Time-Frequency Analysis of Non-stationary Phenomena in Electrical Engineering

A. Bracale, G. Carpinelli
Dipartimento di Ingegneria Elettrica
Universita degli Studi di Napoli "Federico II"
Via Claudio 21, 89100 Napoli – Italy
carpinelli@unicas.it

K. Wozniak, T. Sikorski, Z. Leonowicz
Department of Electrical Engineering
Wroclaw University of Technology
Wybrzeze Wyspianskiego 27, 50-370 Wroclaw, Poland
tomasz.sikorski@pwr.wroc.pl

Abstract - **This paper serves the idea of applying joint time-frequency representations in electrical engineering. Main directions of researches are concentrated around Cohen's class of transformation which gives some possibilities of adaptation for analysed signal by choosing appropriate kernel function. Additionally, novel approach delivered by S-transform is also introduced. In order to investigate the methods several experiments were performed using simulated phenomena of switching on the capacitor banks in distribution system. Firstly some aspect of S-transform application were present. Then the influence of different kernel functions were investigated when Wigner-Ville, Choi-Williams and Zhao-Atlas-Marks distributions were applied. Obtained results were supplemented by comparison to classical spectrogram. Proposed methods allowed to track instantaneous frequency as well as energy with better time-frequency precision than classical spectrogram. It leads to applications in diagnosis and power quality area.**

Keywords: **Power system harmonics, signal analysis, time-frequency analysis, electric variables, estimation.**

1. INTRODUCTION

Nonstationarity brings a new challenges for signal processing area. Natural direction to calculate only the spectrum of the signal can be insufficient, providing only general information with loss of time-varying nature of analysed phenomena. The violation of main assumption of spectral analysis, the stationarity, can be solved by introducing the joint time-frequency domain.

The standard method for study time-varying signals is the short-time Fourier transform (STFT) that is based on the assumption that for a short-time basis signal can be considered as stationary. The spectrogram utilizes a short-time window whose length is chosen so that over the length of the window signal is stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the centre of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the nonstationarities occurring during this interval will be smeared in time and frequency [1],[4],[13]. This inherent relationship between time and frequency resolution becomes more important when one is dealing with signals whose frequency content is changing rapidly. A time-frequency characterization that would overcome above drawback became a major goal for alternative development based on non-parametric, bilinear transformations.

The first suggestions for designing non-parametric, bilinear transformations were introduced by Wigner, Ville and Moyal at the beginning of nineteen-forties in the context of quantum mechanics area. Next two decades beard fruit of significant works by Page, Rihaczek, Levin, Mark, Choi and Williams [6], Born and Jordan, who provided unique ideas for time-frequency representations, especially reintroduced to signal analysis [5],[12]. At last in nineteen-eighties Leon Cohen employed concept of kernel function and operator theory to derive a general class of joint time-frequency representation. It can be shown that many bilinear representations can be written in one general form that is traditionally named Cohen's class. The fundamental goal is to devise a joint function of time and frequency, which represents the energy or intensity per unit time and per unit frequency. Such joint distribution $TFC(t,\omega)$ means intensity at time t and frequency ω or $TFC(t,\omega)\Delta t\Delta\omega$ means fractional energy in time-frequency cell $\Delta t\Delta\omega$ at t,ω [5],[8],[9].

Next alternative approaches to analysis of time-varying spectrum delivers S-transform [7],[10]. The S – transform is a time-frequency spectral localization method, similar to the short-time Fourier transform, based on moving and scalable localizing Gaussian window. It uniquely combines a frequency dependent resolution with simultaneous location of the real and imaginary spectra. The basis functions for the S-transform are Gaussian modulated cosinusoids whose width varies inversely with the frequency.

Although the main directions of time-frequency representation has evaluated since nineteen-forties only present increase of computational power made them attractive and possible to apply. The main novel fields of application consist speech processing, seismic, economic and biomedical data analysis [1],[3],[13]. Recently some efforts was also made to introduce time-frequency analysis in electrical engineering area [1],[3]. The authors perceive a crucial need for better estimation of distorted electrical signal that can be achieved by applying the time-frequency analysis [14],[15]. Mentioned goal is strongly supported by an issue of energy quality and its wide-understood influence on energy consumers as well as producers. The proper estimation of signal components is very important for control and protection tasks. The authors see the chance of improvement in estimation effect with the ability of investigated methods to track the time-varying spectrum with better

time-frequency resolution than classical spectrogram.

