HIGH RESOLUTION SPECTRUM ESTIMATION METHODS FOR SIGNAL ANALYSIS IN POWER ELECTRONICS AND SYSTEMS

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ABSTRACT

Modern frequency power converters generate a wide spectrum of harmonic components. Large converter systems can also generate non-characteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed at 20 ms (50 Hz), while periodicity intervals in the presence of interharmonics are variable and very long. A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. The singular value decomposition (SVD) method as applied for signal analysis was also tested for this purpose. Both new high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonic frequency. To investigate the methods several experiments were performed using simulated signals and the waveforms of a frequency converter current. For comparison, similar experiments were repeated using the FFT with the same number of samples and sampling period. The comparison proved the superiority of the new methods. However, its computation is much more complex than FFT, and requires more extensive mathematical manipulations.

1. INTRODUCTION

Modern frequency power converters generate a wide spectrum of harmonic components, which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converter systems generate not only characteristic harmonics typical for the ideal converter operation, but also a considerable amount of non-characteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage [2][7]. Interharmonics are defined as non-integer harmonics of the main fundamental under consideration.

The estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of harmonic distortion in both current and voltage waveforms. There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, etc. [3]. However, most of them can operate adequately only in the narrow range of frequencies and at moderate noise levels.

The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods". MUSIC (Multiple Signal Classification) harmonic retrieval method is an example of eigenstructure-based methods, which yield high resolution and asymptotically exact results. Its resolution is theoretically independent of the SNR (Signal-Noise Ratio) [8][12].

The MUSIC method, as all of the analyses based on second-order statistics, can also be based on fourth-order statistics. There are several motivations to use higher-order spectra in signal processing. In practical applications, a non-Gaussian signal is received along with additive coloured Gaussian noise of unknown power spectrum. For Gaussian signals all higher-order spectra of an order greater than two are identical to zero. A transformation to a higher-order cumulant domain eliminates the noise. In this paper the diagonal slice of the fourth-order cumulant is used to estimate the signal spectrum with the help of the MUSIC method.

The singular value decomposition (SVD) approach is also a very good tool for frequency estimation [1][6][9]. The SVD technique is a highly reliable, computationally stable mathematical tool to solve the rectangular overdetermined system of equation. The method can also be applied for frequency estimation of the fundamental component of very distorted periodical signals.

The known non-linear least square methods [4][10] are developed and investigated for periodic signals, without interharmonics, and seem to be efficient for the narrow range of harmonics frequencies. In some cases, the proposed method can be used for calculating starting values for the maximum likelihood solution.

To investigate the ability of the methods several experiments were performed. We investigated simulated waveforms as well as current waveforms at the output of a simulated three-phase frequency converter supplying an induction motor. For comparison, similar experiments were repeated using the FFT with the same number of samples and the same sampling period.

2. CUMULANT-BASED APPROACH

In the case of real harmonic signals the fourth-order cumulant is given by [8]

$$C_{4,x} = -\tfrac{1}{8}\sum_{k=1}^{p}\alpha_k^4\left[\cos\omega_k(\tau_1-\tau_2-\tau_3)+ \right. \\ \left. +\cos\omega_k(\tau_2-\tau_3-\tau_1)+\cos\omega_k(\tau_2-\tau_3-\tau_1)\right] \qquad (1)$$

If the investigated signal is a sum of p real-valued sinusoids, then the diagonal slice of the fourth-order cumulant retains all of the pertinent information about the number of harmonics, their amplitudes and frequencies. Setting $\tau_1 = \tau_2 = \tau_3 = \tau$ in (1) :

$$C_{4,y}(\tau) = -\tfrac{3}{8}\sum_{k=1}^{p}\alpha_k^4\cos(\omega_k\tau) \qquad (2)$$

which is nearly identical with the autocorrelation of the signal.
It is known that signal parameters can be estimated from the output correlation in the white noise case; they can also be estimated from the fourth-order cumulant, which is useful in the additive coloured noise case. It means that existing high-resolution methods, such as MUSIC, can be applied by replacing correlation quantities with the fourth-order cumulant [5][11].

