MODERN SPECTRAL ANALYSIS OF NON-STATIONARY SIGNALS IN ELECTRICAL POWER SYSTEMS

Z. Leonowicz, T. Lobos
Wroclaw University of Technology
Wroclaw, Poland
lobos@elektryk.ie.pwr.wroc.pl

P. Schegner
Technical University of Dresden
Dresden, Germany
schegner@eev.et.tu-dresden.de

Abstract – A novel approach to harmonic and interharmonic analysis, based on the "subspace" methods, is proposed. Min-norm harmonic retrieval method is an example of high-resolution eigenstructure-based methods. The Wigner-Ville representation was also tested for time-frequency analysis of electric signals. The high-resolution methods do not show the disadvantages of the traditional tools and allow exact estimation of the interharmonic frequencies. To investigate the methods several experiments were performed using real and simulated signals, current waveforms at the output of a simulated frequency converter, shunt reactor compensated transmission lines, and compensated AC arc furnace installation. For comparison, similar experiments were repeated using the FFT. The comparison proved the superiority of the new methods.

Keywords: time-frequency representation, subspace methods, Wigner-Ville distribution, arc furnace, frequency converter, series-compensated transmission lines.

1   INTRODUCTION

Spectrum estimation of discretely sampled processes is usually based on procedures employing the FFT. This approach is computationally efficient and produces reasonable results for a large class of signal processes. However, there are several performance limitations of the FFT. The most prominent limitation is that of frequency resolution, i.e. the ability to distinguish the spectral responses of two or more signals. A second limitation is due to windowing of the data, which manifests as leakage in the spectral domain. These performance limitations are particularly troublesome when analysing short data records, which occur frequently in practice, because many measured processes are brief.

Modern frequency converters generate a wide spectrum of harmonic components. Large converter systems and arc furnaces can also generate non-characteristic harmonics and interharmonics, which strongly deteriorate the quality of the power supply voltage [1]. Periodicity intervals in the presence of interharmonics can be very long. Parameter estimation of the components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

In an attempt to alleviate the limitations of the FFT, many alternative spectral estimation procedures have been proposed within the last 3-4 decades. The most recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and have been called subspace methods. Their resolution is theoretically independent of the SNR [2].

Majority of signals encountered in the real world have time-dependent spectra. In many applications, it is far useful to characterise the signal in time and frequency domains simultaneously. However, the joint time-frequency analysis is not widely accepted.

The time-varying spectra of a non-stationary time series commonly used are the spectrograms from the Short-Time Fourier Transform (STFT). Another type of time-frequency distribution is the Wigner-Ville Distribution (WVD). The Wigner-Ville spectrum of signals with time-limited windows shows better frequency concentration and less phase dependence than Fourier spectra [3].

The following signals have been investigated using the subspace Min-Norm method and the Wigner-Ville Distribution:

-current waveforms at the output of an AC arc furnace power supply installation (with interharmonics),

-current waveforms at the output of a frequency converter during a short circuit with fault resistance,

-voltage waveforms of an series-compensated transmission line after switch-off.

In some cases, the modern methods deliver information, which are not accessible when using the Fourier analysis.

In the first case, the Min-Norm method enables the parameter estimation of interharmonics. In the second case – a short circuit in converter – the modern methods enable to detect additional components, which can be applied as a fault indicator. In the case of a fault in series-compensated lines, the voltage waveforms can be resolved into 2-3 components.

2   MODIFIED MIN-NORM METHOD

The MIN-NORM method [2] involves calculation of the correlation matrix of the signal. Smallest eigenvalues of the matrix correspond to the noise subspace and

largest (all greater than the noise variance) correspond to the signal subspace. The matrix of eigenvectors is defined by:

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{M+1} & \mathbf{e}_{M+2} & \cdots & \mathbf{e}_N \end{bmatrix} \qquad (1)$$

N-M smallest eigenvalues of the correlation matrix (matrix dimension $N>M+1$) correspond to the noise subspace and M largest (all greater than σ_0^2) corresponds to the signal subspace.

