Advanced Spectrum Estimation Methods for Signal Analysis in Power Electronics

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***Abstract* — Modern frequency power converters generate a wide spectrum of harmonics components. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed, while periodicity intervals in the presence of interharmonics are variable and very long. A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. The Prony method as applied for signal analysis was also tested for this purpose. Both the high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonics frequencies.**

***Index terms*: — Discrete Fourier Transform, frequency conversion, frequency measurement, harmonic analysis, power system, Prony method, Min-Norm.**

I. INTRODUCTION

Modern frequency power converters generate a wide spectrum of harmonic components, which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate also considerable amount of non- characteristic harmonics and interharmonics, which may strongly deteriorate the quality of the power supply voltage [1]. The estimation of the components parameters is very important for control and protection tasks.

Spectrum estimation of discretely sampled processes is usually based on procedures employing the fast Fourier transform (FFT). This approach is computationally efficient and produces reasonable results for a large class of signals. In spite of the advantages there are several performance limitations of the FFT approach. The most prominent limitation is that of frequency resolution, i.e. the ability to distinguish the spectral responses of two or more signals. A second limitation is due to windowing of the data. Windowing manifests itself as "leakage" in the spectral domain. These two limitations are particularly troublesome when analysing short data records. Short data records occur frequently in practice, because many measured processes are brief in duration or have slowly time-varying spectra that may be considered constant only for short record lengths.

To alleviate the limitations of the FFT approach, many new spectral estimation methods have been proposed during the last decades [2,3,10]. Advantages of the new methods depend strongly upon the signal-to-noise ratio (SNR). Even in those cases where improved spectral fidelity can be achieved, the computational effort of those alternative methods may be significantly higher than FFT processing. Conventional FFT spectral estimation is based on a Fourier series model of the data, that is, the process is assumed to be composed of a set of harmonically related sinusoids. Windowing of data makes the implicit assumption that the unobserved data outside the window are zero. A smeared spectral estimate is a consequence.

If more knowledge about a process is available, or if it is possible to make a more reasonable assumption, one can select a model for the process that is a good approximation. It is then usually possible to obtain a better spectral estimate. Spectrum analysis becomes a three step procedure: to select a time series model, to estimate the parameters of the assumed model and finally to calculate the spectral estimate. The modelling approach enables to achieve a higher frequency resolution. There are three basic time series models: autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA). Prony method is related to the autoregressive spectral estimation [9,10].

The subspace frequency estimation methods rely on the property that the noise subspace eigenvectors of a Toeplitz autocorrelation matrix are orthogonal to the eigenvectors spanning the signal space. The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function. The eigenvectors spanning the noise space are the ones whose eigenvalues are the smallest and equal to the noise power. The earliest application of the property is the Pisarenko harmonic decomposition (PHD). The PHD method does not itself provide reliable frequency estimates. However, it has promoted a big interest in application of eigenanalysis to frequency estimation. One of the most important techniques, based on the concepts of subspaces is the min-norm method [11].

In the paper the frequencies of signal components are estimated using the Prony and min-norm methods. To investigate the ability of the methods several experiments were performed. Current waveforms at the outputs of a simulated and an industrial frequency converter were investigated. For comparison, similar experiments were repeated using the FFT.

II. PRONY METHOD

Assuming the N complex data samples $x[1],..., x[N]$ the investigated function can be approximated by M exponential functions:

$$y[n] = \sum_{k=1}^{M} A_k e^{(a_k + jw_k)(n-1)T_p + jy_k} \qquad (1)$$

where

$n = 1,2,..., N$; $\quad T_p$ – sampling period,

A_k – amplitude; α_k – damping factor,
ω_k – angular velocity; ψ_k – initial phase.

