Modern spectral analysis of non-stationary signals in power electronics

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ABSTRACT
Classical techniques of spectrum estimation of multicomponent signals are based on Fourier-based transformations. The frequency estimates obtained from their spectral peaks are affected by the window length and phase of signal components, thus presenting a large variance even in the absence of noise.
We estimated the spectrum of the signals with the help of min-norm method (subspace method) and we obtained its time-frequency representation with excellent time and frequency resolution. The accuracy and phase dependence of the tested methods were investigated and compared with the parameters of the frequency estimation via FFT. The proposed methods were tested with non-stationary multiple-component signals occurring during the fault operation of inverter-fed drives.

Categories and Subject Descriptors
J.2 **[Physical Sciences and Engineering]**: *Engineering, Electronics, Mathematics and Statistics.*

General Terms
Measurement, Experimentation, Algorithms, Theory

Keywords
Discrete Fourier Transform, frequency conversion, frequency measurement, harmonic analysis, power system, Min-Norm

1. INTRODUCTION
The quality of voltage waveforms is nowadays an issue of the great importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The liberalization of European energy market will strengthen the competition and is expected to drive down the energy prices. This is reason for the requirements concerning the power quality.
The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Modern frequency power converters generate a wide spectrum of harmonics components, which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of non-characteristic harmonics and interharmonics which may strongly deteriorate the quality of the power supply voltage.
Interharmonics are defined as non-integer harmonics of the main fundamental under consideration. The estimation of the harmonic components is very important for control and protection tasks. Interharmonics are considered more damaging than characteristic harmonics components of the distorted signals. Their emission is specified lower than those are for the harmonics.
There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels.
Spectrum estimation of discretely sampled processes is usually based on procedures employing the FFT. This approach is computationally efficient and produces reasonable results for a large class of signal processes. However, there are several performance limitations of the FFT, e.g.:

- frequency resolution, i.e. the ability to distinguish the spectral responses of two or more signals;
- limitations due to windowing of the data.

Fourier algorithms are accurate only when the sampling interval is equal to one period or more periods of the main component. In the presence of interharmonics in power system the period can be very long and change with time. Windowing manifests as leakage in the spectral domain: energy of the main lobe of a spectral response "leaks" into the sidelobes obscuring and distorting other spectral responses.
Analysis of modern motor drives is difficult because of the presence of interharmonics and non-characteristic components due to complex interactions in converter control systems [1, 6].
Limitations of the Fourier analysis are troublesome when analysing short data records, which occur frequently in practice because many processes are brief in duration or have slowly time-varying spectra that may be considered constant only for short record lengths.
The time-varying spectra of a nonstationary time series commonly used are the spectrogram from the short-time Fourier transform (STFT) and the scalogram obtained from the wavelet transform.
The recent methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods". Its resolution is theoretically independent of the SNR. The model of the signal in this case is a sum of random sinusoids in the background of noise of a known covariance function.
Pisarenko first observed that the zeros of the Z-transform of the eigenvector corresponding to the minimum eigenvalue of the covariance matrix lie on the unit circle, and their angular positions correspond to the frequencies of the sinusoids. In a

later development it was shown that the eigenvectors might be divided into two groups, namely, the eigenvectors spanning the signal space and eigenvectors spanning the orthogonal noise space. The eigenvectors spanning the noise space are the ones whose eigenvalues are the smallest and equal to the noise power. One of the most important technique, based on the Pisarenko's approach of separating the data into signal and noise subspaces, is the min-norm method.

To investigate the ability of the methods several experiments were performed. Simulated signals, current waveforms at the output of a simulated three-phase frequency converter as well as current waveforms at the output of an industrial frequency converter were investigated. For comparison, similar experiments were repeated using the FFT.

2. MIN-NORM METHOD

The min-norm method involves projection of the signal vector [8]:

$$\boldsymbol{s_i} = \begin{bmatrix} 1 & e^{jw_i} & \dots & e^{j(N-1)w_i} \end{bmatrix}^T \tag{1}$$

onto the entire noise subspace.

