Improved Recursive Newton Type Algorithm for Frequency And Spectra Estimation in Power Systems

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Abstract – In this paper a new Improved Recursive Newton Type Algorithm suitable for various measurement applications in electric power systems is presented. It is used for the power system frequency and spectra estimation. The recursive algorithm form is improved with a strategy of sequentially tuning of the forgetting factor. By this, the algorithm convergence and accuracy, are significantly improved. To show the main algorithm features the results of computer simulation, laboratory testing (load rejection and unsuccessful synchronisation tests) and field data processing are given.

Keywords: Power systems, nonlinear estimation, recursive algorithms, measurement, frequency, spectrum, computer simulation, laboratory testing.

I. INTRODUCTION

Fast computers with parallel architectures connected through efficient communication networks allow real-time measurement of basic quantities (voltage or current phasors and spectra, local system frequency, time constants etc.), as well as the calculation of the quantities derived from the basic quantities (active and reactive power, total harmonic distortion factor, the rate of change of basic quantities, etc.).

This paper is devoted for solving the problem of digital frequency and spectra measurement. In [1] Discrete Fourier Transform based numerical algorithm for frequency measurement is presented. Its performance can be adversely affected by decaying DC components, or low Signal to Noise Ratio and it requires long measurement windows when frequency deviation is small. In [2] a Recursive Least Error Squares algorithm in which the non-stationary filter gains are off-line calculated and stored in a ring buffer is described. By this, the costly normal matrix inversion is avoided. With the inclusion of the decaying DC component in the signal model, the gains of the recursive filter do not remain cyclic and have to be on-line calculated, requiring the normal matrix inversion. In [3] a two stage algorithm, based on Kalman filtering is used for frequency estimation in underfrequency load-shedding. Here the statistical properties of the input signals processed are the prerequisite for the successful estimation of the unknown model parameters. In [4] Recursive Least Squares and the Least Mean Squares algorithms for dynamic frequency estimation is described. Frequency is estimated by a finite derivative of the phase derivation, followed by a moving-average filter.

In [5] a robust and efficient numerical algorithm derived from the non-recursive Least Error Squares algorithm is presented, whereas in [6] non-recursive Newton Type Algorithm (NTA) is given. Both algorithms require the costly on-line normal matrix inversion.

This paper describes a new Improved Recursive Newton Type (IRNTA) numerical algorithm. Particularly, it is suitable for phasors, harmonics, time constants and for the local system frequency measurement.

Hitherto developed numerical algorithms aimed to frequency estimation in power systems can be divided into two groups: 1) non-recursive and 2) recursive algorithms. As it is known, non-recursive numerical algorithms are computationally more efficient, avoiding directly normal matrix inversion by providing it recursively. The corresponding recursive estimator does not have the same features as the non-recursive one. It is also questionable whether the new, recursive estimator, satisfies power system measuring requirements. These questions regarding IRNTA are answered in this paper.

In order to improve the properties of IRNTA, a new *forgetting factor tuning procedure* is designed and implemented. Instead of selecting forgetting factor to have a constant values (e.g. 0.99), it is heuristically tuned to the system dynamics, providing the algorithm with the faster convergence and the better accuracy.

Through the algorithm application in frequency, phasor and spectrum estimation, computational, accuracy and tracking properties of IRNTA are obtained. After discussing various tuning details, IRNTA is tested using computer simulated, laboratory and field data. Under laboratory conditions load rejection and unsuccessful synchronisation tests are executed. A sudden generator unit disconnection from a real power system is presented in a real-life testing. The results obtained are compared with other known methods.

II. SIGNAL MODEL REPRESENTATION

Let us assume the following observation model of the input signal (arbitrary voltage, or current):

$$s(t)=h(\mathbf{x},t)+\xi(t) \tag{1}$$

where *s(t)* is an instantaneous signal at time *t*, *ξ(t)* a random noise, **x** a suitable parameter vector and *h(.)* is expressed as follows:

$$h(\mathbf{x},t)=S_0 e^{-\delta\, t}+\sum_{k=1}^{M} S_k \sin(k\omega t+\varphi_k) \tag{2}$$

For the generic model (2), a suitable vector of unknown parameters is given by:

$$\mathbf{x}=[S_0,\delta,\omega,S_1,\ldots,S_M,\varphi_1,\ldots,\varphi_M]^T \tag{3}$$

where S_0 is the magnitude of the decaying DC component at $t=0$, $\delta=1/T$, T being the time constant, M is the highest order of the harmonics presented in the signal, ω is the fundamental angular velocity, equal to $2\pi f$, f being frequency, S_k is the magnitude of the k-th harmonic and φ_k is the phase angle of the k-th harmonics. The number of unknowns, i.e. the model order is $n=2M+3$. The model (2) can be simplified, e.g. containing only the fundamental harmonic. The model selection depends on the application, i.e. on the features of the input signal processed.