The discrete-time function may be concisely expressed in the form

$$y[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{2}$$

where $h_k = A_k e^{j\psi_k}$; $z_k = e^{(\alpha_k + j\omega_k)T_p}$

The estimation problem bases on the minimization of the squared error over the N data values:

$$\delta = \sum_{n=1}^{N} |e[n]|^2 \tag{3}$$

where

$$e[n]=x[n]-y[n]=x[n]-\sum_{k=1}^{M} h_k z_k^{n-1} \tag{4}$$

This turns out to be a difficult non-linear problem. It can be solved using the Prony method that utilizes linear equation solutions.

If as many data samples are used as there are exponential parameters, then an exact exponential fit to the data may be made.

Consider the discrete-time function:

$$x[n]=\sum_{k=1}^{M} h_k z_k^{n-1} \tag{5}$$

The p equations of (5) may be expressed in matrix from as:

$$\begin{bmatrix} z_1^0 & z_2^0 & \cdots & z_M^0 \\ z_1^1 & z_2^1 & \cdots & z_M^1 \\ \vdots & \vdots & & \vdots \\ z_1^{M-1} & z_2^{M-1} & \cdots & z_M^{M-1} \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} x[1] \\ x[2] \\ \vdots \\ x[M] \end{bmatrix} \tag{6}$$

The matrix equation represents a set of linear equations that can be solved for the unknown vector of amplitudes.

Prony proposed to define the polynomial that has the z_k exponents as its roots:

$$F(z)=\prod_{k=1}^{M}(z-z_k)= \\ =(z-z_1)(z-z_2)\ldots(z-z_M) \tag{7}$$

The polynomial may be represented as the sum:

$$F(z)=\sum_{m=0}^{M} a[m]z^{M-m} = \\ =a[0]z^M + a[1]z^{M-1}+\ldots+a[M-1]z+a[M] \tag{8}$$

Shifting the index on (5) from n to n-m and multiplying by the parameter $a[m]$ yield:

$$a[m]x[n-m]=a[m]\sum_{k=1}^{M} h_k z_k^{n-m-1} \tag{9}$$

The (9) can be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m]= \\ =\sum_{k=1}^{M} h_k z_k^{n-M}\left\{\sum_{m=0}^{M} a[m]z_k^{M-m-1}\right\} \tag{10}$$

The right-hand summation in (10) may be recognized as polynomial defined by (8), evaluated at each of its roots z_k yielding the zero result:

$$\sum_{m=0}^{M} a[m]x[n-m]=0 \tag{11}$$

The equation can be solved for the polynomial coefficients. In the second step the roots of the polynomial defined by (8) can be calculated. The damping factors and sinusoidal frequencies may be determined from the roots z_k.

For practical situations, the number of data points N usually exceeds the minimum number needed to fit a model of exponentials, i.e. $N > 2M$, In the over-determined data case, the linear equation (11) must be modified to:

$$\sum_{m=0}^{M} a[m]x[n-m]=e[n] \tag{12}$$

The estimation problem bases on the minimization of the total squared error:

III. MIN-NORM METHOD

The min-norm method involves projection of the signal vector:

$$\boldsymbol{s_i} = \begin{bmatrix} 1 & e^{j\omega_i} & \ldots & e^{j(N-1)\omega_i} \end{bmatrix}^T \tag{13}$$

onto the entire noise subspace.

A random sequence $\boldsymbol{x}$ made up of M independent signals in noise is considered.

$$\boldsymbol{x}=\sum_{i=1}^{M} A_i \boldsymbol{s_i} + \boldsymbol{h}\ ; \quad A_i = |A_i| e^{j\phi_i} \tag{14}$$

If the noise is white, the correlation matrix is

$$\mathbf{R_x}=\sum_{i=1}^{M} E\{A_i A_i^*\}\boldsymbol{s_i}\boldsymbol{s_i^T} + \sigma_0^2\mathbf{I} \tag{15}$$

N-*M* smallest eigenvalues of the correlation matrix (matrix dimension N>M+1) correspond to the noise subspace and M largest (all greater than σ_0^2) corresponds to the signal subspace. The matrix of eigenvectors is:

$$\mathbf{E}_{noise} = [\mathbf{e}_{M+1} \quad \mathbf{e}_{M+2} \quad \ldots \quad \mathbf{e}_N] \tag{16}$$