Min-norm method uses one vector $\mathbf{d}$ for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element equal to one. We can present $\mathbf{E}_{noise}$ in the form

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{c}^{*T} \\ \mathbf{E}'_{noise} \end{bmatrix} \qquad (2)$$

where $\mathbf{c}^{*T}$ is the upper row of the matrix. Hence $\mathbf{c} = \mathbf{E}_{noise}^{*T}\ell$, where $\mathbf{d}^{*T}\ell = 1$. These conditions are expressed by the following equation

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T}\mathbf{c}}\mathbf{E}_{noise}\mathbf{c} = \begin{bmatrix} 1 \\ \left(\mathbf{E}'_{noise}\mathbf{c}\right)/\left(\mathbf{c}^{*T}\mathbf{c}\right) \end{bmatrix} \qquad (3)$$

Pseudospectrum defined with the help of $\mathbf{d}$ is defined as:

$$\hat{P}\left(e^{j\omega}\right) = \frac{1}{\left|\mathbf{w}^{*T}\mathbf{d}\right|^2} = \frac{1}{\mathbf{w}^{*T}\mathbf{d}\mathbf{d}^{*T}\mathbf{w}} \qquad (4)$$

where $\mathbf{w}$ is defined as: $\mathbf{w} = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$

Since each of the elements of the signal vector is orthogonal to the noise subspace, the quantity (4) exhibits sharp peaks at the signal component frequencies.

In order to adapt this high-resolution method for analysis of non-stationary signals we use similar approach as in short-time Fourier transform (STFT). The time varying signal is broken up into minor segments (with the help of the temporal window function) and each segment (possibly overlapping) is analysed.

The denominator of (4) is estimated for the each time instant. Instantaneous estimates of (4) can be used as estimates of the instantaneous frequency of the signal [4].

3   WIGNER-VILLE REPRESENTATION

The Wigner-Ville distribution (WVD) [3] is a time-frequency representation given by:

$$W_x\left(t,\omega\right) = \int_{-\infty}^{\infty} x\left(t+\tfrac{\tau}{2}\right) x^*\left(t-\tfrac{\tau}{2}\right) e^{(-j\omega\tau)} d\tau \qquad (5)$$

where t is a time variable, ω is a frequency variable. The Wigner distribution is a two-dimensional function describing the frequency content of a signal as a function of time [5] and possesses many advantageous properties, among them:

1. Instantaneous frequency property

2. Energy contained in the $W_x\left(t,\omega\right)$ is equal to the energy possessed by the original signal.

The short-time Fourier transform (STFT) was the first tool for analysing the signal in joint time-frequency domain. The crucial drawback inherent to the STFT is a trade-off between time and frequency resolution. WVD does not suffer from interaction between time and frequency resolutions, but presents some other undesired properties. One main deficiency of the WVD is the cross-term interference. WVD of the sum of components is a linear combination of auto- and cross-terms, thus the desired time-frequency representation may be confusing. The way of lowering cross-term interference is to apply smoothing to the WVD. We use Gaussian smoothed Wigner distribution to analyse the signals in subsection 4.2.

4   SIGNALS IN TRANSMISSION LINES

4.1 Single line to ground fault

The investigation results when estimating the parameters of the voltage positive sequence fundamental component during single phase to earth fault are shown in [6]. The results have been obtained using the Fourier and Kalman algorithms with a sliding sampling window. The same case was investigated using the Wigner-Ville Distribution. The simulated voltage waveform of the faulted phase is shown in Figure 1. The waveform is composed of the exponentially damped oscillating component with the frequency 260 Hz, assuming that the fundamental component is equal to zero. Fault appears after the 20th sample (sampling frequency 2,5 kHz).
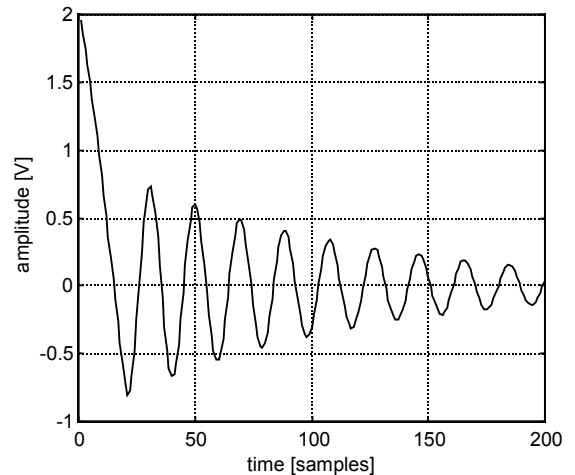

Figure 1: Voltage waveform in faulted phase during single line to ground short circuit.