If the input signal is uniformly sampled with the sampling frequency f_s and the sampling period $T_s=1/f_s$, then the value of t at a discrete time index is given by $t_m=mT_s$ and the following discrete representation of the signal model should be used:

$$s_m=h(\mathbf{x}_m,t_m)+\xi_m \qquad m=1,2,3,\ldots \tag{4}$$

where all unknown parameters from (3) have a subscript m.

III. ALGORITHM DEVELOPMENT

The vector of unknown model parameters (3) can be estimated by applying non-recursive NTA numerical algorithm [6] given by:

$$\hat{\mathbf{x}}_{k+1}=\hat{\mathbf{x}}_k+\left(\mathbf{J}_k^T\mathbf{J}_k\right)^{-1}\mathbf{J}_k^T(\mathbf{s}-\mathbf{h}(\hat{\mathbf{x}}_k)) \tag{6}$$

where k us an iteration index, **J** is an $(m\cdot n)$ Jacobian matrix, **s** is an $(m\cdot 1)$ measurement vector, **h** is an $(m\cdot 1)$ vector of nonlinear functions determined by the model assumed and m is the number of signal samples belonging to the data window. The use of the NTA necessitates the normal-equation matrix $\mathbf{N}=\left(\mathbf{J}_k^T\mathbf{J}_k\right)^{-1}$ inversion at each iteration. This requires much of CPU time, particularly when the model order is high. The real-time computation can be significantly reduced by applying the following recursive form of the NTA algorithm:

$$\hat{\mathbf{x}}_{k+1}=\hat{\mathbf{x}}_k+\mathbf{P}_{k+1}\mathbf{j}_{k+1}(s_{k+1}-h(\hat{\mathbf{x}}_k)) \tag{7}$$

$$\mathbf{P}_{k+1}=\frac{1}{\lambda_{k+1}}\left(\mathbf{P}_k-\frac{\mathbf{P}_k\mathbf{j}_{k+1}\mathbf{j}_{k+1}^T\mathbf{P}_k}{\lambda_{k+1}+\mathbf{j}_{k+1}^T\mathbf{P}_k\mathbf{j}_{k+1}}\right) \tag{8}$$

where $\mathbf{j}_m^T=[j_1,j_2,\ldots,j_n]$ is the m-th row of the Jacobian matrix. For example, the 3[rd] element (i.e. the first derivative of angular velocity) is given by the following equation:

$$j_3=\frac{\partial h(\mathbf{x})}{\partial\omega}=\sum_{k=1}^{M} S_k kt\cos(k\omega t+\varphi_k) \tag{9}$$

Other elements can be in the same way simply derived.

The new recursive algorithm presented requires the appropriate selection of the sampling frequency, the initial guess for $\mathbf{x_0}$ and the forgetting factor λ. Normally, one selects λ near 1.0 (e.g. 0.95). Here, λ is tuned to the dynamics of the input signal processed.

The quality of estimation depends on the pre-selected forgetting factor lambda. The faster convergence and the bigger sensitivity to random noise can be achieved by setting λ less than 1.0 (e.g. $\lambda=0.5$). On the other hand, the slower convergence and less sensitivity to random noise follows for λ near 1.0 (e.g. $\lambda=0.99$). In the IRNTA presented, λ is tuned according to the parameter changes.

