous motor were simulated using the EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program)[5].

Three-phase systems can be described with complex phasors [3]. On the plane of complex numbers we obtain a rotating phasor. Unsymmetrical conditions of the machine operation are reflected in the spectrum of the voltage or current space-phasor as positive or negative sequence systems.

The spectrum of the space-phasor was estimated with the help of the Wigner-Ville distribution (WVD). The Wigner spectrum of the signals in the supply installation of compensated AC arc furnace was also estimated.

A huge amount of work was already done in analysis of stationary signals [4], but the real signals in vast majority are non-stationary. This has lead to the development of tools that are designed specifically to deal with non-stationary situations. Time-frequency methods explicitly consider the time dependence of the frequency contents of the signal. The WVD is especially appropriate for the analysis of non-stationary multicomponent signals due to its good temporal resolution, excellent performance in the presence of noise, better frequency concentration and less phase dependence than Fourier spectra [8],[9].

II. WIGNER-VILLE REPRESENTATION

The Wigner-Ville distribution (WVD) is a time-frequency representation given by [2]:

Authors are with the Institute of Electrical Engineering Fundamentals, Wroclaw University of Technology, pl. Grunwaldzki 13, 50-370 Wroclaw, Poland, e-mail: *lobos@ipee.pwr.wroc.pl*

$$W_x(t,\omega) = \int_{-\infty}^{\infty} x\left(t+\tfrac{\tau}{2}\right) x^*\left(t-\tfrac{\tau}{2}\right) e^{(-j\omega\tau)} \mathrm{d}\tau \qquad (1)$$

where t is a time variable, ω is a frequency variable and * denotes complex conjugate. The Wigner distribution is a two-dimensional function describing the frequency content of a signal as a function of time [6] and possesses many advantageous properties, among them:

1. Instantaneous frequency property which says that, at the time instant t, the mean instantaneous frequency of the $W_x(t,\omega)$ is equal to the mean instantaneous frequency of the signal.
2. Because the Wigner-Ville distribution satisfies both time and frequency marginal conditions, it can be shown, that energy contained in the $W_x(t,\omega)$ is equal to the energy possessed by the original signal.

For a discrete-time signal $x(n)$ the discrete pseudo-Wigner-Ville distribution (PWD) is evaluated using a sliding symmetrical finite-length analysis window $h(\tau)$.

$$W_{xh}(n,k) = 2\sum_{\tau=-L}^{L} x(n+\tau)x^*(n-\tau)\times h(\tau)h^*(-\tau)e^{-j4\pi k\tau/N} \qquad (2)$$

where $h(\tau)$ is a windowing function that satisfies the condition: $h(\tau)=0;\ |\tau|>L$. Variables n and k correspond respectively to the discrete time and frequency variables.

The short-time Fourier transform (STFT) was the first tool for analysing the signal in joint time-frequency domain. The crucial drawback inherent to the STFT is a trade-off between time and frequency resolution. WVD does not suffer from interaction between time and frequency resolutions, but presents some other undesired properties. One main deficiency of the WVD is the cross-term interference. WVD of the sum of multicomponents is a linear combination of auto- and cross-terms. Each pair of the signal components creates one additional cross-term in the spectrum, thus the desired time-frequency representation may be confusing. Traditionally, the cross-terms are considered as something undesired in the WVD and should be removed [7]. It is not entirely correct. Some types of cross-terms (e.g. caused by two Gaussian functions whose time and frequency centres are far apart when the cross-term is highly oscillating) have near-zero average and limited influence on the time-marginal conditions as well as other useful properties.

In other situations when a cross-term is discarded, the resulting representation will leave significant energy out.

One way of lowering cross-term interference is to apply a low-pass filter to the WVD. The smoothing, however, will reduce the frequency resolution of the WVD and cause the loss of many useful properties of the transformation.