In this paper some efforts were made to apply time-frequency transformations for analysis of nonstationary electrical signals. In order to carry out complete researches the investigations were divided into two groups. Firstly some aspect of S-transform were introduced. Then the influence of different kernel functions on representations from Cohen's class were investigated when constant unity, Gaussian and cone-shaped kernels were applied. Obtained results were supplemented by comparison to classical spectrogram. Obtained results confirm ability to track instantaneous frequency as well as energy with better time-frequency precision than classical spectrogram. General purpose of the work is to emphasize the advantages and disadvantages of proposed methods in point of their application for time-varying spectral estimation.

2. PROPOSED APPROACH

2.1. Short-Time Fourier Transform and S-Transform

The short-time Fourier transform is the classical method of time-frequency analysis. Investigated signal x(t) is multiplied, with an analysis window
$\gamma(\tau-t)$, which is an arbitrarily chosen, and then the Fourier transform of the windowed signal is calculated [1],[13]:

$$STFT(\tau,f) = \int_{-\infty}^{+\infty} x(t)\gamma(\tau-t)e^{-i2\pi ft}dt$$

where: t and f denote time and frequency, and τ gives the position of γ on the t-axis.

The action of this window is to localize the time, and the resulting spectrum is the "local spectrum". This localizing window moves on the time axis to produce local spectra for the entire range of time. Although this approach gives time localization of the spectrum, the STFT is not always sufficient solution. The width of analysis window is fixed, which causes fixed time-frequency resolution for

Representation	Kernel function $\phi_{\omega t}(\theta,\tau)$
Wigner (WD) (constant unity kernel)	1
Wigner-Ville (WVD) (constant unity kernel)	1
Choi - Williams (CWD) (Gaussian kernel)	$\exp\left(-(\theta\tau)^2/\sigma\right)$
Born-Jordan (BJD) ("sinc" kernel)	$\sin\left(\dfrac{\theta}{2}\tau\right)/\dfrac{\theta}{2}\tau$
Zhao-Atlas-Marks (ZAMD) (cone-shaped kernel)	$h(\tau)\cdot\|\tau\|\cdot\left(\sin\left(\dfrac{\theta}{2}\tau\right)/\dfrac{\theta}{2}\tau\right)$

all spectral components. This is fundamental limitation of STFT. Non-stationary signals characterized by wide range of frequency spectrum with transient and sub-harmonic components are difficult to analyze with STFT [11].

The S – transform is a time-frequency spectral localization method, similar to the short-time Fourier transform

(STFT), however based on moving and scalable localizing Gaussian window [11]. It uniquely combines a frequency dependent resolution with desirable information about real and imaginary part of spectrum. The basis functions for the S-transform are Gaussian modulated with cosinusoids, which width varies inversely with the frequency. The S transform of a time series x(t) is defined as [7],[10]:

$$S(\tau,f,\alpha) = \int_{-\infty}^{+\infty} x(t)\gamma(\tau-t,\alpha)\,e^{-i2\pi ft}dt$$

where the localizing Gaussian window $\gamma(t)$ is given by:

$$\gamma(t) = \frac{1}{\alpha\sqrt{2\pi}}e^{-(t^2/2\alpha^2)}, \quad \alpha = \frac{k}{|f|}$$

Parameter α is a dilatation responsible for window width or scaling. The window width α is assumed proportional to the inverse of frequency. The primary purpose of the α is to increase the width of the window function for lower frequency and vice versa. Parameter k is the factor controlling the frequency resolution. If k is above 1, the frequency resolution is increased. Likewise, if k is below 1, the time resolution is improved. The final expression becomes:

$$S(\tau,f) = \int_{-\infty}^{\infty} x(t)\frac{|f|}{k\sqrt{2\pi}}e^{-((\tau-t)^2 f^2/2k^2)}e^{-i2\pi ft}dt$$

where f is the frequency, t and τ are time and time-shift variables.

The multiresolution S-transform output is a complex matrix, where the rows of corresponds with frequencies and the columns with time. Each column thus represents the "local spectrum" for that point in time. Since $S(\tau,f)$ is complex valued. In practice, usually the modules $|S(\tau,f)|$ is plotted that this gives time-frequency S spectrum. The S-transform improves the STFT in that it has a better resolution in phase space (i.e. a more narrow time window for higher frequencies), giving a fundamentally more sound time frequency representation.