A random sequence **x** made up of M independent signals in noise is considered.

$$\boldsymbol{x} = \sum_{i=1}^{M} A_i \boldsymbol{s_i} + \boldsymbol{h}\ ; \quad A_i = |A_i| e^{jf_i} \tag{2}$$

If the noise is white, the correlation matrix is

$$\mathbf{R_x} = \sum_{i=1}^{M} E\{A_i A_i^*\} \boldsymbol{s_i} \boldsymbol{s_i^T} + s_0^2 \mathbf{I} \tag{3}$$

N-M smallest eigenvalues of the correlation matrix (matrix dimension *N>M+1*) correspond to the noise subspace and *M* largest (all greater than s_0^2) correspond to the signal subspace.

The matrix of eigenvectors is defined as:

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{M+1} & \mathbf{e}_{M+2} & \dots & \mathbf{e}_N \end{bmatrix} \tag{4}$$

Min-norm method uses one vector **d** for frequency estimation. This vector, belonging to the noise subspace, has minimum Euclidean norm and his first element equal to one.

Vector **d** is equal to

$$\mathbf{d} = \frac{1}{\mathbf{c}^{*T}\mathbf{c}} \mathbf{E}_{noise}\mathbf{c} = \begin{bmatrix} 1 \\ (\mathbf{E}'_{noise}\mathbf{c})/(\mathbf{c}^{*T}\mathbf{c}) \end{bmatrix} \tag{5}$$

Pseudospectrum defined with the help of **d** is defined as.

$$\hat{P}(e^{jw}) = \frac{1}{|\mathbf{w}^{*T}\mathbf{d}|^2} = \frac{1}{\mathbf{w}^{*T}\mathbf{d}\mathbf{d}^{*T}\mathbf{w}} \tag{6}$$

where **w** is defined as in (1).

To investigate the time-varying signals with the min-norm method, we use similar approach as in short-time Fourier transform (STFT) [7]. The time varying signal is broken up into small time segments (with the help of the temporal window function) and each segment (possibly overlapping) is analysed. The denominator of (6) is estimated for the each time instant. [4]

3. DRIVE SIMULATION

In the recent years, simulation programs for complex electrical circuits and control systems have been improved essentially. The simulation of characteristic transient phenomena concerning the electrical quantities became feasible without any arrangement of hardware. The EMTP-ATP (Electromagnetic Transients Program - Alternative Transients Program) as a FORTRAN based and to MS-DOS/WINDOWS adapted program serves for modelling complex 1- or 3-phase networks occurring in drive, control and energy systems [2, 5].

In the paper we show investigation results of a 3kVA-PWM-converter with a modulation frequency of 1 kHz supplying a 2-pole, 1 kW asynchronous motor (U_n=220 V, P_n=1,1 kW, s=6 %, cos f =0,81, n=1410 min^{-1} (Figure 1)

4. TIME-FREQUENCY REPRESENTATION OF THE SPACE-PHASOR

The motor is provided with a positive-sequence 3-phase voltage system f_R, f_S, f_T .

Complex space-phasor $\boldsymbol{f_p} = f_a + j \cdot f_ß$ of a three-phase system f_R, f_S, f_T is given by [3]:

$$\begin{bmatrix} f_a \\ f_b \end{bmatrix} = \sqrt{\tfrac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \sqrt{\frac{3}{2}} & -\sqrt{\frac{3}{2}} \end{bmatrix} \begin{bmatrix} f_R \\ f_S \\ f_T \end{bmatrix} \tag{7}$$

It describes, in addition to the positive-sequence component, existing negative-sequence component, harmonic and non-harmonic frequency components of the signal.

The complex space-phasor of the converter output voltages is investigated using min-norm method. Each line of the time-frequency representation is obtained from the time interval of 40 samples.

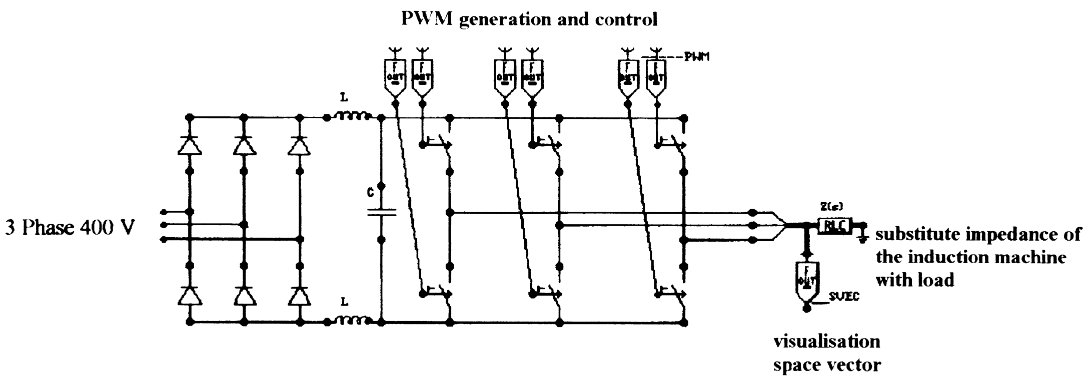

Figure 1. Simulated PWM converter.