Wigner-Ville representation offers the possibility to track the frequency and amplitude changes of non-stationary signals. In this case, 200 samples were taken into calculation. In Figure 2 the estimated instantaneous frequency of the oscillating component and in Figure 3 the estimated instantaneous amplitude (power) of the investigated signal are shown. Both characteristics show

good correlation with the true changes of investigated signal. Many experiments showed that unmodified WVD is most suitable for one-component signals. In other cases, due to the cross-term interference, it is difficult to interpret the obtained results.
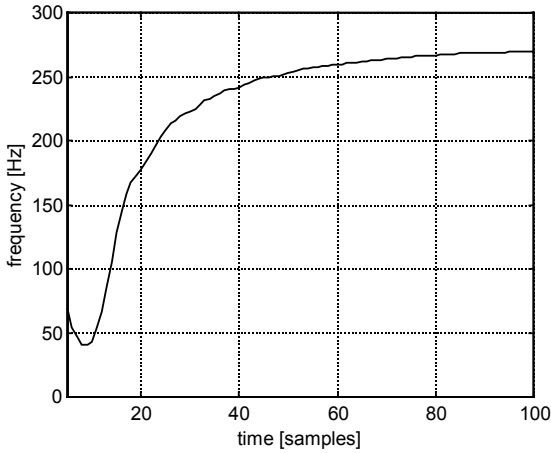

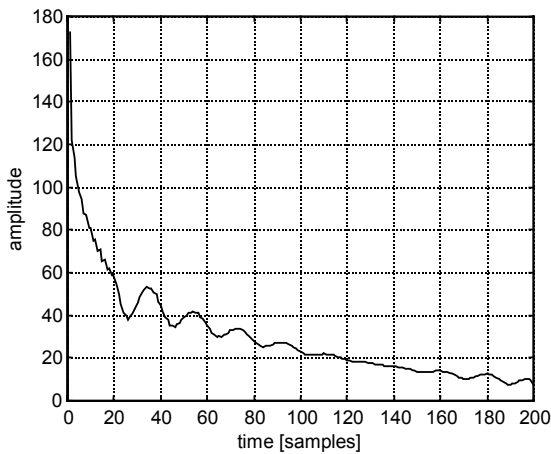**Figure 2:** Estimated frequency of the signal in Figure 1.

Figure 3: Estimated amplitude of the signal in Figure 1.

4.2 Shunt Reactor Compensated Transmission Lines

The signal on the transmission line is composed of several frequency components. The line capacitance and the shunt reactors determine one of them; the second is dependent on the line transposition conditions and, additionally, the phase inter-coupling phenomena are present [8].

In the chapter, investigation results of the simulated 500 kV transmission line, shunt compensated with low compensation degree are shown. Only voltage waveform at the line side of the circuit breaker after deenergisation is considered. In the first case, the compensation degree is 30 % in all phases. The signal consisting of two components with frequencies 27,6 and 34 Hz is shown in Figure 4. In the second case (Figure 5) the signal consists of three components with frequencies 27,6 ; 28,9 and 35,2 Hz due to the uneven compensation in three phases (phase 1: 30%, phase 2: 35%, phase 3: 30%). With noise level (SNR) of 70 dB, it was possible to identify all the components with good accuracy in

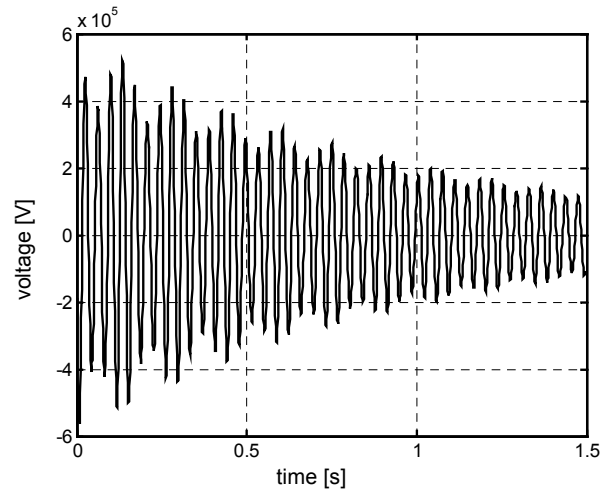both cases, taking only 200 samples (sampling frequency 1000 Hz) into calculation.