Min-norm method uses one vector $\mathbf{d}$ for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element equal to one. These conditions are expressed by:

$$\mathbf{d}=\mathbf{E}_{noise}\mathbf{E}_{noise}^{*T}\mathbf{d} \\ \mathbf{d}^{*T}\ell=1 \tag{17}$$

and

$$\mathbf{d}^{*T}\ell = \left(\mathbf{E}_{noise}\mathbf{E}^{*T}_{noise}\mathbf{d}\right)^{*T}\ell = \mathbf{d}^{*T}\mathbf{E}_{noise}\mathbf{E}^{*T}_{noise}\ell = 1 \qquad (18)$$

The lagrangian is formed:

$$\begin{aligned} \mathrm{L} = \mathbf{d}^{*T}\mathbf{d} + m\left(1 - \mathbf{d}^{*T}\mathbf{E}_{noise}\mathbf{E}^{*T}_{noise}\ell\right) + \\ + m^{*}\left(1 - \ell^{T}\mathbf{E}_{noise}\mathbf{E}^{*T}_{noise}\mathbf{d}\right) \end{aligned} \qquad (19)$$

Gradient of (19) has the form

$$\nabla_{\mathbf{d}^*}\mathrm{L} = \mathbf{d} - m\mathbf{E}_{noise}\mathbf{E}^{*T}_{noise}\ell = \mathbf{0} \qquad (20)$$

where m is chosen in such way that the first element of the vector is equal to one.

From (20) results that the first element of the vector **d** is equal to $m\mathbf{c}^{*T}\mathbf{c}$.

Finally, vector **d** is equal to

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T}\mathbf{c}}\mathbf{E}_{noise}\mathbf{c} = \begin{bmatrix} 1 \\ \left(\mathbf{E}^{'}_{noise}\mathbf{c}\right) / \left(\mathbf{c}^{*T}\mathbf{c}\right) \end{bmatrix} \qquad (21)$$

Pseudospectrum defined with the help of **d** is defined as.

$$\hat{P}\left(e^{jw}\right) = \frac{1}{\left|\mathbf{w}^{*T}\mathbf{d}\right|^{2}} = \frac{1}{\mathbf{w}^{*T}\mathbf{d}\mathbf{d}^{*T}\mathbf{w}} \qquad (22)$$

where **w** is defined as in (14).

IV. SIMULATION OF A FREQUENCY CONVERTER

In the paper we show investigation results of a 3 kVA – PWM – converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U = 380 V, I = 2.8 A). The simulated converter can change the output frequency within a range 0.1 ÷ 150 Hz.

Figure 1 shows the current waveform at the converter output for the frequency 100 Hz. The sampling interval was 0.2 ms.

The Prony and min-norm methods enable to estimate the frequencies of all the signal components using 100 samples. The following frequencies have been detected: ca. 100, 800, 1000, 1200 Hz. The estimation accuracy is better than when using the Fourier algorithm.

V. INDUSTRIAL FREQUENCY CONVERTER

The investigated drive represents a typical configuration of industrial drives, consisting of a three-phase asynchronous motor and a power converter composed of a single-phase half-controlled bridge rectifier and a voltage source converter. The waveforms of the converter output current under normal conditions were investigated using the Prony, min-norm and FFT methods for the sampling windows equal to 20 ms (Fig.2) and 40 ms (Fig. 3). The main frequency of the waveform was 40 Hz. When using the window 40 ms the estimation results are a little more accurate than for the window 20 ms. However, already the smaller window makes it possible to detect the main harmonic components. Using the Prony and min-norm methods the following harmonics have been detected: 5^{th}, 7^{th}, 17^{th}, 19^{th}, 25^{th},35^{th} and 41^{st}. It is also possible to estimate the frequency of the fundamental component.

