HIGH RESOLUTION SPECTRUM ESTIMATION METHODS FOR SIGNAL ANALYSIS IN POWER SYSTEMS

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Introduction

- Modern frequency power converters generate a wide spectrum of harmonic components

- In some cases, large converter systems generate not only characteristic harmonics typical for the ideal converter operation, but also a considerable amount of non-characteristic harmonics and interharmonics

  - this may strongly deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of the power system.

The estimation of the signal parameters is important for control and protection tasks.

Most of commonly used approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, operate adequately only in the narrow range of frequencies and at moderate noise levels.

The Fourier algorithms are accurate only when the sampling interval is equal to one period or more periods of the main component. In the presence of interharmonics the period can be very long and change with time.

Authors propose the use of:

- ♦ Methods of spectrum estimation based on the linear algebraic concepts of subspaces like **MUSIC** (Multiple Signal Classification) and on **higher order statistics** (cumulants).

Advantages:

- ❖ transformation to a higher-order cumulant domain eliminates the noise
- ❖ MUSIC is an example of eigenstructure-based methods, which yield high resolution and asymptotically exact results

- ♦ The **singular value decomposition** (SVD) approach

Advantages:

- ❖ computationally stable mathematical tool to solve the rectangular overdetermined system of equation. The method can be applied for frequency estimation of all the components of very distorted periodical signals.

Subspace spectrum estimation methods based on cumulants

1. *Cumulant-based approach*

The second-, third-, and fourth-order cumulants of a zero-mean stationary random process $x(t)$ are

$$
\begin{aligned}
&C_{1,x} = E\{x(t)\} = 0 \\
&C_{2,x}(t) = E\{x(t)x(t+t)\} \\
&C_{3,x}(t_1,t_2) = E\{x(t)x(t+t_1)x(t+t_2)\} \\
&C_{4,x}(t_1,t_2,t_3) = E\{x(t)x(t+t_1)x(t+t_2)x(t+t_3)\} + \\
&-C_{2,x}(t_1)C_{2,x}(t_2-t_3) - C_{2,x}(t_2)C_{2,x}(t_3-t_1) + \\
&\qquad -C_{2,x}(t_3)C_{2,x}(t_1-t_2)
\end{aligned}
$$

In the case of real harmonic signals the fourth-order cumulant is given by:

$$
\begin{aligned}
C_{4,x} = -\tfrac{1}{8}\sum_{k=1}^{p} a_k^4 \,[&\cos w_k(t_1-t_2-t_3) + \\
&+\cos w_k(t_2-t_3-t_1) + \cos w_k(t_3-t_1-t_2)]
\end{aligned}
$$

The second-order cumulant is the autocorrelation.

In the case of symmetrical distribution of a signal the third-order cumulant is equal to zero.

The **diagonal slice** of the fourth-order cumulant retains all of the information about the number of harmonics, their amplitudes and frequencies.

Existing high-resolution methods, such as MUSIC, can be applied by replacing correlation quantities with the fourth-order cumulant.

SVD approach

The waveform of the voltage or current is assumed as the sum of harmonics of unknown magnitudes and phases:

$$x(t) = \sum_{k=1}^{N} X_k \cos(w_k t + j_k)$$

The number of measurements is usually higher than the number of harmonics. Estimation of harmonics is then equivalent to solving the overdetermined system of algebraic equations.

$$\mathbf{Ah} = \mathbf{b}$$

where the matrix **A** and vectors **h** and **b** are given as follows

$$\mathbf{A} = \begin{bmatrix} x_l & x_{l-1} & \cdots & x_1 \\ x_{l+1} & x_l & \cdots & x_2 \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-1} & x_{n-2} & \cdots & x_{n-l} \\ x_2 & x_3 & \cdots & x_{l+1} \\ x_3 & x_4 & \cdots & x_{l+2} \\ \cdots & \cdots & \cdots & \cdots \\ x_{n-l+1} & x_{n-l+2} & \cdots & x_n \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ h_l \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} x_{l+1} \\ x_{l+2} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ x_{n-l} \end{bmatrix}$$

l is the order of predicted **AR** model of the data ($N \le l \le n - N/2$). The vector **h** is composed of the coefficients of the impulse response of this model

$$H(z) = 1 - \sum_{i=1}^{l} h_i \; z^{-i}$$

The solution for vector **h** of is possible in the least square (LS) sense,

$$E = \frac{1}{2} \left\| \mathbf{Ah} - \mathbf{b} \right\|_2^2$$

To solve the most suitable method is the application of singular value decomposition. In this approach the rectangular matrix **A** is the product of three matrices.