2.2. Cohen' class of time-frequency transformations

Cohen defined a general class of bilinear transformation (TFC) introducing kernel function, $\phi_{\omega t}(\theta,\tau)$ [5],[8],[9],[12]:

$$TFC_x(t,\omega) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x\left(u+\frac{\tau}{2}\right)x^*\left(u-\frac{\tau}{2}\right)\cdot$$
$$\cdot\phi_{\omega t}(\theta,\tau)e^{j\theta t}e^{-j\omega\tau}e^{-j\theta u}du d\tau d\theta$$

where: t – time, ω – angular frequency, τ – time lag, θ – angular frequency lag, u – additional integral time variable.

According to Cohen's generalization it is possible to obtain any bilinear time-frequency distribution by choosing suitable kernel function. Some examples of connections between distributions and their kernel functions were presented in Table I. Performing the transformations brings two dimensional planes which represent the changes of frequency component, here called auto-terms (a-t).

Unfortunately, bilinear nature of discussed transformations manifests itself in existing of undesirable compo-

nents, called cross-terms (c-t). Cross-terms are located between the auto-terms and have an oscillating nature. It reduces auto-components resolution, obscures the true signal features and make interpretation of the distribution difficult. One crucial matter of kernel function is smoothing effect of the cross-terms with preservation useful properties of designed distribution.

Thus, the selection of appropriate kernel function reveals as the first level of adaptation for electrical engineering area. Most cases of nonstationary phenomena in electrical engineering are characterized by wide rang of frequency components which involves great numbers of undesirable cross-terms in time-frequency representations. Therefore main criterion for choosing the kernel function became efficiency of smoothing the cross-terms on the equation level. Carried out investigations allowed to separate some subclass of Cohen's transformations which is most effective for analysis of nonstatinary electrical engineering signals. Mentioned subclass preserves the affinity feature. Affinity is the feature of the transformations which consists in preservation of time scaling and time shift properties. All transformations of Cohen's class preserve time shift and frequency shift properties. Thus, it is possible to separate some subclass of Cohen's transformation which would preserve time shifts, frequency shifts and time scaling, simultaneously. It leads to affine subclass, sometimes called: "shift-scale invariant class". Characteristic for mentioned subclass is very specific structure of the kernel function which can be treated as a one dimensional function of product of $\theta\tau$:

$$\phi_{\omega t}\left(\theta,\tau\right)=\hat{\phi}_{\omega t}\left(\theta\tau\right)=\hat{\phi}_{\omega t}\left(\beta\right)\Big|_{\theta\tau=\beta}$$

It brings possibilities for suppressing effect of the undesirable cross-terms. If $\hat{\phi}_{\omega t}(\theta\tau)$ is concentrated around $\beta=0$ then we obtain representations which suppress the cross-terms on the definition level. Thus, the most prominent influence of cross-terms is observed in case of Wigner-Ville Distribution with constant unity kernel. Applying Gaussian kernel - Choi-Williams Distribution, "sinc" kernel – Born-Jordan Distribution or cone-shaped kernel – Zhao-Atlas-Marsk Distribution brings mentioned smoothing effect [5],[6].

3. INVESTIGATIONS

Investigated phenomena concerned simulated operation of switching on the capacitor banks in electrical distribution system. The first capacitor bank (900kVar), installed 0.2km from the station (110/15kV, 25MVA), was switched on at time 0.03s. Second bank (1200kVar), installed 1.2km

Fig. 1. Current waveform at the beginning of the feeder during subsequent switching of two capacitor banks (a) and its spectrum (b).

from the station, was switched on at time 0.09s. Current signal measured at the beginning of the feeder and its spectrum was illustrated in Fig. 1. Clearly visible violation of stationarity in investigated signal allows to treat classical Fourier spectrum only as a preliminary view of observed phenomena. Thus, appropriate approach should consider joint time-frequency analysis.

3.1. Short-Time Fourier Transform and S-Transform

Classical spectrogram allowed detecting basic component 50Hz and transient components 270Hz, 475Hz, appearing after switching operations. In order to preserve sufficient time resolution Hamming window with width equalled two periods of basic component was applied. Mentioned inherent tradeoff between time and frequency resolution can be observed in Fig 3a.

Applying S-transform with moving scalable Gaussian window gives possibilities to adopt time-frequency resolution to analysed phenomena. Fig. 3b underlines effect of attuning time- frequency resolution in order to observe variability of investigated distortion. Especially, comparing STFT with S-Transform in point of dynamism of tracking the transient components, we can note improvement in algorithm response, Fig. 4.