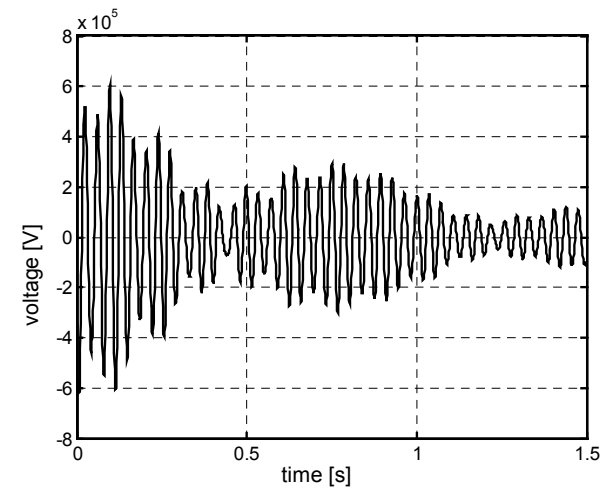**Figure 4:** Voltage waveform at the line side, compensation level 30 %.

Figure 5: Voltage waveform at the line side, uneven compensation level 30 %, 35% and 30%.
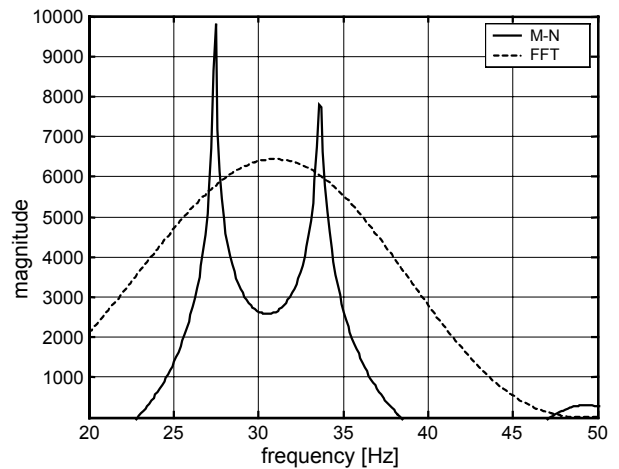

Figure 6: Spectrum of the signal in Figure 4. Comparison between min-norm and DFT spectrum for 200 ms data vector.
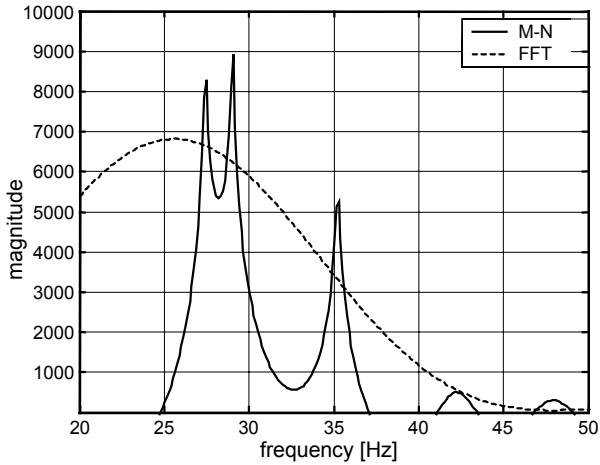
Figure 7: Spectrum of the signal in Figure 5. Comparison between min-norm and FFT spectrum for 200 ms data vector.

However, when using the subspace method, only the frequencies of the signal components can be estimated. This enables to choose an appropriate sampling window and to estimate the amplitudes using the DFT algorithm. Parameter estimation of the voltage signal components makes it possible to predict the course of the voltage waveform, which can be helpful by choosing the time point of the circuit breakers switching off.

5   SIGNALS IN ARC FURNACE INSTALLATION

To investigate the efficiency of the proposed approaches several experiments were performed with the signal waveform recorded in Germany at an industrial arc furnace (66 MVA) installation (Figures 8, 9, 10).
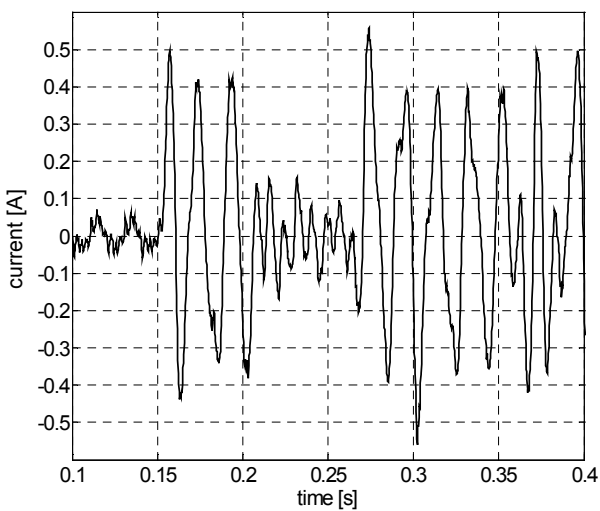