Another way of lowering cross-term interference is to use the WVD of analytical signal. Analytical function of the signal is a half hand function and therefore the resulting WVDa (WVD of analytical signal) avoids all cross-terms associated with negative frequency components. The analytical function, however, differs from the original signal in different ways; e.g. its instantaneous properties may substantially differ from that of the original signal [6]. Also the WVDa reduces the cross-term interference at the cost of loosing some useful properties.

III. INVESTIGATIONS

A. Single line to ground fault

The investigation results when estimating the parameters of the voltage positive sequence fundamental component during single phase to earth fault are shown in [3]. The results have been obtained using the Fourier and Kalman algorithms with a sliding sampling window. The same case was investigated using the Wigner-Ville Distribution. The simulated voltage waveform of the faulted phase is shown in Figure 1. The waveform is composed of the exponentially damped oscillating component with the frequency 260 Hz, assuming that the fundamental component is equal to zero. Fault appears after the 20th sample (sampling frequency 2,5 kHz).

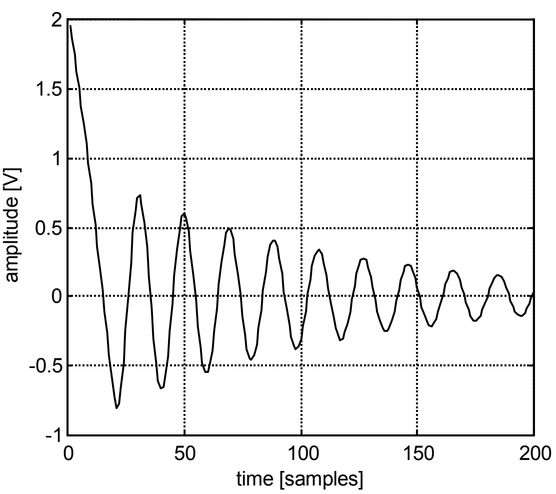

Fig. 1. Voltage waveform in faulted phase during single line to ground short circuit.

Wigner-Ville representation offers the possibility to track the frequency and amplitude changes of non-stationary signals. In this case, 200 samples were taken into calculation. In Figure 2 the estimated frequency of the investigated signal is shown. The characteristic

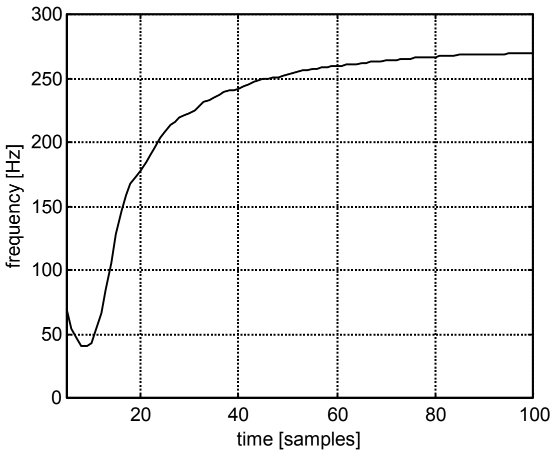

Fig. 2. Estimated instantaneous frequency of the signal in Figure 1.

shows good correlation with the true changes of investigated signal.

Many experiments showed that unmodified WVD is most suitable for one-component signals. In other cases, due to the cross-term interference, it is difficult to interpret the obtained results.

B. Industrial frequency converter

In the recent years, simulation programs for complex electrical circuits and control systems have been improved essentially. The EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program) as a FORTRAN based and to MS-DOS/WINDOWS adapted program serves for modelling complex 1- or 3-phase networks occurring in drive, control and energy systems.