Fig. 3. Time-varying spectrum obtained using Short-Time Fourier Transform (a) and S-Transform (b).

Fig. 4. Comparison of STFT and S-Transform on the basis of tracking the time-varying energy of transient component.

3.2. Cohen' class of time-frequency transformations

Following the specific structure of kernel functions the authors suggest applying Cohen's transformations belonging to affine subclass. Firstly, the Wigner-Ville Distribution which is not weighted of any kernel function was applied. Fig. 5a illustrates time-frequency plane with detected basic component 50Hz and two transient components 475Hz and 270Hz. Interactions between above auto-
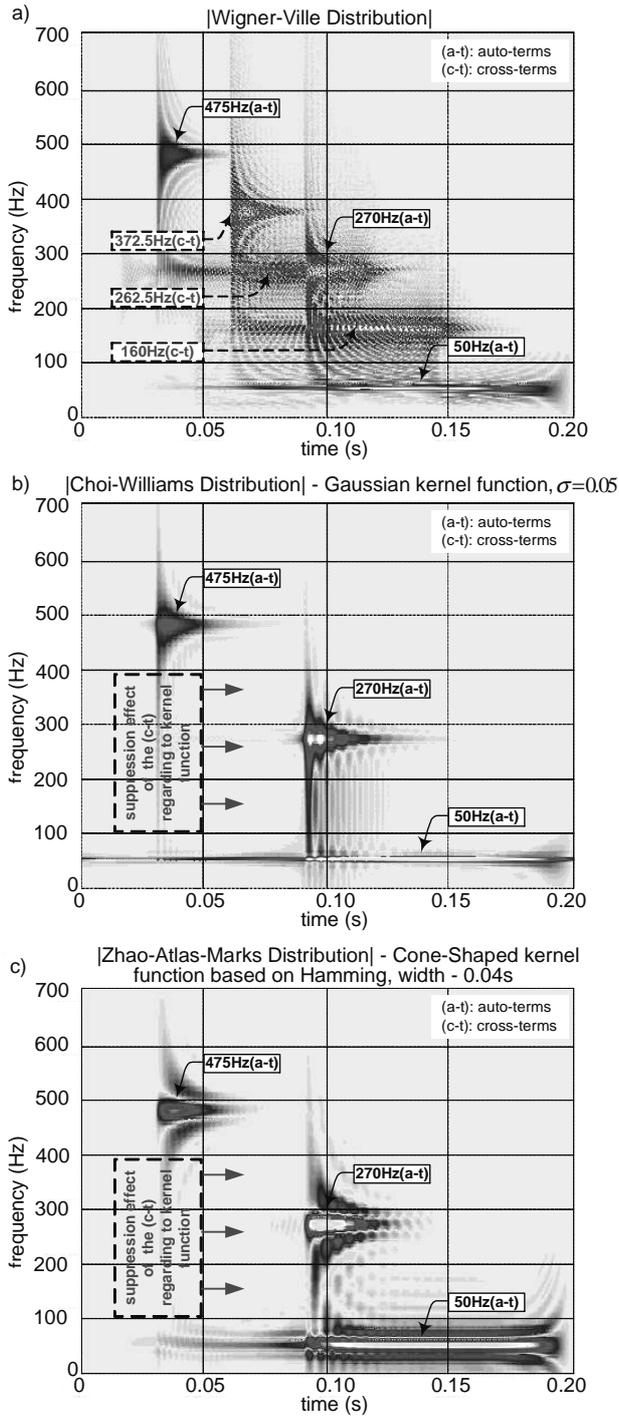
a) |Wigner-Ville Distribution|

b) |Choi-Williams Distribution| - Gaussian kernel function, $\sigma = 0.05$

c) |Zhao-Atlas-Marks Distribution| - Cone-Shaped kernel function based on Hamming, width - 0.04s

Fig. 5. Time-varying spectrum obtained using Wigner-Ville Distribution (a) Choi-Williams Distribution (b) and Zhao-Atlas-Marks Distribution (c).