$$\mathbf{A} = \mathbf{U}\,\mathbf{S}\,\mathbf{V}^t$$

The essential information of the system is contained in the first non-zero singular values and first ***p*** singular vectors, forming the orthogonal matrices **U** and **V**. By cutting the appropriate matrices to this size and denoting them by $\mathbf{U_r}$, $\mathbf{S_r}$, and $\mathbf{V_r}$, respectively, the solution is obtained in the form

$$\mathbf{h} = \mathbf{V_r}\,\mathbf{S_r^{-1}}\,\mathbf{U_r^{t}}\,\mathbf{b}$$

$$\mathbf{S}^{-1} = \mathrm{diag}\left[\frac{1}{s_1}, \frac{1}{s_2}, \ldots, \frac{1}{s_p}\right].$$

The phases of the roots closest to the unit circle denote the angular frequencies of the sinusoids forming the waveform.

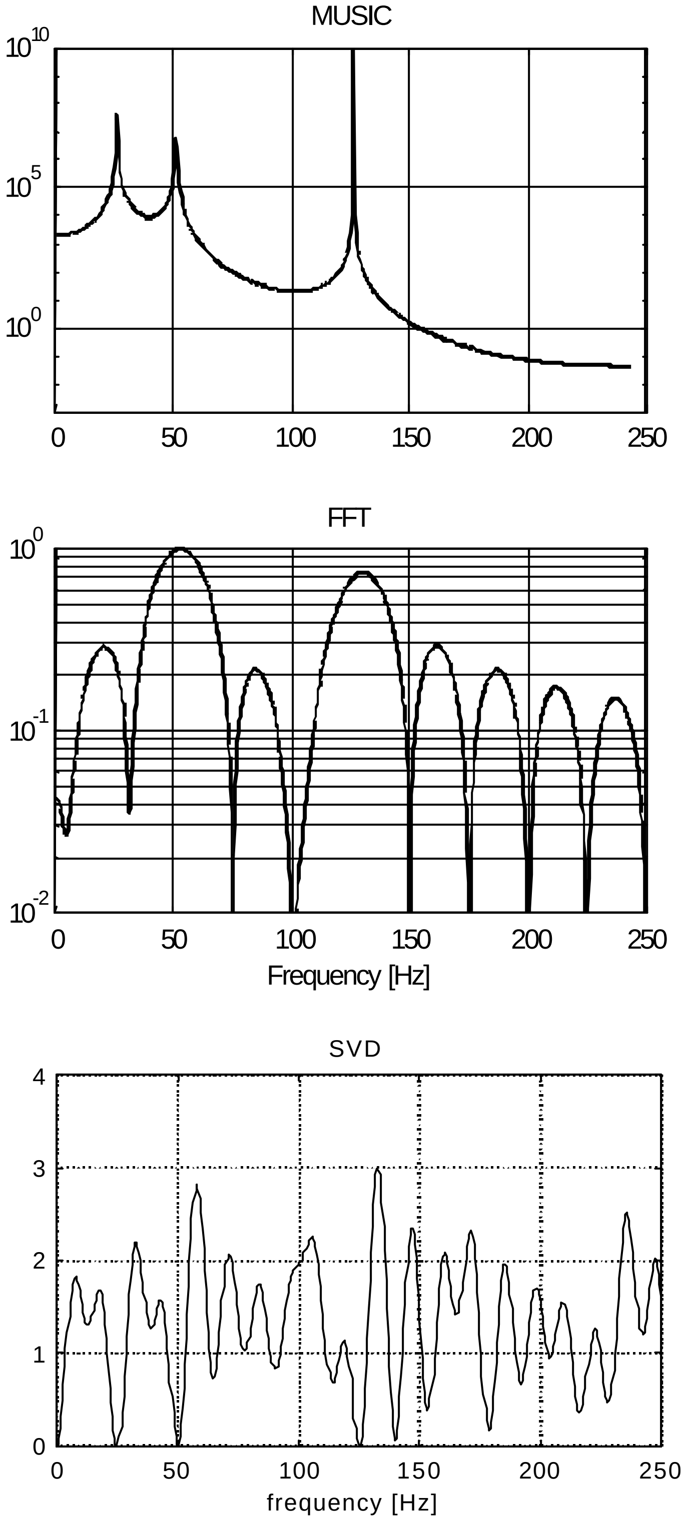


Fig. 1. Magnitude characteristics of the signal calculated by the MUSIC, FFT algorithms (n=80) and SVD (l=80, n=90).

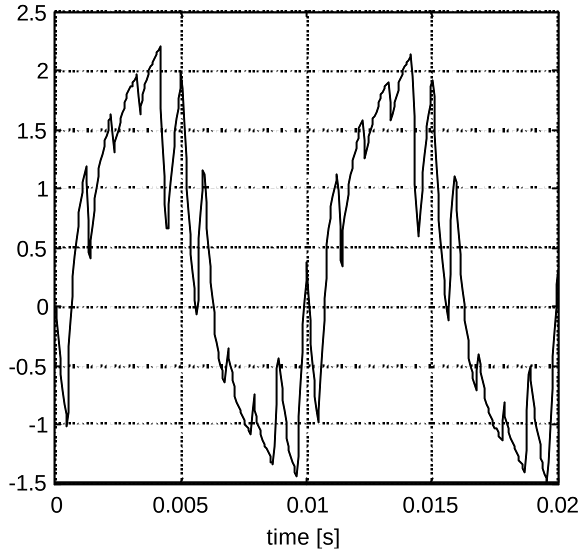


Fig. 2. Current waveform at the output of a simulated frequency converter, f = 100 Hz.

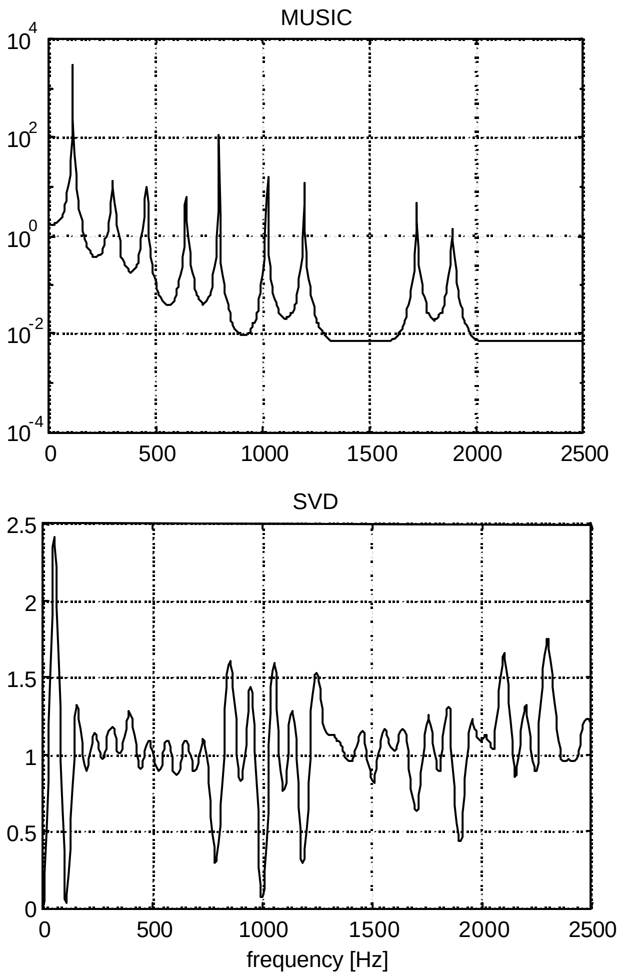


Fig. 3. Magnitude characteristics of the signal in Figure 2 calculated by the MUSIC (n=100) and SVD (l= 254, n= 260) method.