In the paper investigation results of a 3 kVA-PWM-converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U=380V, I=2.8 A) are shown. To design the intermediate circuit, the L, C values of a typical 3-kVA converter were chosen [1]. Figure 3 shows current waveform at the converter output (phase R) for the frequency 40 Hz during a short circuit between two phases with fault resistance $100\,\Omega$. The motor is provided with a positive-sequence 3-phase voltage system f_R, f_S, f_T

Complex space-phasor $\underline{f_p} = f_\alpha + j \cdot f_\beta$ of a three-phase system f_R, f_S, f_T is given by [2]

$$\begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} f_R \\ f_S \\ f_T \end{bmatrix} \tag{3}$$

It describes, in addition to the positive-sequence component, an existing negative-sequence component, harmonic and non-harmonic frequency components of the signal. The complex space-phasor of the converter output voltages is investigated using WVD.

Figures 4 and 5 show the estimated frequency representation of the space-phasor. In this case the investigated signal was sampled with the frequency of 5 kHz and 200 samples were taken into calculation. Before the fault the fundamental component frequency

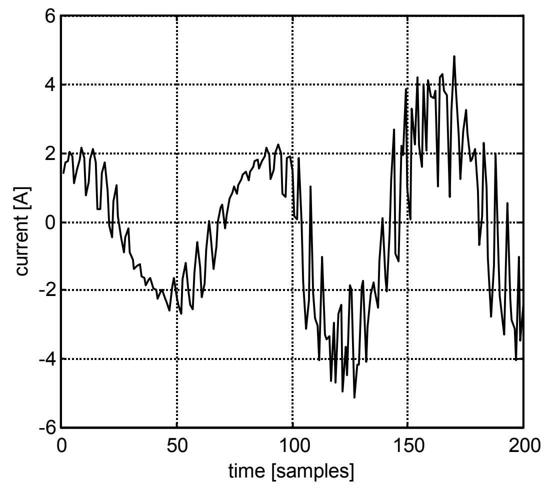

Fig. 3. Current waveform (phase R) at the motor input during a short circuit. Fault occurs after the 100th sample.

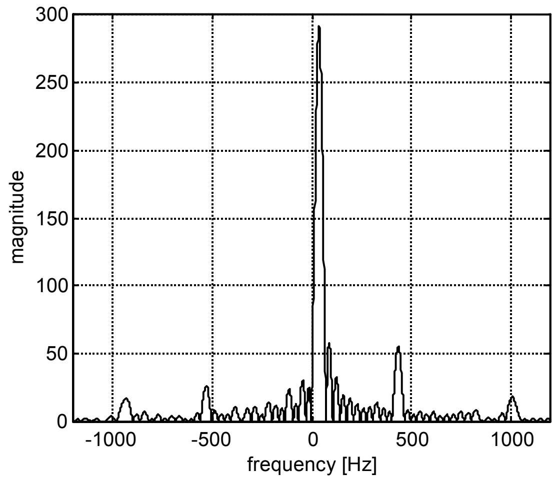

Fig. 4. Estimated WVD of the space-phasor (signal in Fig. 3), at the time point t=0.015 s. (before the fault).

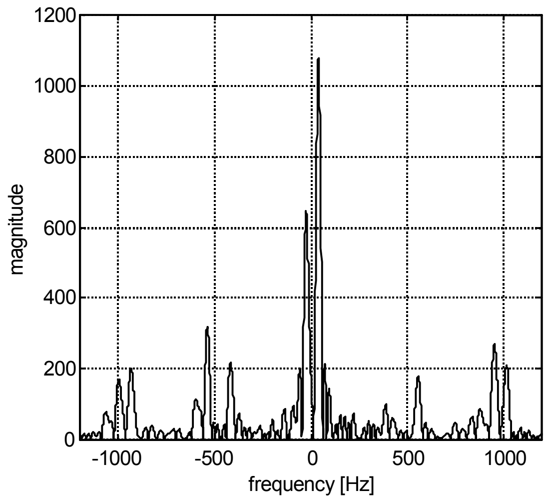

Fig. 5. Estimated WVD of the space-phasor (signal in figure 3), at the time point t=0.06 s. (during the fault).

40 Hz and the 10th harmonic are present, both positive-sequence components (situated in the right half plane). Under fault conditions the negative-sequence component (situated in the left half plane) of the fundamental frequency appears and also irregular frequencies 440 and 560 Hz (mainly negative-sequence components) have also been detected. Detection of the negative-sequence components (with negative frequencies) can be applied as a fault indicator.