In (7) the value of the residual error r at (k+1)-st iteration can be calculated as $r_{k+1}=s_{k+1}-h(\hat{\mathbf{x}}_k)$. Let r_{k+1} and $R_{k+1}=\sum_{p=k+1-L}^{k+1}|r_p|$ be the residual error and the sum of the residual error absolute values, belonging to the moving data window with L old samples, respectively. The following heuristic strategy for iteratively tuning of λ is defined by letting:

$$\lambda_{k+1}=g_{k+1}(\lambda_0,R_0,R_{k+1})=\lambda_0+(1-\lambda_0)e^{-R_{k+1}/R_0} \tag{10}$$

where λ_{k+1} and R_{k+1} are the forgetting factor and the sum of the residual error absolute values at the (k+1)-st iteration, and λ_0 and R_0 are the tuning parameters defining the tuning function $g_{k+1}(\lambda_0,R_0,R_{k+1})$. If the error signal R is small, then λ will be near 1.0, allowing IRNTA to use more the previous information. If, on the other hand, R is large, than λ will be near λ_0, allowing the parameters to be estimated using the most recent data and this will improve the speed of convergence and at the same time the accuracy.

The length of the residual error data window, L, can be selected to be proportional to the fundamental period (half of it, equal, larger). The error signal R could be calculated as a sum of squares of the residual errors *r*, as well. Normally, residual error should be a small number. That means that the sum of squares of *r* should be the small number, too. In order to avoid the calculations with very small numbers, the sum of residual errors absolute values is selected for the calculation of *R*.

The presented strategy of tuning λ is rather general and can be also used in some other designs and applications of recursive algorithms. That means that the ordinary Recursive Least Error Squares algorithm can be extended with the tuning λ, too, and in this more efficient form implemented in some other estimation and identification problems. In particular, the better results are expected at the beginning of the estimation/identification procedures where one usually selects $\mathbf{x_0} = \mathbf{0}$ (the zero vector) and when the expected error is maximal, when λ should be much smaller than 1.0. Generally, the better estimates are expected particularly in the processes in which the step changes of parameters are expected (faults on the elements of power systems, the sudden changes of the network topology and states, etc.).

IV. COMPUTER SIMULATED TESTS

First, a set of static computer simulated tests are performed. The sampling frequency is selected to be $f_s = 600$ Hz and the forgetting factor $\lambda = 0.95$. With the initial guess for $\mathbf{x_0}$ correct selected/calculated, true estimates are obtained in the frequency range of $\pm f_s/2 = \pm 300$ Hz.

Second, through the dynamics tests the importance of tuning λ is investigated. An input sinusoidal test signal with frequency step change at $t = 0.02$ s from 50 to 45 Hz is processed with $\lambda_1 = 0.50$, $\lambda_2 = 0.99$ and $\lambda_3 = g(0.5, 0.06, R)$. For λ_3 the error signal *R* is calculated from $L = 12$ old samples. In Fig. 1 true and estimated frequencies are presented. For λ_1 the fastest convergence and the limited accuracy are obtained. For λ_2 the convergence is too slow. With λ_3 a compromise between the fast convergence and a good accuracy is achieved. In Fig. 2 the changes of λ_3 and *R* are plotted. At the very beginning of signal processing *R* increases and λ_3 takes the minimal values (0.5) increasing the speed of algorithm convergence. Later on, when the true estimates are reached, *R* goes to zero and λ near 1.0.

In the next dynamic test two sinusoidal test signals with the frequencies: $f_1 = 50 - 5t - 5t^2$ and $f_2 = 50 + 5t + 5t^2$ are processed with $\lambda = g(0.5, 0.06, R)$. In Fig. 3 the estimated frequencies are depicted. At $t = 2$ s the rate of frequency changes is 25 Hz/s and the errors are lower than 0.025 Hz. Due to large inertia such values are not encountered in power systems.

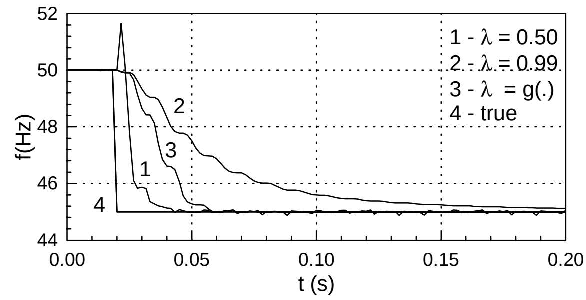

Fig. 1: Frequency estimates for the step changes of the signal frequency.