2.1 MUSIC (MUltiple SIgnal Classification)

The MUSIC method [12] involves calculation of the correlation matrix of the signal. Smallest eigenvalues of the matrix correspond to the noise subspace and largest (all greater than the noise variance) correspond to the signal subspace. The matrix of eigenvectors is defined by:

$$\mathbf{E}_{noise} = \left[\mathbf{e}_{M+1} \quad \mathbf{e}_{M+2} \quad \cdots \quad \mathbf{e}_N\right] \qquad (3)$$

and can be used to form a projection matrix for the noise subspace

$$\mathbf{P}_{noise} = \mathbf{E}_{noise}\mathbf{E}_{noise}^{*T} \qquad (4)$$

The MUSIC pseudospectrum is defined as:

$$\hat{P}\left(e^{j\omega}\right) = \left[\mathbf{w}^{*T}\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{w}\right]^{-1} \qquad (5)$$

where $\mathbf{w} = \left[1 \quad e^{j\omega_i} \quad \cdots \quad e^{j(N-1)\omega_i}\right]$ is an auxiliary vector. Since each of the elements of the signal vector is orthogonal to the noise subspace, the quantity (5) exhibits sharp peaks at the signal component frequencies.

3. PRINCIPLES OF THE SVD APPROACH

Let us assume the waveform of the voltage or current as the sum of harmonics of unknown magnitudes and phases

$$x(t) = \sum_{k=1}^{N} X_k\cos(\omega_k t + \varphi_k) + k_s e(t) \qquad (6)$$

in which X_k, ω_k and φ_k are the unknown amplitude, angular frequency and phase of the *k-th* harmonic and N is the number of these harmonics. The variable $e(t)$ represents the additive Gaussian noise with unity variance and k_s - the gain factor. Further, let us consider the set of n measured samples $x_1, x_2, \ldots x_n$ of the waveform. The number of measurements is usually higher than the number of harmonics. Estimation of harmonics is then equivalent to solving the overdetermined system of algebraic equations [4].

$$\mathbf{Ah} = \mathbf{b} \qquad (7)$$

where the matrix **A** and vectors **h** and **b** are given as follows

$$\mathbf{A} = \begin{bmatrix} x_l & x_{l-1} & \cdots & x_1 \\ x_{l+1} & x_l & \cdots & x_2 \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-l} \\ x_2 & x_3 & \cdots & x_{1+1} \\ x_3 & x_4 & \cdots & x_{1+2} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-l+1} & x_{n-l+2} & \cdots & x_n \end{bmatrix} \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ h_l \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} x_{l+1} \\ x_{l+2} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ x_{n-l} \end{bmatrix}$$

l is the order of predicted **AR** model of the data ($N \le l \le n\text{-}N/2$). The vector **h** is composed of the coefficients of the impulse response of this model

$$H(z) = 1 - \sum_{i=1}^{l} h_i\ z^{-i} \qquad (8)$$

The solution for vector **h** of (7) is possible in least square (LS) sense that is by minimising the summed squared error between the left and right hand sides of the equation. The objective function to be minimised may be expressed in the norm - 2 vector notation form as

$$E = \frac{1}{2}\left\|\mathbf{Ah} - \mathbf{b}\right\|_2^2 \qquad (9)$$

To solve (9) the most suitable method seems to be the application of singular value decomposition. In this approach we represent the rectangular matrix **A** as the product of three matrices