In figure 6 for comparison, the power spectrum of the signal is shown. Despite the dramatic change in the signal shape after the fault its power spectrum remains nearly identical before and after the fault. It proves the superiority of the proposed approach over the conventional FFT-based tool.

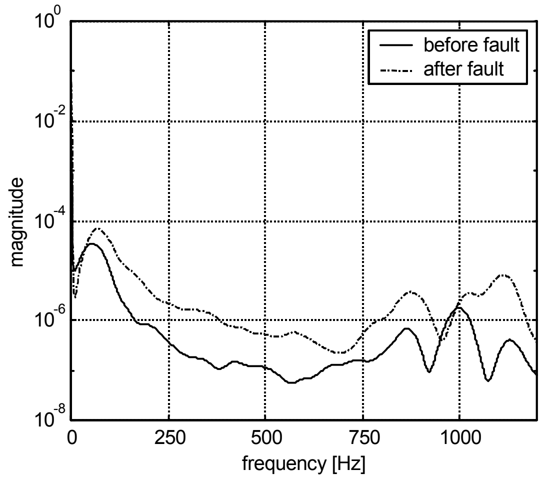

Fig. 6. Power spectrum of the signal in Figure 3.

Another set of investigations concerned the motor lead to ground fault. The Wigner-Ville representation of the space phasor computed from the inverter output currents is presented in the Figures 7 and 8. In the case of lower short circuit current (Figure 7), the negative-sequence 50 Hz component is visible as a result of the asymmetry of the supply. In the second case (Figure 8) with lower short-circuit resistance, additional sub-harmonic components can be detected.

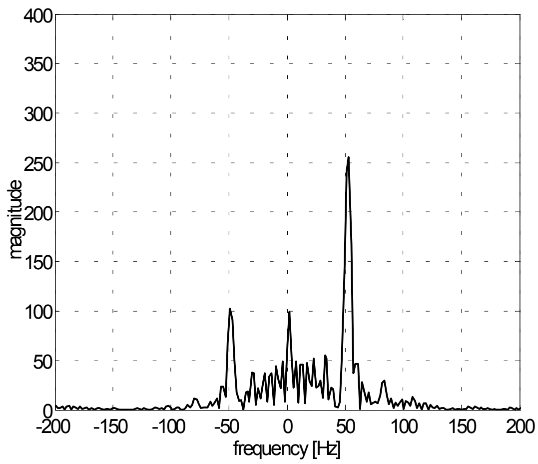

Fig. 7. Estimated WVD of the space-phasor of the fault signal during the motor lead to ground short-circuit in the converter drive (high short-circuit resistance).

C. Arc furnace installation

To investigate the efficiency of the proposed approaches several experiments were also performed with the signals recorded at Riesa (Germany) in the supply installation of an industrial arc furnace (66 MVA) (Figures 9, 10 and 11).

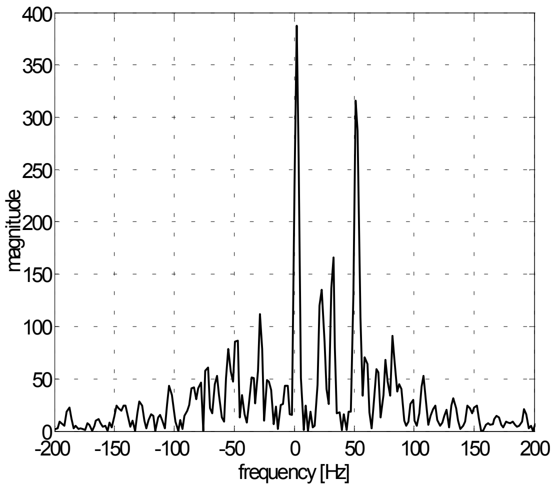

Fig. 8. Estimated WVD of the space-phasor of the fault signal during the motor lead to ground short-circuit in the converter drive (low short-circuit resistance).