The following fault operations of the inverter drive have been investigated:

¦ Increase of the motor lead resistance in one phase to 100 O. Main output frequency of the inverter is 120 Hz, sampling frequency 20 kHz. Beginning of the fault – 10 ms. (Figure 2).

¦ Short-circuit between motor leads. Main output frequency of the inverter is 60 Hz, sampling frequency 20 kHz. Beginning of the fault – 19 ms (Figure 3)

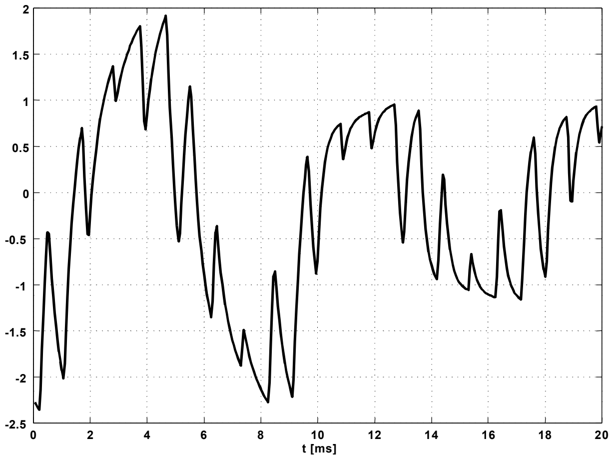

Figure 2. Current waveform in the case of increase of the motor lead resistance.

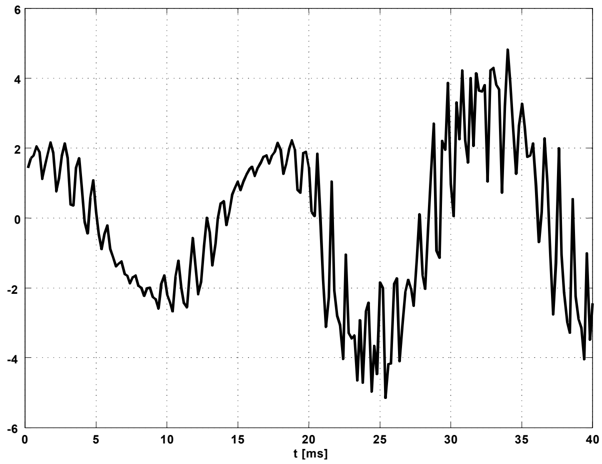

Figure 3. Current waveform in the case of short-circuit between two motor leads.