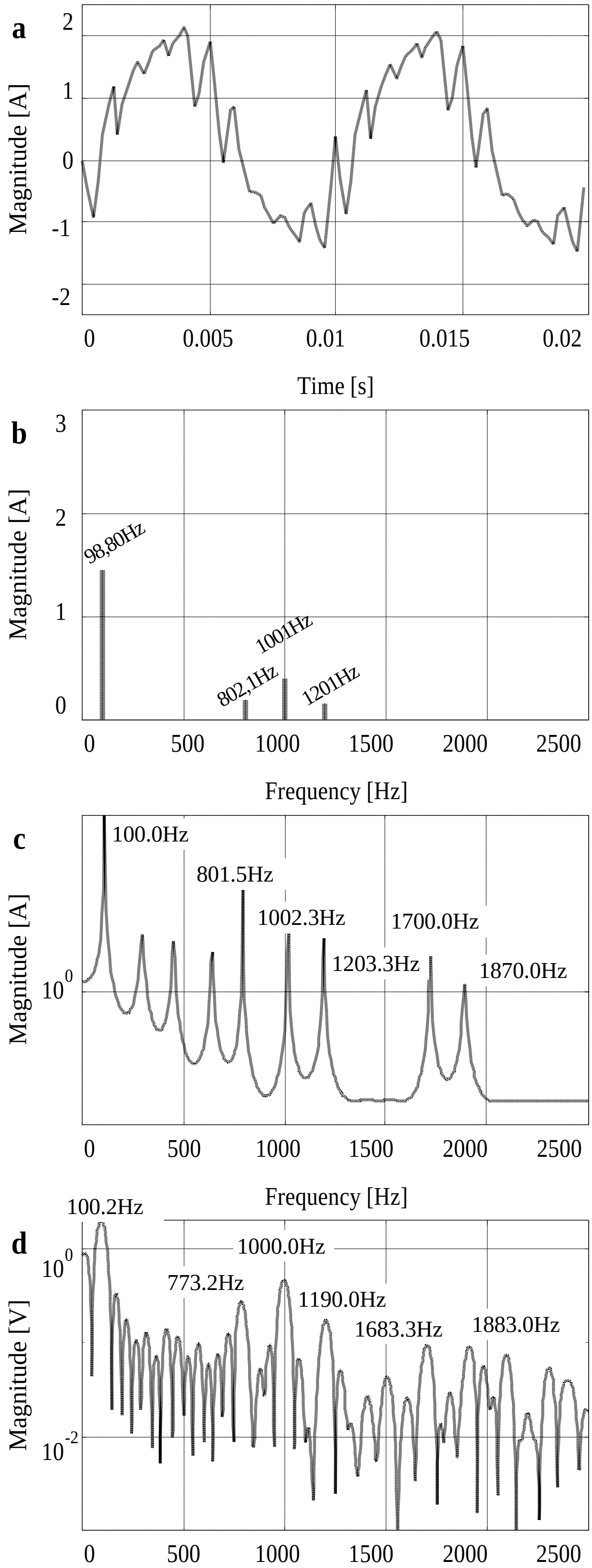

Figure 1: Current waveform at the output of a simulated frequency converter (a); investigation results: Prony N=80,M=40 (b); min-norm N=100 (c); FFT N=80 (d); fp=5000Hz.