Real industrial frequency converter

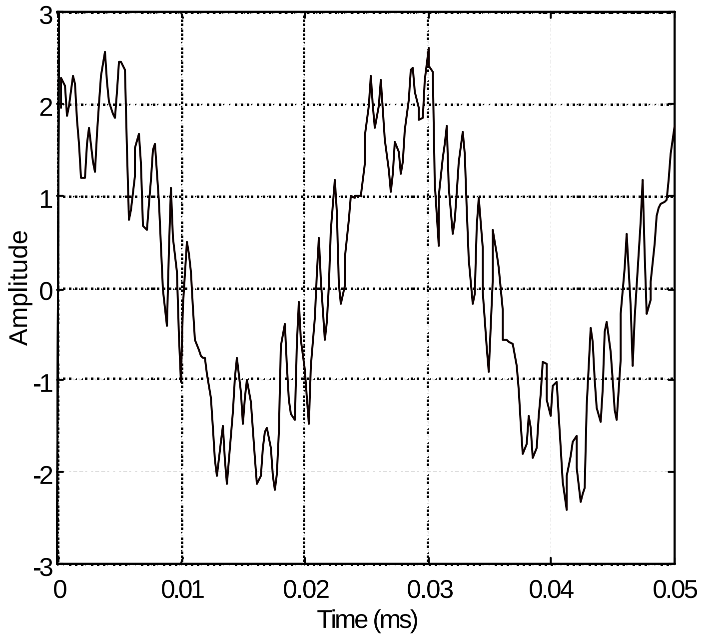
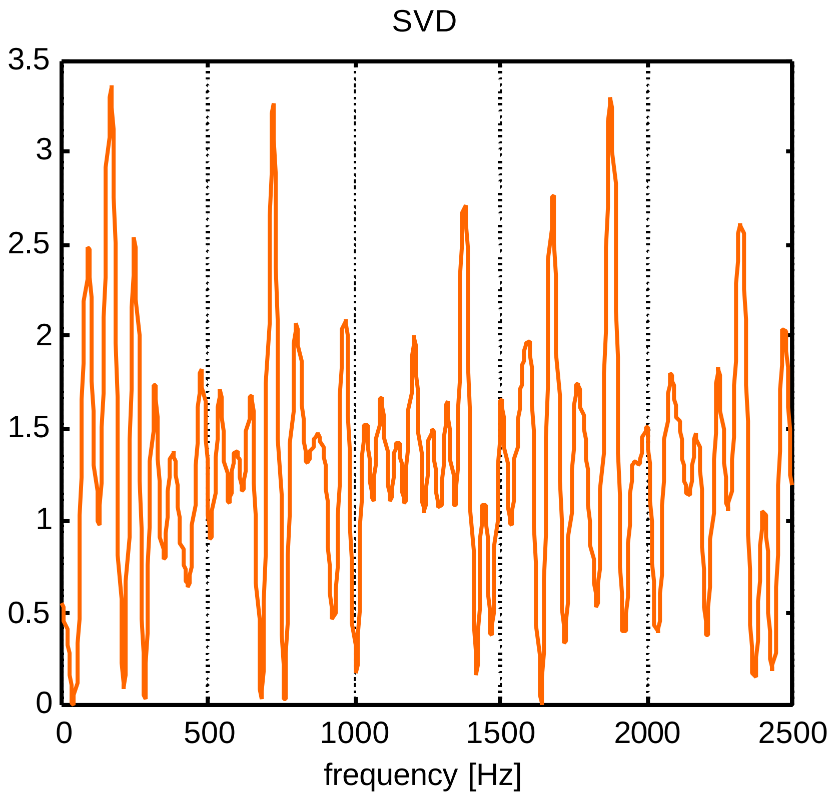


Fig. 4. Current waveform at the output of frequency converter.