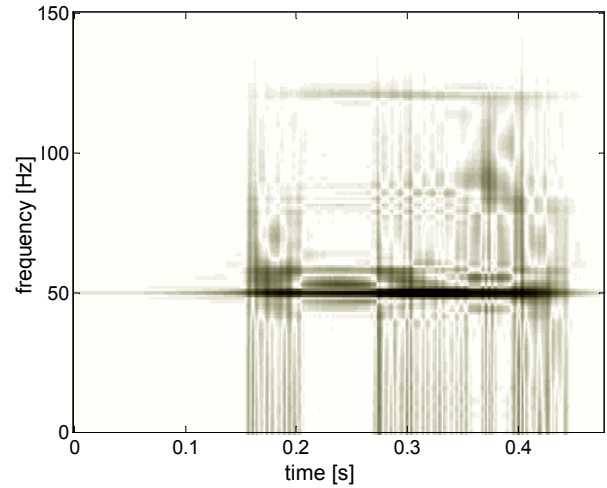**Figure 8:** Signal recorded at the output of an AC arc furnace power supply installation

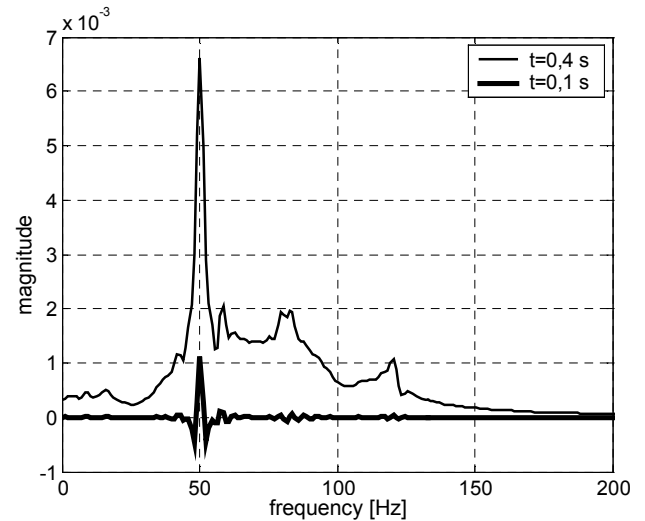**Figure 9:** Gaussian smoothed Wigner-Ville representation of the signal in Figure 8.

Figure 10: Cross-sections of the Wigner-Ville representation in Figure 9 for time slices *t*=0,1 and 0,4 s
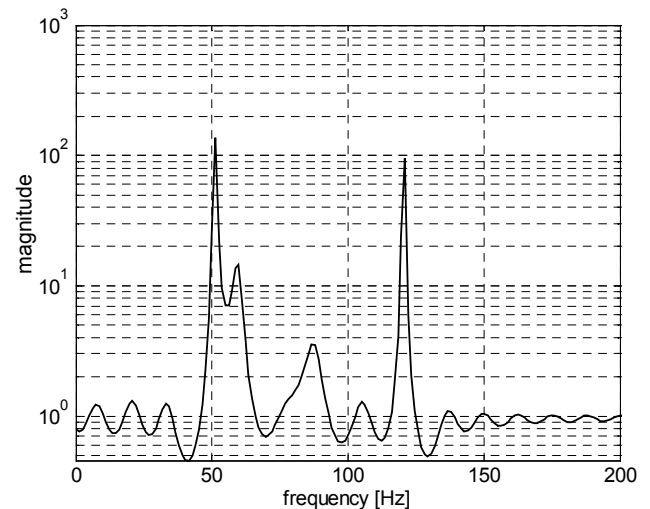

Figure 11: Min-norm spectrum of the signal in Figure 8.

For comparison, in Figure 11 the min-norm spectral estimate of the signal is presented. Accuracy of both

WVD and min-norm methods is comparable, but the WVD allows, additionally to compare directly amplitudes (energy) of the signal components, which cannot be performed using the min-norm representation.

The knowledge about frequency characteristics of the signals is necessary for development of filters and compensation apparatuses.