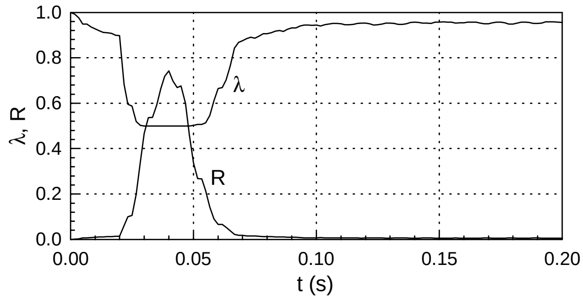

Fig. 2: The changes of λ and *R*.

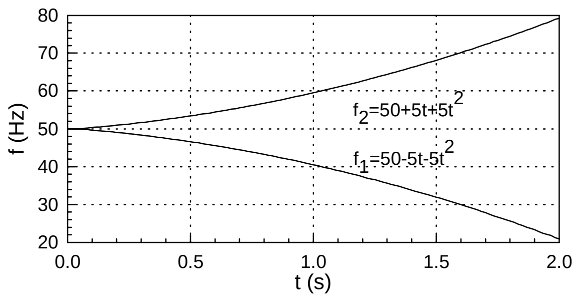

Fig. 3: Dynamic test frequency estimates.

Tests with the random noise corrupted signals are provided, too. A sinusoidal 50 Hz input test signal with the additive white zero-mine Gaussian random noise is processed. The random noise is selected to obtain a prescribed value of the Signal to Noise Ratio, k_{SNR}, defined as $k_{SNR} = 20\log(A/2\sigma)$, where A is the magnitude of the signal fundamental harmonics and σ is the noise standard deviation. For λ = const. the sensitivity to random noise decreases if the values of λ are closer to 1.0 and vice versa. If λ is sequentially tuned, the sensitivity to noise depends on the tuning function *g*(.) selected. As presented in Fig. 4, IRNTA posses lower sensitivity to the random noise than the DFT algorithm [1]. In the testing forgetting factor was selected to be $\lambda = 0.90$.

A series of tests are provided with distorted input signals, too. A signal consisted of the fundamental (1 p.u.), the 3rd (0.3 p.u.), the 5th (0.2 p.u.) and the 7th (0.1 p.u.) harmonics is processed. Until $t = 20$ ms it is a pure sinusoidal 50 Hz signal. At $t = 20$ ms the signal frequency is step changed from 50 to 49 Hz, the fundamental harmonics amplitude is

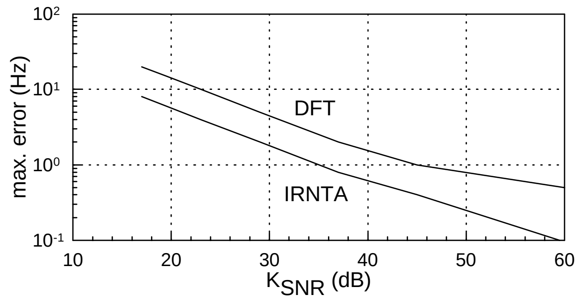

Fig. 4: Sensitivity to the random noise.

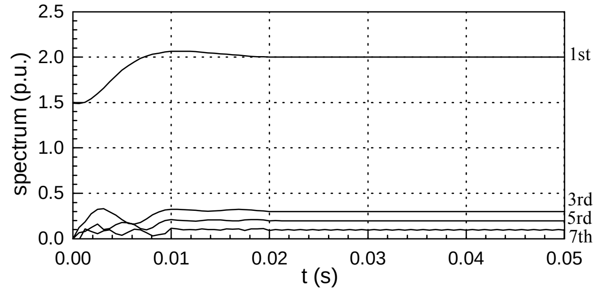

Fig. 5: Estimated amplitudes of harmonics.

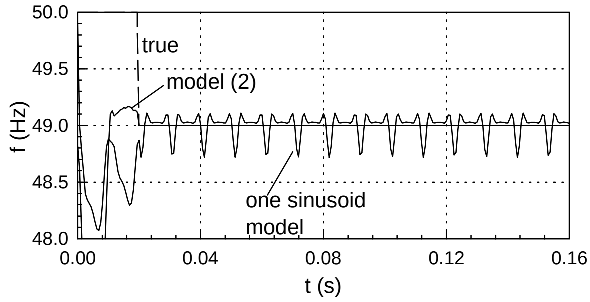

Fig. 6: True and estimated frequencies.

step changed from 1.0 to 2.0 p.u. and the signal distorted with the higher harmonics. Fig. 5 shows the estimated amplitudes of all harmonic components detected. All estimates are exact. Fig. 6 depicts the true frequency, frequency estimated by using the model (2) (in Fig. 6 the exact results) and frequency estimated by using the model containing only one sinusoid (in Fig. 6 the incorrect results). The test confirmed the importance of the model selection, as well as the efficiency of IRNTA.