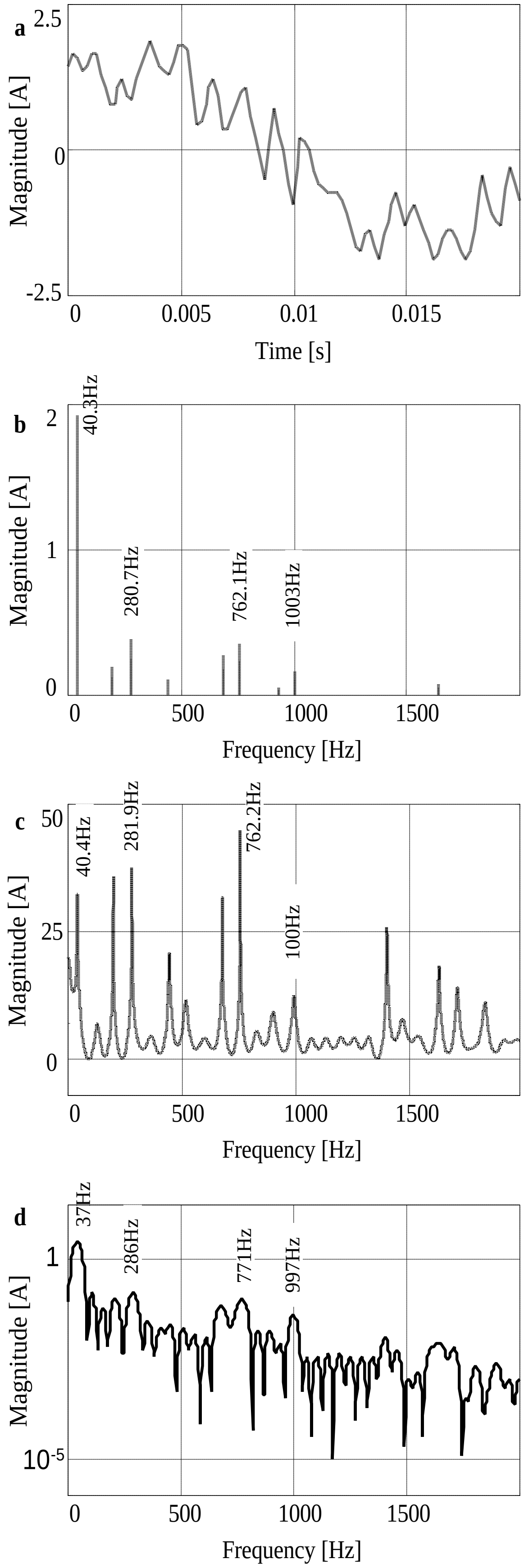

Figure 2: Current waveform at the output of a real frequency converter (a); investigation results: Prony M=40 (b); min-norm (c); FFT (d), N=100, f_p=5000Hz