$$\mathbf{A} = \mathbf{U}\ \mathbf{S}\ \mathbf{V}^{\mathbf{t}} \qquad (10)$$

where **U** and **V** are orthogonal matrices of the dimension $n\times n$ and $l\times l$ respectively, while **S** is the quasidiagonal $n\times l$ matrix of singular values $s_1, s_2, \ldots, s_p$ ordered in a descending way, i.e. $s_1 \ge s_2 \ge \ldots \ge s_p \ge 0$. The essential information of the system is contained in the first nonzero singular values and first p singular vectors, forming the orthogonal matrices **U** and **V**. Cutting the appropriate matrices to this size and denoting them by $\mathbf{U_r}$, $\mathbf{S_r}$, and $\mathbf{V_r}$, respectively, we get the solution of the (7) in the form

$$\mathbf{h} = \mathbf{V_r}\ \mathbf{S_r^{-1}}\ \mathbf{U_r^{t}}\ \mathbf{b} \qquad (11)$$

where

$$\mathbf{S^{-1}} = \mathrm{diag}\left[\frac{1}{s_1}, \frac{1}{s_2}, \ldots, \frac{1}{s_p}\right]. \qquad (12)$$

Based on the determined coefficients **h** the zeros of polynomial (8) can be found. The phases of the roots, closest to the unit circle denote the angular frequencies of the sinusoids forming the waveform (6). These frequencies can also be determined based on the frequency characteristics of the model (8). They correspond to the frequencies in the range $-0.5 \le f \le 0.5$ for which the magnitude response $|H(e^{j2\pi f})|$ is equal or closest to zero.

4. EXPERIMENTS

To investigate the efficiency of the proposed approaches several experiments were performed with the signal waveform described in [7]. The investigated signal is characteristic for DC arc furnace installations without compensation. It consists of basic harmonic (50 Hz), one higher harmonic (125 Hz), one interharmonic (25 Hz) and is additionally distorted by 5% random noise. The sampling interval was 0.5 ms.

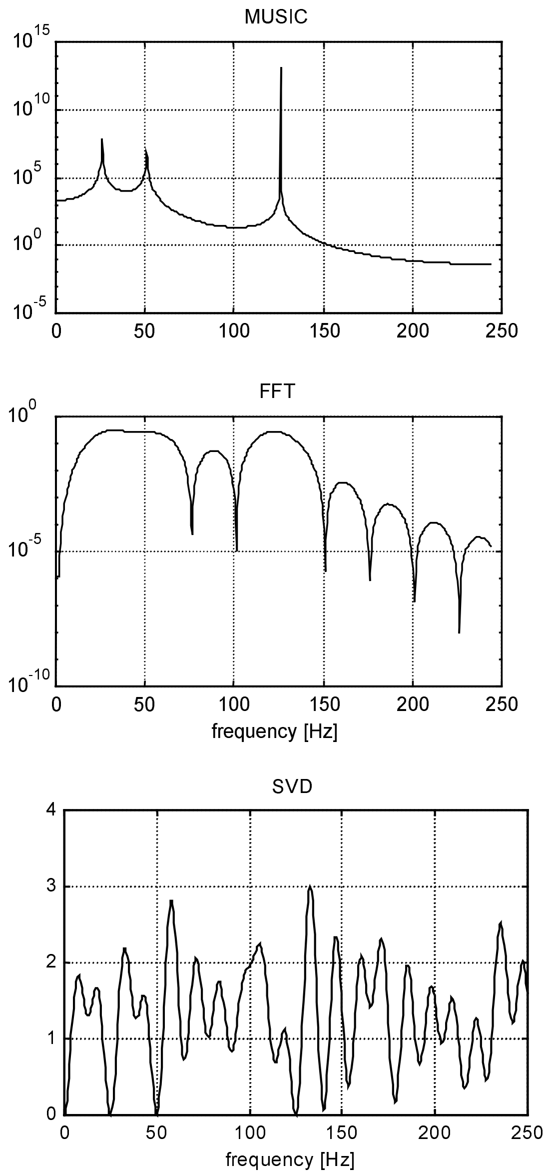

Figure 1. Magnitude characteristics of the signal calculated by the MUSIC, FFT algorithms (n=80) and SVD (l=80, n=90).