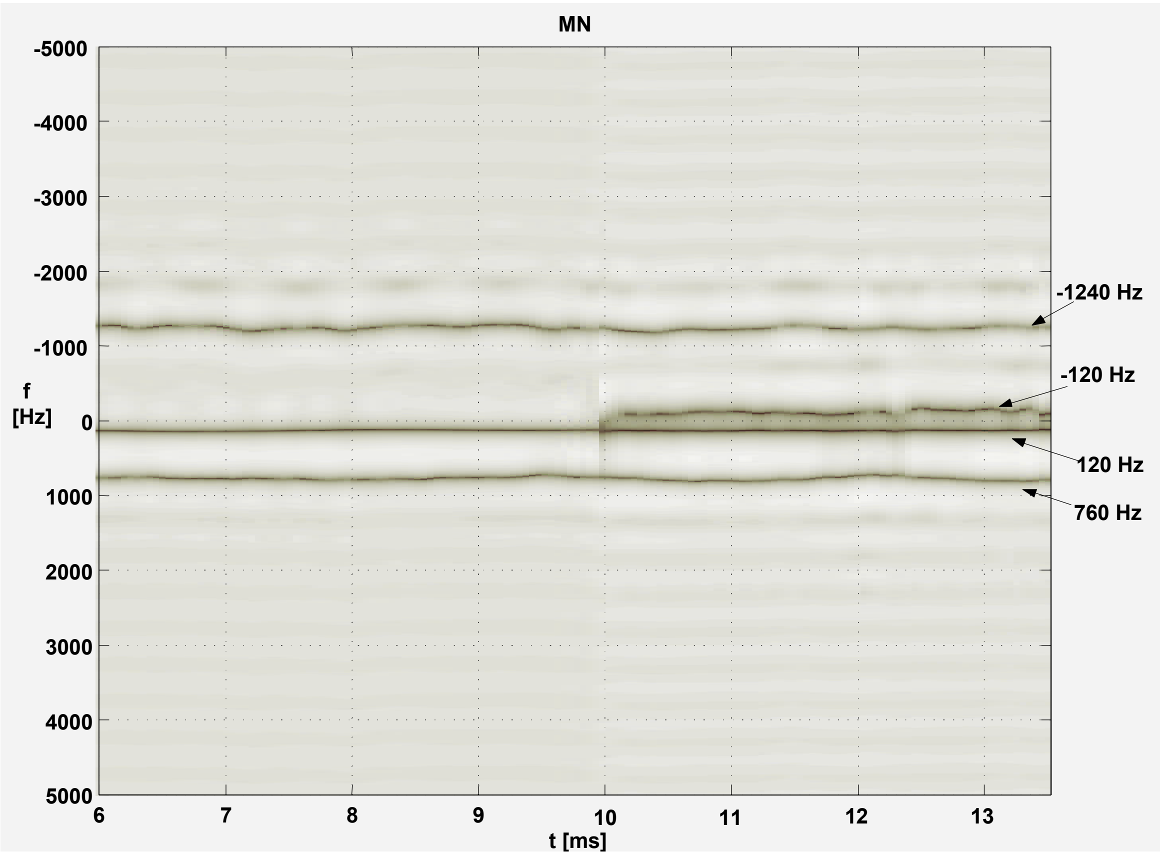

Figure 4. Min-Norm representation of the signal from Figure 2.