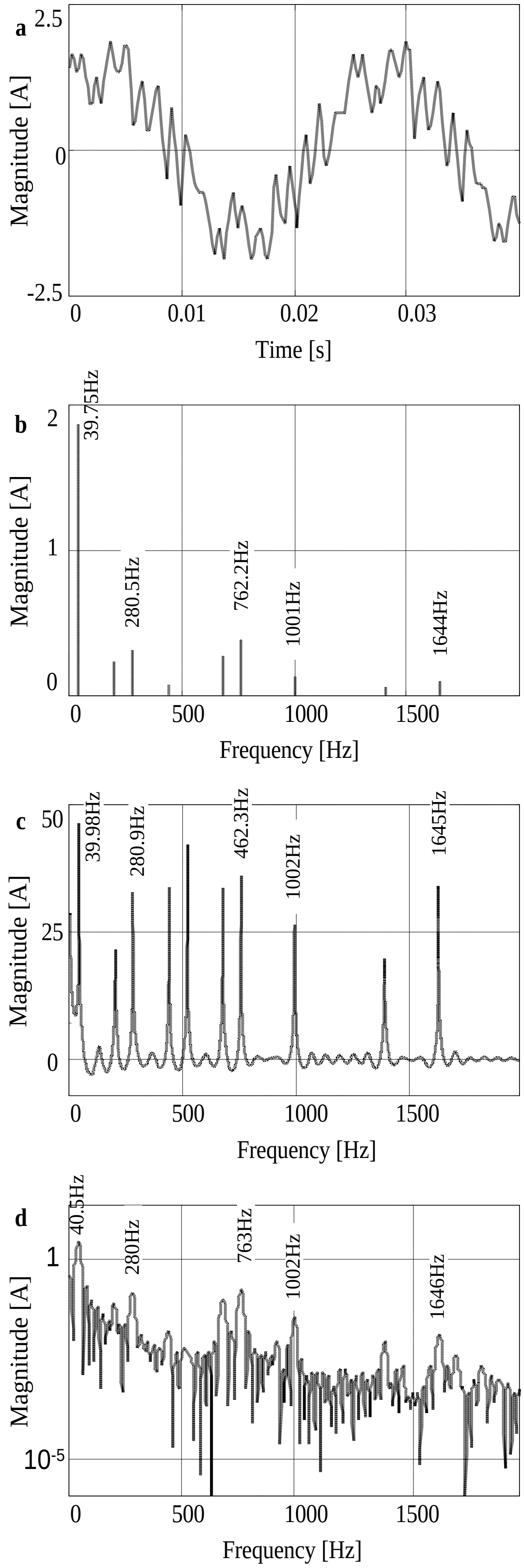

Figure 3: Current waveform at the output of a real frequency converter (a); investigation results: Prony M=80 (b); min-norm (c); FFT (d), N=200, f_p=5000Hz

VI. CONCLUSIONS

It has been shown that a high-resolution spectrum estimation method, such as min-norm could be effectively used for parameter estimation of distorted signals. The Prony method could also be applied for estimation of the frequencies of signal components and offers more exact results than Fourier algorithms. The accuracy of the estimation depends on the signal distortion, the sampling window and on number of samples taken into the estimation process. Unfortunately, the computational effort of the high-resolution methods is significantly higher than FFT processing.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of interharmonics.

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REFERENCES

[1] Carbone R et al, "Iterative harmonics and interharmonic analysis in multiconverter industrial systems", *8th Int. Conference on Harmonics and Quality of Power*, Athens (Greece), 1998, pp. 432-438.

[2] Cichocki A and Lobos T, "Artificial neural networks for real-time estimation of basic waveforms of voltages and currents", *IEEE Trans. Power Systems*, vol. 9, no. 2, September 1994, pp 612-618.

[3] S. M. Kay, *Modern Spectral Estimation: Theory and Application*, Englewood Cliffs: Prentice-Hall, 1988, pp. 224-225.

[4] Leonowicz Z et al, "Application of higher-order spectra for signal processing in electrical power engineering", *Int. Journal for Computation and Mathematics in Electronic Engineering COMPEL*, 1998, vol. 17, no. 5/6, pp. 602-611.

[5] Lobos T et al, "Power system harmonics estimation using linear least squares method and SVD", *16th IEEE Instrumentation and Measurement Technology Conf.*, vol. 2, Venice (Italy), 1999, pp. 789-794.

[6] Lobos T. and Eichhorn K, "Recursive real-time calculation of basic waveforms of signals", *IEE Proceedings Pt.C*, vol. 138, no 6, November 1991, pp. 469-470.

[7] T. Lobos, T. Kozina and S. Osowski, "Detection of remote harmonics using SVD", *8th Int. Conference on Harmonics and Quality of Power*, Athens (Greece), 1998, pp. 1136-1140.

[8] T. Lobos, T. Kozina and H.J. Koglin, "Power system harmonics estimation using linear least squares method and SVD", 16th IEEE Instrumentation and Measurement Technology Conf., Venice (Italy), 1999, vol. 2, pp. 789-794.

[9] Lobos T and Rezmer J, "Real time determination of power system frequency", *IEEE Trans. on Instrumentation and Measurement* vol. 46, no 4, August 1997, pp. 877-881.

[10] M. Meunier and F. Brouage, "Fourier transform, wavelets, Prony analysis: tool for harmonics and quality of power", *8th Int. Conference on Harmonics and Quality of Power*, Athens (Greece), 1998, pp. 71-76.

[11] Therrien C W, *Discrete random signals and statistical signal processing*, Prentice-Hall, Englewood Cliffs, New Jersey, 1992, pp. 614-655.