The signal was investigated using the MUSIC and SVD methods. The MUSIC method enables us to detect all the signal components already using 40 samples. The more samples taking into the calculation the more exact are the results. Figure 1 shows the results for n=80. For comparison similar experiments were repeated using Fourier algorithm with the same number of samples and the same sampling interval. For detection the 25 Hz component using the Fourier algorithm about 500 samples were needed. When applying the SVD method, taking into account only the dominant singular values, we can approximately assess the number of components existing in the measured waveform. For detection all the components at least n=90 samples were needed. Figure 1 presents the obtained magnitude frequency characteristic of the system. The zeros of the characteristic or the points closest to zero determine the exact values of the frequencies. The superiority of the MUSIC and SVD methods over Fourier algorithms is visible.

5. SIMULATION OF A FREQUENCY CONVERTER

In recent years, simulation programs for complex electrical circuits and control systems have been essentially improved. The simulation of characteristic transient phenomena concerning the electrical quantities becomes feasible without any arrangement of hardware. Among many available simulation programs, the EMTP – ATP as a Fortran based program adapted to DOS/Windows serves for modelling complex 1- and 3-phase networks occurring in drive, control and energy systems.

In the paper we show investigation results of a 3 kVA – PWM – converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U=380V, I=2.8A). The simulated converter can change the output frequency within a range 0.1 ÷ 150 Hz.

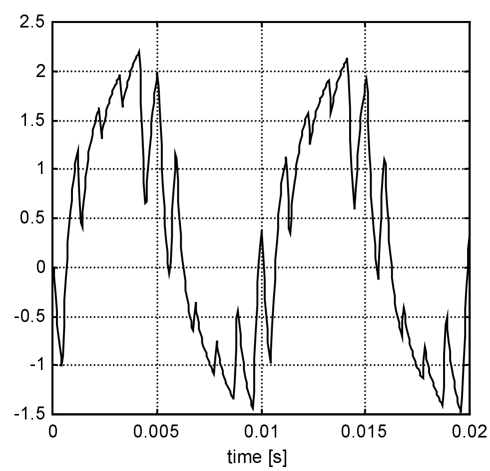

Figure 2. Current waveform at the output of a simulated frequency converter, f = 100 Hz.

Figure 2 shows the current waveform at the converter output for the frequency 100 Hz. The sampling interval was 0,05 ms. The MUSIC method enables us to estimate the frequencies of all the signal components already using 100 samples. The number of samples corresponds to a half period of the main component.. When applying the SVD method, for detection all the components n=260 samples were needed. Using the MUSIC and SVD methods with l=254 and n=260 we can detect the following frequencies: 100, 800, 1000 and 1200 Hz.

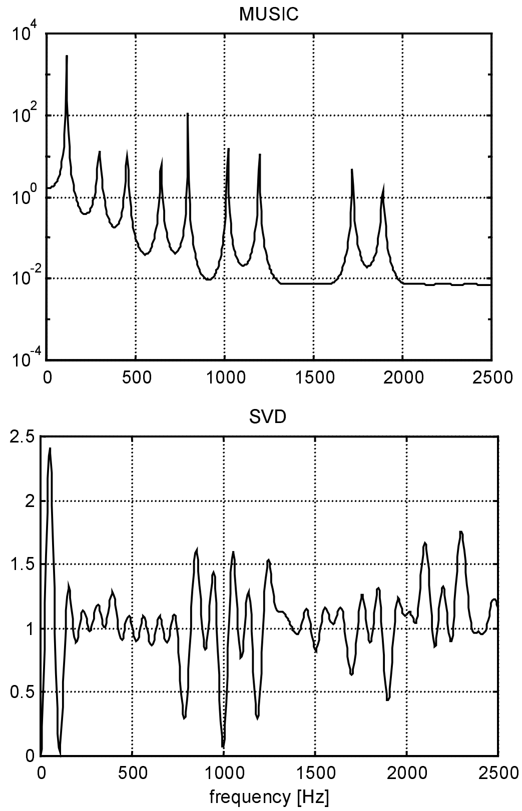

Figure 3. Magnitude characteristics of the signal in Figure 2 calculated by the MUSIC (n=100) and SVD (l= 254, n= 260) method.