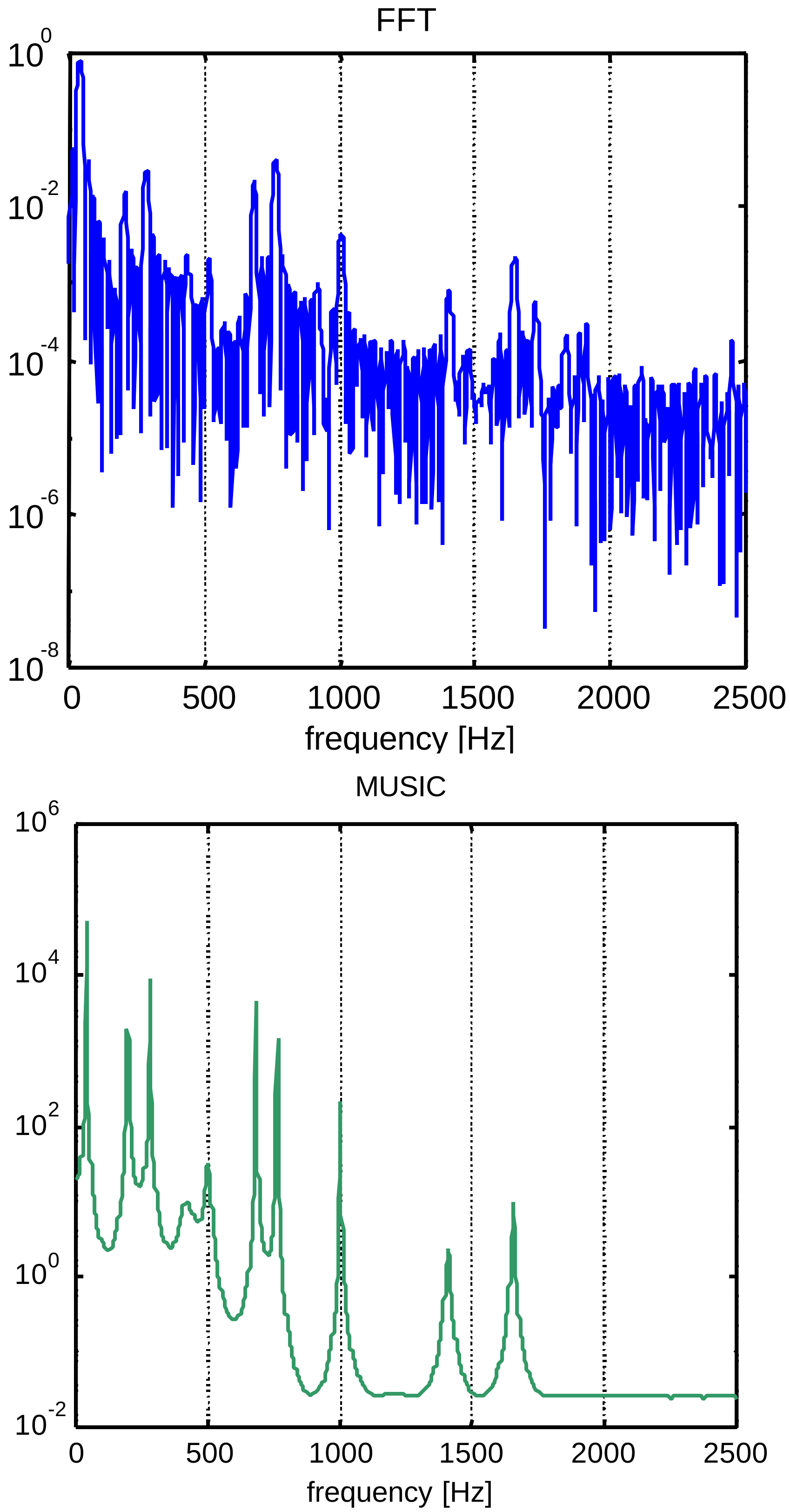


Fig. 5. Magnitude characteristic SVD method, the FFT algorithm and by the MUSIC method of the signal in Fig. 4, l = 70, n=100.

Conclusions

It has been shown that:

- A high-resolution spectrum estimation method, such as MUSIC combined with the use of higher–order statistics could be effectively used for parameter estimation of distorted signals.

  First of all, the method has been applied for frequency estimation of harmonic and interharmonic signal components. The frequency estimation of the main harmonic makes it possible to calculate the stationary space-phasor for system visualisation and to determine an appropriate window for a DFT-analysis.

- The linear least square method for harmonics and interharmonics detection in a power system has also been investigated in this paper using the singular value decomposition (SVD).

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection and location of all higher harmonics existing in the system.