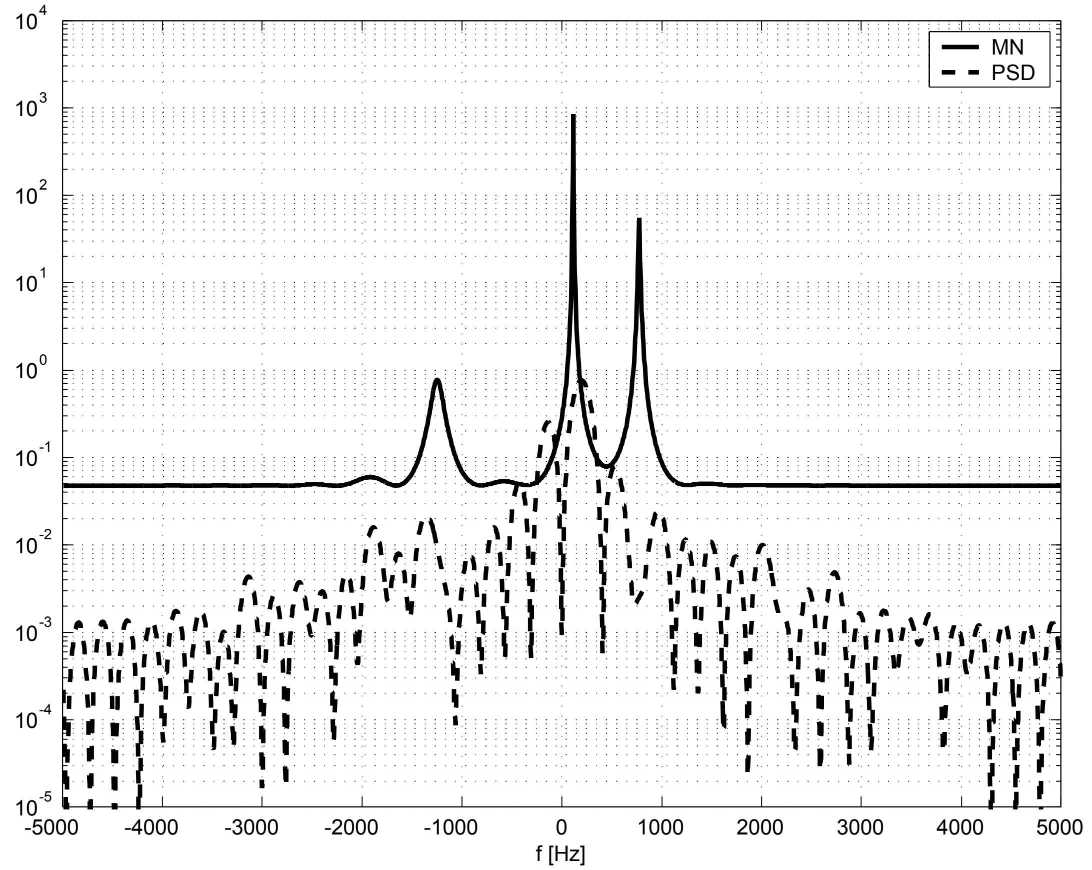

Figure 5. Slice of the MN (min-norm) and PSD (power spectrum) representation of the signal from Figure 2 for t=8 ms

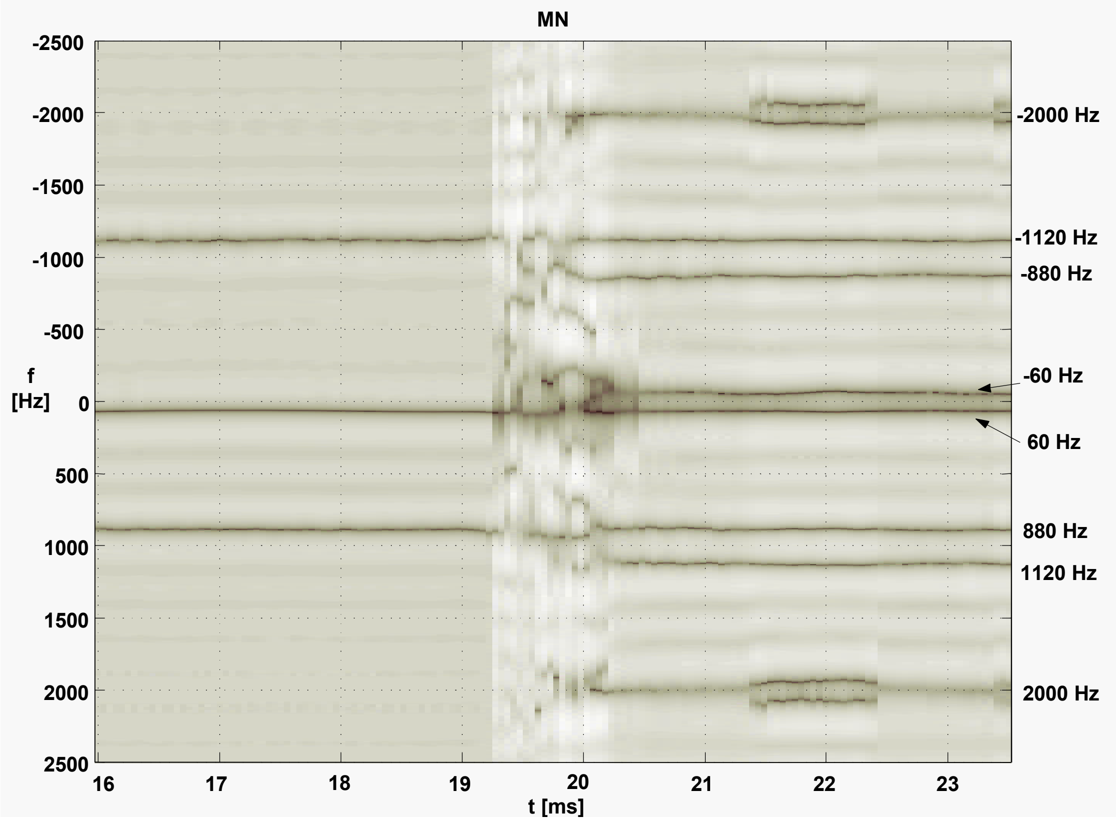

Figure 6. MN representation of the signal from Figure 3.