In some measurement applications, e.g. in power system protection, it is important to dispose of algorithms insensitive to decaying DC component, which is typical in processing currents during faults on elements of power system. Fig. 7 plots the frequency estimates of 50 Hz signal containing decaying DC component. The following three methods are used: DFT, RLS without DC assumed in the signal model and IRNTA with model (2). The last method delivered the best estimates. DFT and RLS delivered inaccurate results during the existence o decaying DC component.

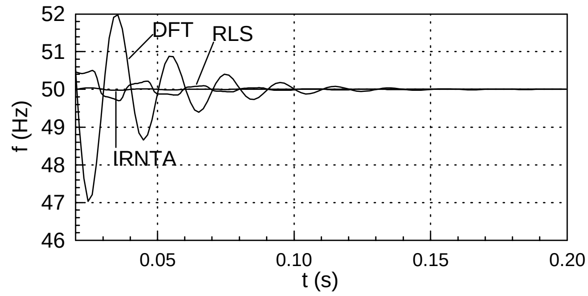

Fig. 7: Estimated frequency of the signal containing decaying DC component.

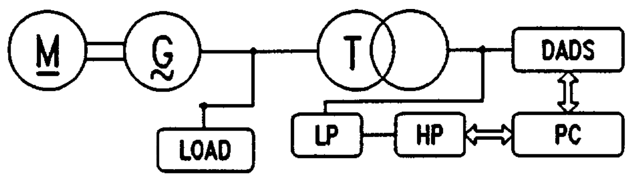

Fig. 8: Laboratory setup (load rejection test).

V. LABORATORY TESTING: LOAD REJECTION TEST

In order to check the validity of IRNTA, at University of Belgrade (Yugoslavia) the laboratory setup depicted in Fig. 8 has been prepared. Data records are obtained from the Motor-Generator (**M-G**) Set. First, the generator terminal voltage is transformed to the required voltage level and then digitised using 12-bit Data Acquisition Digital System (**DADS**) with sampling frequency $f_s = 1600$ Hz. The true frequency is measured using HP 3457A Multimeter (**HP**). It is connected to the circuit over a low-pass analog filter (**LP**) with a cut-off frequency $f_c = 125$ Hz. Both HP and DADS are connected to personal computer (**PC**) over IEEE-488-Bus. M-G Set is operated in a steady-state and the frequency is maintained at approximately 50 Hz. The output generator voltage was slightly corrupted with the 3^{rd}, the 5^{th} and the 7^{th} harmonics. A series of disturbances are provided. Two cases, in which a portion of load is rejected, will be presented. Before the load rejection, generator was loaded with $P_L = 4 \cdot 75 = 300$ W of a pure resistive load. In the first test the whole load is rejected at $t = 1.8$ s. In the second test 25 % of load (75 W) is rejected at $t = 3.2$ s. In Fig. 9 the estimated generator frequencies are plotted. The expected frequency acceleration, proportional to the amount of the load rejected, followed the load rejection. The shaft speed, $n(t)$ in r/min, is proportional to the frequency estimated, according to relation $n(t) = 60 f(t)/p$, where p is the pole pairs number. In the experiment p=2 and consequently $n(t) = 30 f(t)$ r/min.

By comparing the results obtained using HP Multimeter, the maximum difference with the estimated values was less than 0.001 Hz.

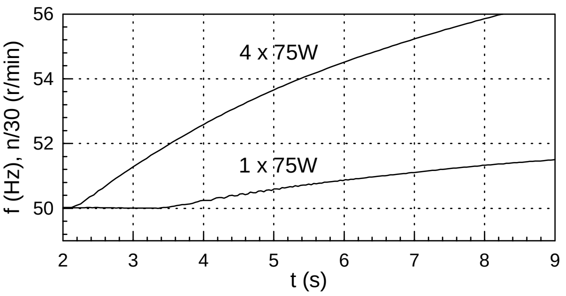

Fig. 9: Estimated generator output voltage frequency.