6 SIGNALS IN INDUSTRIAL FREQUENCY CONVERTER

In this section, some results of simulation investigations of a converter-fed induction motor drive are shown. PWM converters supplying asynchronous motor were simulated using the EMTP-ATP.

Exact estimation of signal parameters at the motor input and among others, the estimation of the basic component frequency is essential to determine an appropriate sampling window for a DFT-analysis [7]. The signals are much distorted due to PWM modulation of a converter. Fast determination of the frequency contents of distorted voltages and currents is a very difficult task. For this purpose various numerical algorithms have been developed. In the paper, investigation results of a 3kVA-PWM-converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U=380 V, I=2.8 A) are shown. Characteristic RC-damping components at the rectifier bridge and at the converter valves are considered. To design the intermediate circuit, the L, C values of a typical 3-kVA converter are chosen. The simulated converter can change the output frequency within a range 0.1-150 Hz. Figure 12 shows current waveform at the converter output (phase R) for the frequency 40 Hz during a short circuit between two phases with fault resistance 100 Ω. The motor is provided with a positive-sequence 3-phase voltage system u_A, u_B, u_C. Complex space-phasor $\underline{u}_p = u_\alpha + j \cdot u_\beta$ of a three-phase system u_A, u_B, u_C is given by [8]:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} \quad (6)$$

It describes, in addition to the positive-sequence components, existing negative-sequence components, harmonic and non-harmonic frequency components of the signal. The complex space-phasor of the converter output voltages is investigated using WVD allowing fast and compact analysis of the three-phase system.

Figures 12 and 13 show the estimated frequency representation of the space-phasor. The signal was sampled with the frequency of 5 kHz and 200 samples from each phase were taken into calculation. First, the samples of the space phasor and its complex conjugate are calculated, according to (6). After that, the calculated samples are used for calculation of the Wigner-Ville spectrum according to the discrete version of (5).
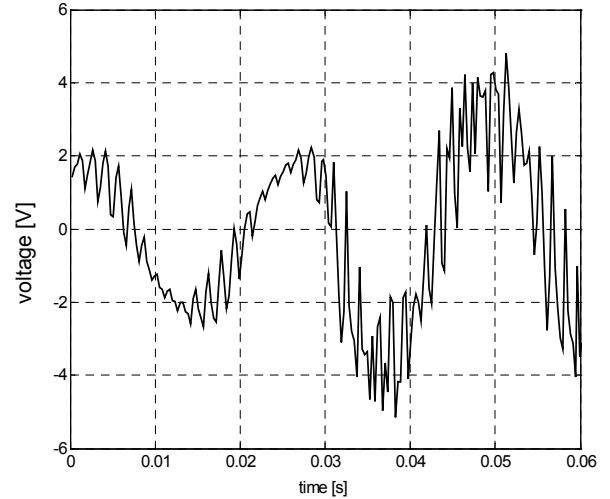

Figure 12: Voltage waveform (phase A) at the motor input during a short circuit. Fault occurs after 0.03 s.
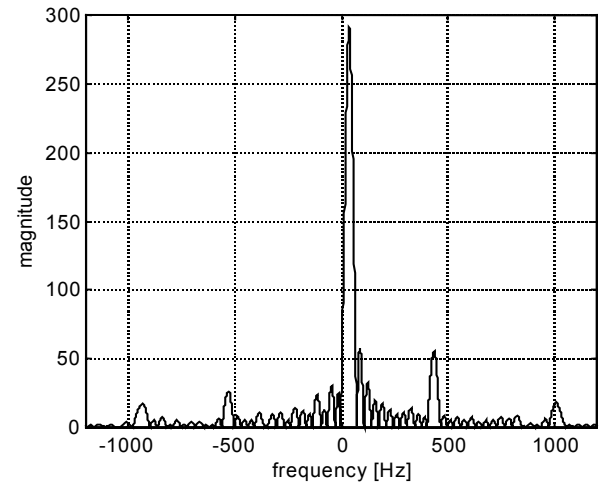