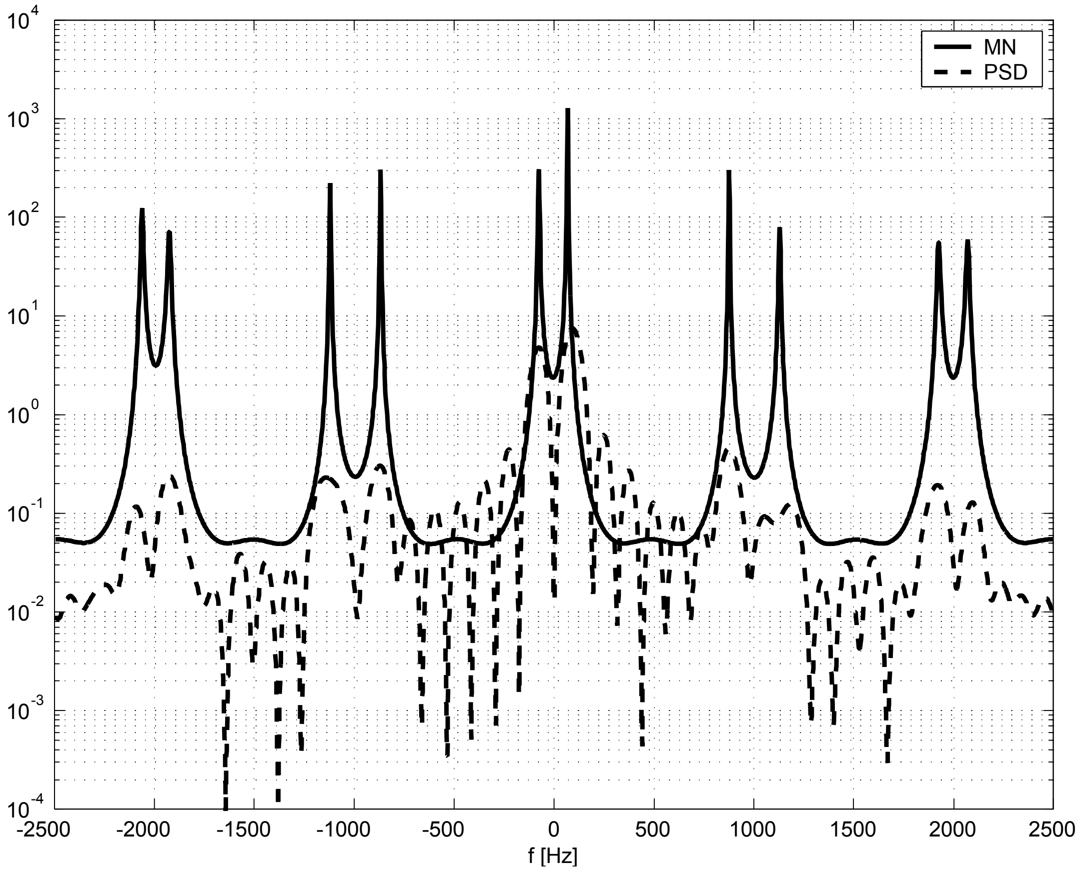

Figure7. Slice of the MN (min-norm) and PSD (power spectrum) representation of the signal from Figure 3 for t=22 ms.

In Figure 4 we can see a simple case of supply asymmetry of the motor. After the fault appears a negative component with frequency 120 Hz caused by asymmetry state. We can see also positive intermodulation component with frequency 760 Hz (1000 Hz (PWM modulation frequency) - 2·120 Hz = 760 Hz) and negative intermodulation component with frequency -1240 Hz (1000 Hz + 2·120 Hz = 1240 Hz).
In the case, where a short circuit occurs between the motor leads, (Figure 6) we can see additional intermodulation components with frequencies –880, 1120 and ±2000 Hz. It appears also a negative component with frequency 60 Hz.

5. CONCLUSIONS

Modern frequency power converters generate a wide spectrum of harmonic components. Large converter systems can also generate non-characteristic harmonics and interharmonics. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present and the periodicity intervals are fixed at 20 ms (50 Hz), while periodicity intervals in the presence of interharmonics are variable and may be very long.
Visualisation of frequency converter supplied drives by means of a static space-phasor is a very useful and compact observation and diagnosis method. Spectrum of the space-phasor has been investigated under different operation conditions using the min-norm time-frequency representation.
It was shown that the approach based on the space-phasor and high-resolution spectrum estimation methods, such as min-norm, could be effectively used to identify different faults in inverter-fed drives.

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