VI. LABORATORY TESTING: UNSUCCESSFUL SYNCHRONISATION

Synchronisation of two separate electrical power systems is a normal and a standard procedure provided during the operation of any multimachine power system. Very often, a single generator unit is synchronised to the external huge network. In order to avoid overcurrents, response of protective devices, instability, or damage, synchronisation must be provided carefully. The prerequisite for a successful synchronisation is that the corresponding phasors of two systems are equal. At the same time the frequencies of two systems must be the same. Nowadays a synchronisation is provided automatically using digital devices. In the next test an example of a dangerous unsuccessful synchronisation and the analyses of the following transients by using IRNTA, will be given. The testing is provided in the Laboratory of Saarland University (Germany).

In Fig. 10 one line diagram of a synchronous generator **SG** connected over a block transformer **T** to the load is presented. By closing the circuit breaker **B** at $t = 0.5$ s, the single generator system is synchronised to the infinite bus, $\mathbf{V_{inf}}$, over the line. The synchronisation was intentionally unsuccessful, i.e. the phasors and the frequencies of two decoupled systems was not the same at the instant of the synchronisation. As a consequence, a severe dangerous transients occurred.

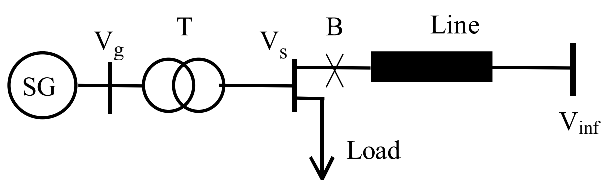

Fig. 10: Laboratory setup (unsuccessful synchronisation test).

In Fig. 11 generator current and its estimated amplitude are presented. An unallowable heavy current, equivalent to fault currents, is experienced. From the current amplitude estimated, the dumped synchronising oscillations can be noticed. Before the synchronisation, SG delivered power only to its local load Y. After it, SG delivered power to the external network, too (see the increase in the current amplitude in the new state reached after the synchronisation).

In Fig. 12 the estimated amplitude of the generator terminal voltage is plotted. During the transient ($t = 0.5$-2.5 s) the expected severe voltage drop is encountered. From the generator terminal voltage the generator frequency is estimated and presented in Fig. 13. First, generator frequency was 50.18 Hz. After the synchronisation, severe synchronising oscillations occurred. At $t = 3$ s generator frequency converged to the frequency of the infinite bus (50 Hz). During the transient, generator frequency oscillated around 50 Hz. The oscillations are dumped.

From Figs. 11-13 the fast convergence and accuracy of IRNTA can be noticed. The changes of the current and voltage amplitudes are severe. They are followed with the changes in the frequency of the signals processed. In spite of such a fast dynamics, the unknown parameters are estimated fast and correctly.

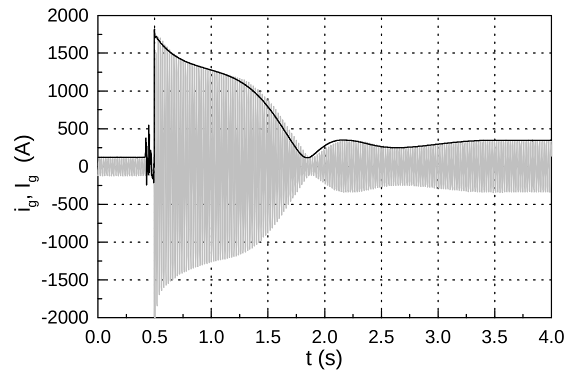

Fig. 11: Generator current and its estimated amplitude.

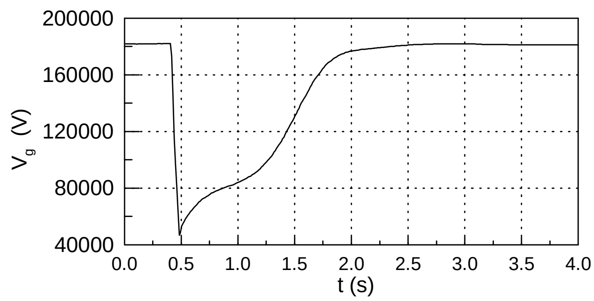

Fig. 12: Generator terminal voltage estimated amplitude.