a) Dynamism of tracking the energy changes - 475Hz component

b) Dynamism of tracking the energy changes - 270Hz component

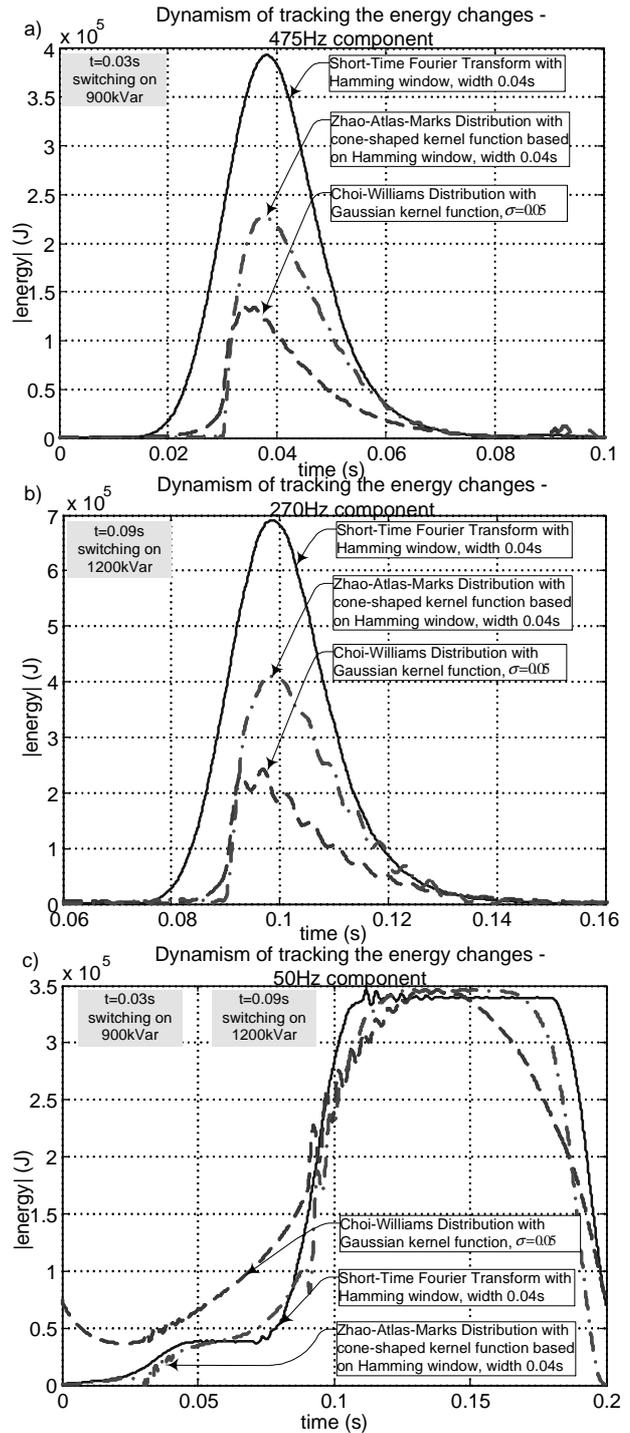c) Dynamism of tracking the energy changes - 50Hz component

Fig. 6. Dynamism of tracking the energy changes of detected components, 475Hz(a), 270Hz(b), 50Hz(c), for transformations based on Gaussian and cone-shaped kernel function in comparison with classical spectrogram.

terms brought undesirable cross-term components in the form of oscillation in 372.5Hz, 262.5Hz and 160Hz. Great number of cross-terms obscures obtained time-frequency plane and makes representation unuseful for application in electrical engineering. However, comparing Fig. 5a to Fig. 3a, detection of the components is more sharp than in case of classical spectrogram. Remaining on the equation level next approach considers applying kernel function.

Especially Choi-Williams Distribution with Gaussian kernel and Zhao-Atlas-Marks Distribution with cone-

shaped kernel turned out to be most prominent for analysis of investigated phenomena. As it was illustrated in Fig. 5b and 5c, obtained representations preserves time-frequency resolution on the equation level with additional suppression effect of the cross-terms. In order to emphasize properties of presented methods some comparison with classical spectrogram was also made on the basis of tracked energy changes of detected components. Observing Fig. 6 better time resolution of applied methods is