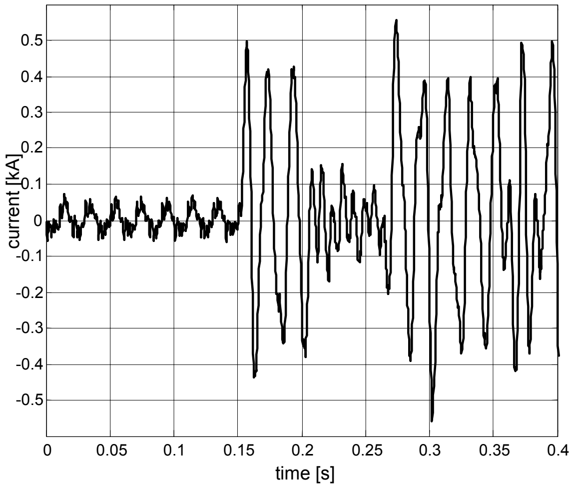

Fig. 9. Signal recorded at the output of an AC arc furnace power supply installation

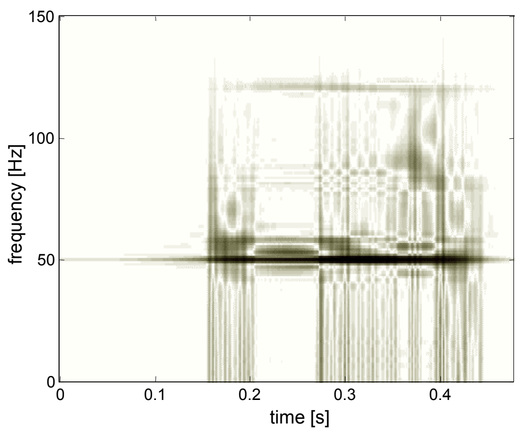

Fig. 10 Gaussian smoothed Wigner-Ville representation of the signal in Fig. 9

7. CONCLUSIONS

Modern semiconductor converters and arc furnaces can generate not only characteristic harmonics but also

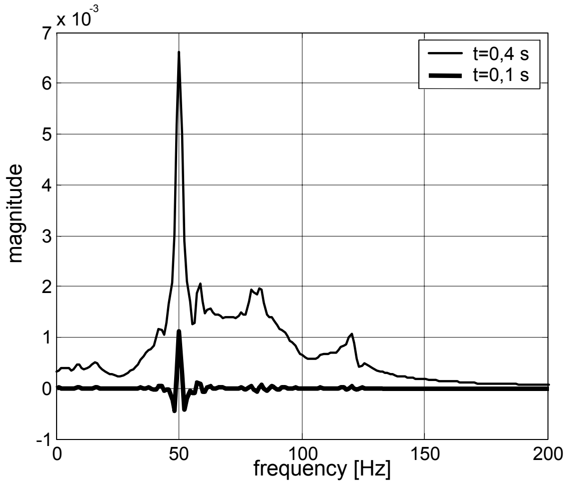

Fig. 11. Cross-sections of the Wigner-Ville representation of the signal in Fig. 7 for time slices t=0,1 and 0,4 s

non-characteristic harmonics and interharmonics. Parameter estimation of the signal components is very important for control and protection tasks. The design of filters and compensation apparatuses relies on the frequency characteristics of distorted currents and voltages. The usually used FFT has some important limitations.

Visualisation of frequency converter supplied drives by means of a static space-phasor is a very useful and compact observation and diagnosis method. Spectrums of both the space-phasor and the real-valued signal have been investigated under different operation conditions using the Wigner-Ville distribution. Superiority of the proposed approach over the conventional FFT-based tool was shown. Detection of irregular frequencies may be useful for diagnosis of some drive faults. WVD offers also the possibility to track the frequency and amplitude changes of non-stationary signals.

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