5. CONCLUSIONS

It has been shown that a high-resolution spectrum estimation method, such as MUSIC combined with the use of higher–order statistics could be effectively used for parameter estimation of distorted signals. The frequency estimation makes it possible to calculate the stationary space-phasor and to determine an appropriate window for a DFT-analysis The accuracy of the estimation method depends on the signal distortion, the sampling window and on the number of samples taken into the estimation process.

The linear least square method for harmonics detection in a power system has been investigated in this paper using the singular value decomposition (SVD).

The proposed methods were investigated under different conditions and found to be very variable and efficient tools for detection and location of all higher harmonics existing in the system. They also make it possible the estimation of interharmonics. The comparison to the standard FFT technique has proved the superiority of the approaches for signals buried in noise.

6. REFERENCES

[1] Bakamidis S., Dendrinos M. and Carayannis G., SVD analysis by synthesis of harmonic signals, *IEEE Trans. Signal Processing*, vol. 39, no. 2, February 1991, pages 472 - 477.

[2] Carbone R., Menniti D., Sorrentino N. and Testa A, Iterative Harmonic and Interharmonic Analysis In Multiconverter Industrial Systems, *8th International Conference on Harmonics and Quality of Power*, Athens (Greece), 1998, pages 432-438.

[3] Cichocki A. and Łobos T., Artificial neural networks for real-time estimation of basic waveforms of voltages and currents, *IEEE Trans. Power Systems*, vol. 9, no. 2, May 1994, pages 612 - 618.

[4] James B., Anderson B.D.O. and Willamson R.C., Conditional Mean and Maximum Likelihood Approaches to Multiharmonic Frequency Estimation, *IEEE Trans. On Signal Processing*, vol. 42, no. 6, June 1994, pages1366-1375.

[5] Leonowicz Z., Łobos T., Ruczewski P., Szymanda J. Application of Higher-Order Spectra for Signal Processing in Electrical Power Engineering, International Journal for Computation and Mathematics in Electronic Engineering COMPEL, 1998, Vol. 17, No. 5/6, pages 602-611

[6] Łobos T., Kozina T. and Koglin H.-J., Power System Harmonics Estimation Using Linear Least Squares Method and SVD, *16th IEEE Instrumentation and Measurement Technology Conf.*, Venice, Italy 1999, vol.2, pages789-794.

[7] Mattavelli P., Fellin L., Bordignon P. and Perna M. Analysis of Interharmonics in DC Arc Furnace Installations, *8th Int. Conf. on Harmonics and Quality of Power*, Athens, Greece 1998, vol.2, pages1092-1099.

[8] Mendel J.M. Tutorial on Higher-Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical Results and Some Applications, Proc. of the IEEE, 1991, Vol. 79, pages 278-305.

[9] Osowski S., SVD technique for estimation of harmonic components in a power system; a statistical approach, *IEEE Proc.-Gener. Transm. Distrib.*, vol. 141, no. 5, September 1994, pages 473 - 479.

[10] Pintelon R. and Schoukens J., An Improved Sine – Wave Fitting Procedure for Characterising Data Acquisition Channels, *IEEE Trans. on Instr. and Measurement*, vol. 45, no. 2, April 1996, pages 588-593.

[11] Swami A. Cumulant-Based Approach to the Harmonic Retrieval and Related Problems, IEEE Trans. on Signal Processing, 1991, Vol. 39, pages 1099-1108.

[12] Therrien C.W. Discrete Random Signals and Statistical Signal Processing, Prentice-Hall, Englewood Cliffs, New Jersey, 1992, pages 614-655.

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