Figure 13: Estimated WVD of the space-phasor (signal in Figure 11), at the time point t=0.015 s. (before the fault). been detected.
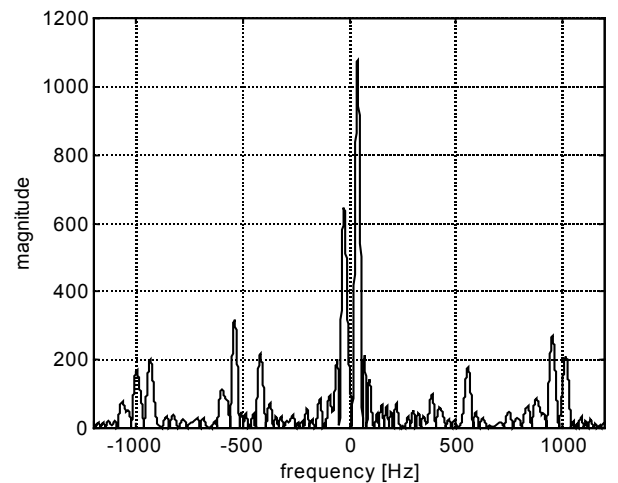

Figure 14: Estimated WVD of the space-phasor (signal in Figure 11), at the time point t=0,05 s. (after the fault).

Before the fault the fundamental component with frequency 40 Hz and the 10th harmonic are present, both positive-sequence components (situated in the right half plane).

Detection of the negative-sequence components (with negative frequencies) can be applied as a fault indicator.

Under fault conditions the negative-sequence component (situated in the left half plane) of the fundamental frequency appears and irregular frequencies 440 and 560 Hz (mainly negative-sequence components) has also been detected.

Detection of irregular frequencies may be useful for diagnosis of some drive faults.

7   CONCLUSIONS

Modern semiconductor converters and arc furnaces can generate not only characteristic harmonics but also non-characteristic harmonics and interharmonics. Parameter estimation of the signal components is very important for control and protection tasks. The design of filters and compensation apparatuses relies on the frequency characteristics of distorted currents and voltages. The usually used FFT has some important limitations.

It has been shown that a high-resolution spectrum estimation method, such as min-norm could be effectively used for estimation of spectral components of distorted signals.

Min-norm method is especially suitable for identification and parameter estimation of signal components, which frequencies differ slightly.

Visualisation of frequency converter supplied drives by means of a static space-phasor is a very useful and compact observation and diagnosis method. Spectrums of the space-phasor of the three-phase signal have been investigated under different operation conditions using the Wigner-Ville distribution.

Superiority of the proposed approaches over the conventional FFT-based tool was shown. Detection of irregular frequencies may be useful for diagnosis of some drive faults. WVD offers also the possibility to track the frequency and amplitude changes of non-stationary signals.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of signal components, which frequencies differ insignificantly. Investigation results confirm the high resolution of the methods. However, their computation is much more complex than FFT.

REFERENCES

[1] C. Mattavelli, at al., "Analysis of Interharmonics in DC Arc Furnace Installations", 8th Int. Conf. on Harmonics and Quality of Power", Athens, Greece, vol.2, pp.1092-1099, 1998.

[2] C.W. Therrien "Discrete Random Signals and Statistical Signal Processing", Englewood Cliffs, Prentice-Hall, 1992, ISBN 0-13-852112-3, pp. 614-655.

[3] M. Sun, C. Chung Li, L. H. Sekhar and R.J. Sclabassi, "A Wigner Spectral Analyser for Nonstationary Signals", IEEE Trans. on Instr. and Meas. 38 (5), pp. 961-966, 1989.

[4] P. Shan, A. A. Beex "High-Resolution Instantaneous Frequency Estimation Based on Time-Varying AR Modelling", Proc. of the IEEE-SP Int. Symposium on Time-Frequency and Time-Scale Analysis, Pittsburgh, USA, pp. 109-112, Oct. 1998.

[5] S. Quian, D. Chen, "Joint Time-Frequency Analysis", Upper Saddle River, Prentice Hall PTR, 1996, ISBN 0-13-254384-2, pp 45-75 & 101-131.

[6] T. Lobos, "Fast Estimation of Symmetrical Components in Real-Time", IEE-Proc.-Gener. Trasm. Distrib.139 (1), p. 27-30, 1992.

[7] T. Lobos, Z. Leonowicz, J. Rezmer: Harmonics and Interharmonics Estimation Using Advanced Signal Processing Methods, 9th Int. IEEE Conf. on Harmonics and Quality of Power, Orlando, USA, vol. I, pp.335-340, 2000.

[8] G. Hosemann, "Space-Phasors for Dynamic Minimum-Time Computing and Digital Real-Time Three-Phase Display". Proceedings of the Int. 8th PSCC Conference, Helsinki, p. 737-745, 1984.