Influence of noise level on accuracy of frequency estimation

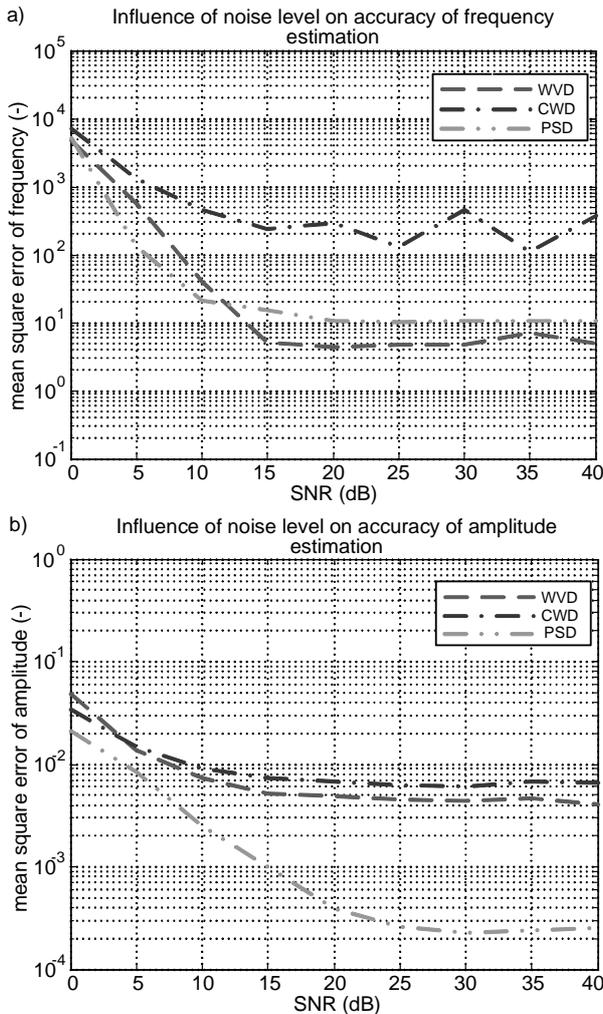## Influence of noise level on accuracy of amplitude estimation

Fig. 7. Influence of noise level on accuracy of frequency (a) and amplitude (b) estimation.

visible. Proposed representations allow to detect the beginning time of occurring transient state with better precision than classical spectrogram.

The comparison of uncertainty of measurements using methods from Cohen's family, as well as a widely used power spectrum estimator (FFT-based method), can be useful and interesting. The following experiment was designed to compare the uncertainty in time and frequency of parameter estimation (amplitude and frequency of each signal component). Testing signals were prepared to belong to a class of waveforms often present in power systems. The signals have following parameters: one 50 Hz main harmonic with unit amplitude, random number(from 0 to 8) of higher odd harmonic components with random amplitude (lower than 0.5) and random initial phase, sampling frequency 5000 Hz, each signal generation repeated 1000 times with re-initialization of random number generator, signal length 200 ms if not otherwise specified. Each run of frequency and amplitude estimation were repeated many times (Monte Carlo approach) and the mean-square error (MSE) of parameter estimation was computed.

Figure 7 illustrates influence of noise level on accuracy of frequency and amplitude estimation when Wigner-Ville and Choi-Williams distribution weighted by Gaussian ker-

nel were applied. Additionally, the effect of classical spectrogram is presented. Observing the details in Fig. 7a we can note increased error of frequency estimation when Gaussian kernel function is used. However, influence of this kernel function is not significant, when amplitude of noisy signals are estimated, Fig. 7b.

4. CONCLUSIONS

Delivered by S-transform and Cohen's generalization novel approaches to analysis of nonstationary signals bring new possibilities in electrical engineering area. Presented scope of research was supposed to specify the qualitative abilities of investigated methods in comparison with popular Short-Time Fourier Transform.

First level of investigations concerned applying S-transform as a scalable tool for analysis of transient phenomena. Presented results contributes possibilities to adaptation for required time-frequency precision. After appropriate selection of the scale improvement of dynamism of tracking the transient components changes is achieved.

Second level of investigations was scoped around Cohen's equation and further effects of applying different kernel functions. Basic representation in this group is Wigner-Ville distribution defined with constant unity kernel. This transformation is characterized by most sharp time-frequency resolution and the most prominent number of undesirable cross-terms at the same time. Effective direction is appropriate choose of kernel function. It allows to keep sharp time-frequency resolution parallel to reduction of cross-terms. Mentioned effect brings applying Gaussian or cone-shaped kernel. Presented results show that discussed methods allow to detect the beginning time of occurring transient state with better precision than classical spectrogram.

Carried out investigations point out some influence of chosen kernel function on uncertainty of measurements. Thus, choosing the kernel function should be multi-layered decision in order to correctly adopt the transformation to analysed signal.

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