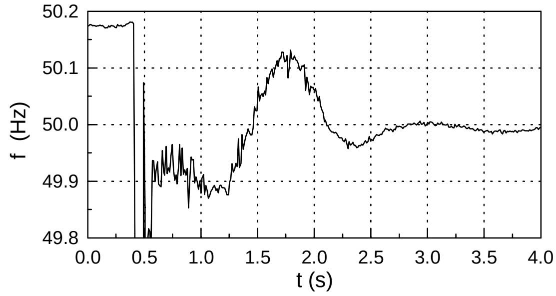

Fig. 13: Estimated generator frequency.

VII. REAL-LIFE CONDITIONS TEST

In the last, real-life test, voltage samples are acquired from the real power system (Balkan Interconnection consisted of Greece, Albanian and Yugoslav pools; the state from 1993, changed in 1995). DADS with the sampling frequency $f_s = 1600$ Hz is used. Data are recorded and off-line processed. First, data are prefiltered by means of the 4th order low-pass Butterworth filter. After that, they are processed by IRNTA. The tuning function is selected to be $\lambda = g(0.9, 0.1, R)$. Frequency is estimated before and after a sudden disconnection of a 200 MW generating unit in Yugoslavia. In Fig. 14 the results obtained are presented. Generator is disconnected at $t = 17.5$ s. From the frequency estimated and presented in Fig. 14, one observes that it oscillates with the low frequency $f = 0.7$ Hz. The value of this frequency is a unique value, inherent to every interconnected power system. The oscillations are dumped and they disappear after approximately 9 seconds after the disturbance inception. The response of prime movers can be noticed, as well. It is manifested through the frequency increase (for $t > 22$ s). The last result, depicted in Fig. 14 can be further used for a complex expert analyses of the system response to the disturbance described.

The system frequency is measured using HP Multimeter, as well. As in the laboratory testing, the maximum difference between the respective values was less than 0.001 Hz.

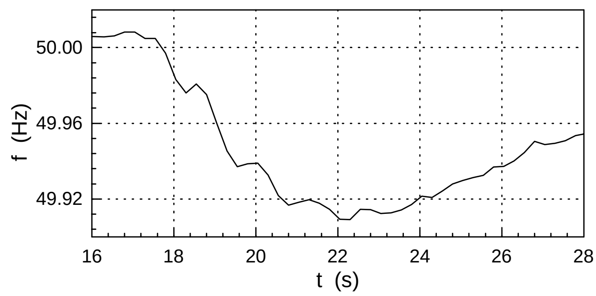

Fig. 14: Frequency estimated in the real-life test.

VIII. CONCLUSION

In this paper a new Improved Recursive Newton Type Algorithm (IRNTA) is presented. In the algorithm a new strategy of sequentially calculating (tuning) forgetting factor is included. Forgetting factor is tuned as a function of the sum of the residual error absolute values, called the error signal. This approach is rather general and could be utilised in some other recursive algorithms which include forgetting of old data. The IRNTA can be utilised for frequency, phasor, spectra, etc. estimation. Through the extensive algorithm testing it is shown that the new algorithm can be successfully applied as a reliable measurement tool. It is proved through computer simulated, laboratory and full-scale real-life tests. The comparison of the estimates obtained using IRNTA and the corresponding values measured using a commercial digital devices, proved that the new method can be used for tracking both slow and fast changes in power systems.

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XI. BIOGRAPHY

Vladimir V. Terzija (M'95) was born in Donji Baraći (Bosnia and Herzegovina) in 1962. He received his B.Sc., M.Sc. and Ph.D. degrees in Electrical Power Engineering from Department of Electrical Engineering, University of Belgrade, Yugoslavia, in 1988, 1993 and 1997, respectively. In 1988 he joined University of Belgrade, where he is now an assistant professor teaching courses in Electric Power Quality, Power System Control, Electromechanical Transient Processes In Power Systems and Estimation Techniques in Power Engineering. At present he is a Research Fellow at Institute of Power Engineering, Saarland University, granted by Alexander von Humboldt Foundation. His areas of scientific interest are power system protection, measurement, monitoring and control, electric power quality and estimation and DSP applications in power systems.