ZBIGNIEW **LEONOWICZ**, TADEUSZ **£OBOS**
PIOTR **RUCZEWSKI**, JAROSŁAW **SZYMAÑDA**
TECHNICAL UNIVERSITY OF WROCLAW, POLAND

APPLICATION OF HIGHER-ORDER SPECTRA FOR SIGNAL PROCESSING IN ELECTRICAL POWER ENGINEERING

CUMULANTS OF RANDOM SIGNALS

Given a set of n real random variables $\left\{x_1, x_2, \ldots, x_n\right\}$, their joint CUMULANTS of order $r = k_1 + k_2 + \ldots + k_n$ are defined as

$$c_{k_1 \ldots k_n} \triangleq (-j)^k \left. \frac{\partial^k \ln \Phi(\omega_1, \omega_2, \ldots, \omega_n)}{\partial \omega^{k_1} \partial \omega^{k_2} \ldots \partial \omega^{k_n}} \right|_{\omega_1 = \omega_2 = \ldots = \omega_n = 0}$$

where $\Phi(\omega_1, \omega_2, \ldots, \omega_n) = \mathsf{E}\left\{ e^{\left[j(\omega_1 x_1 + \omega_2 x_2 + \ldots + \omega_n x_n) \right]} \right\}$ is their joint characteristic function.

HIGHER-ORDER STATISTICS

The second-, third-, and fourth-order cumulants of a zero-mean stationary random process $x(t)$ are

$$C_{1,x} = \mathsf{E}\{x(t)\} = 0$$

$$C_{2,x}(\tau) = \mathsf{E}\{x(t)x(t+\tau)\}$$

$$C_{3,x}(\tau_1,\tau_2) = \mathsf{E}\{x(t)x(t+\tau_1)x(t+\tau_2)\}$$

$$\begin{aligned} C_{4,x}(\tau_1,\tau_2,\tau_3) &= \mathsf{E}\{x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)\} + \\ &- C_{2,x}(\tau_1)C_{2,x}(\tau_2-\tau_3) - C_{2,x}(\tau_2)C_{2,x}(\tau_3-\tau_1) + \\ &- C_{2,x}(\tau_3)C_{2,x}(\tau_1-\tau_2) \end{aligned}$$

MUSIC (MULTIPLE SIGNAL CLASSIFICATION)

$$\mathbf{R}_x = \sum_{i=1}^{M} E\left\{ A_i A_i^* \right\} \boldsymbol{s}_i \boldsymbol{s}_i^T + \sigma_0^2 \mathbf{I}$$

N-*M* smallest eigenvalues of the correlation matrix (matrix dimension $N>M+1$) correspond to the noise subspace and *M* largest (all greater than σ_0^2) corresponds to the signal subspace.

$$\mathbf{R}_x = \mathbf{E}_{signal} \Lambda_{signal} \mathbf{E}_{signal}^{*T} + \mathbf{E}_{noise} \Lambda_{noise} \mathbf{E}_{noise}^{*T}$$

The squared magnitude of the projection of **w** onto the noise subspace is given by

$$\mathbf{P}_{noise} = \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T}$$

Each of the elements of the signal vector is orthogonal to the noise subspace.

The MUSIC pseudospectrum is defined as:

$$\widehat{P}\left(e^{j\omega}\right) = \left[\mathbf{w}^{*T} \mathbf{P}_{szumu} \mathbf{w}\right]^{-1} = \left[\mathbf{w}^{*T} \mathbf{E}_{szumu} \mathbf{E}_{szumu}^{*T} \mathbf{w}\right]^{-1}$$

It exhibits sharp peaks at the signal frequencies where $\mathbf{w}=\mathbf{s}_i$.

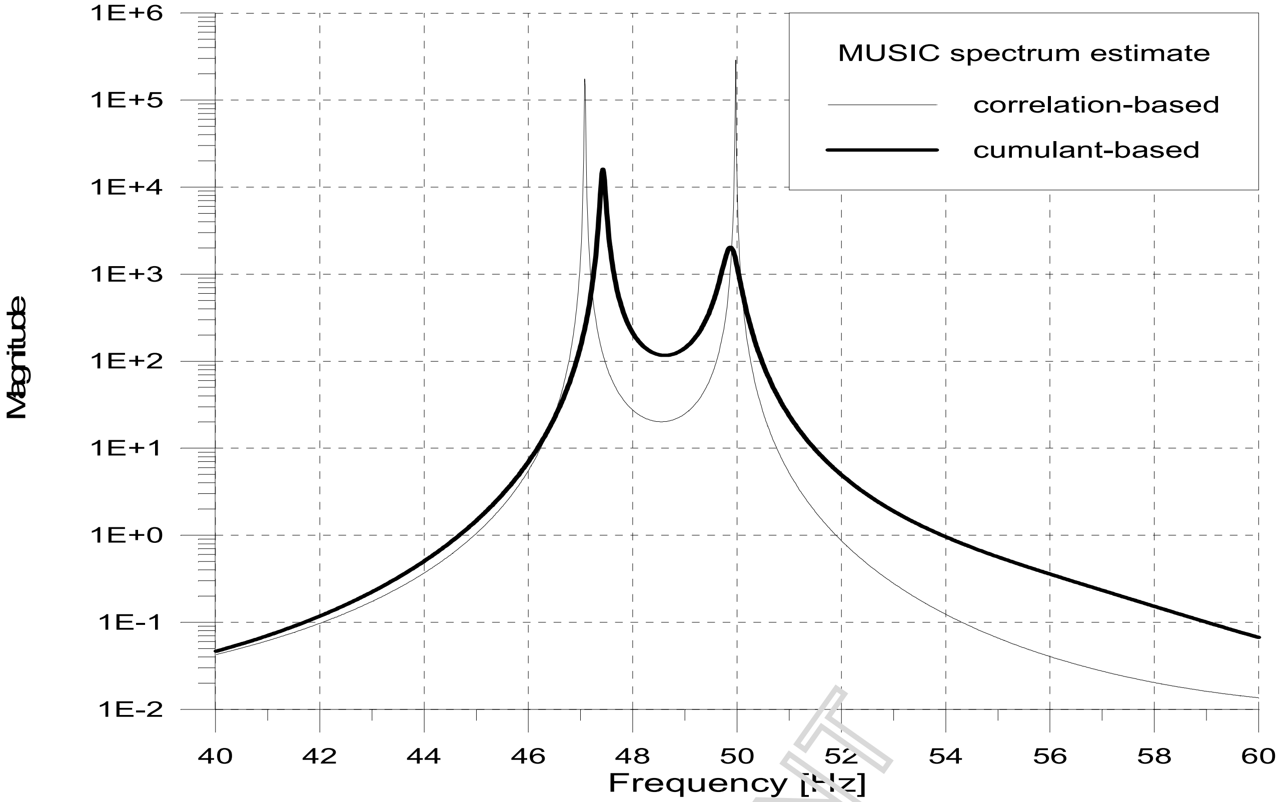
CUMULANT-BASED APPROACH

In the case of real harmonics

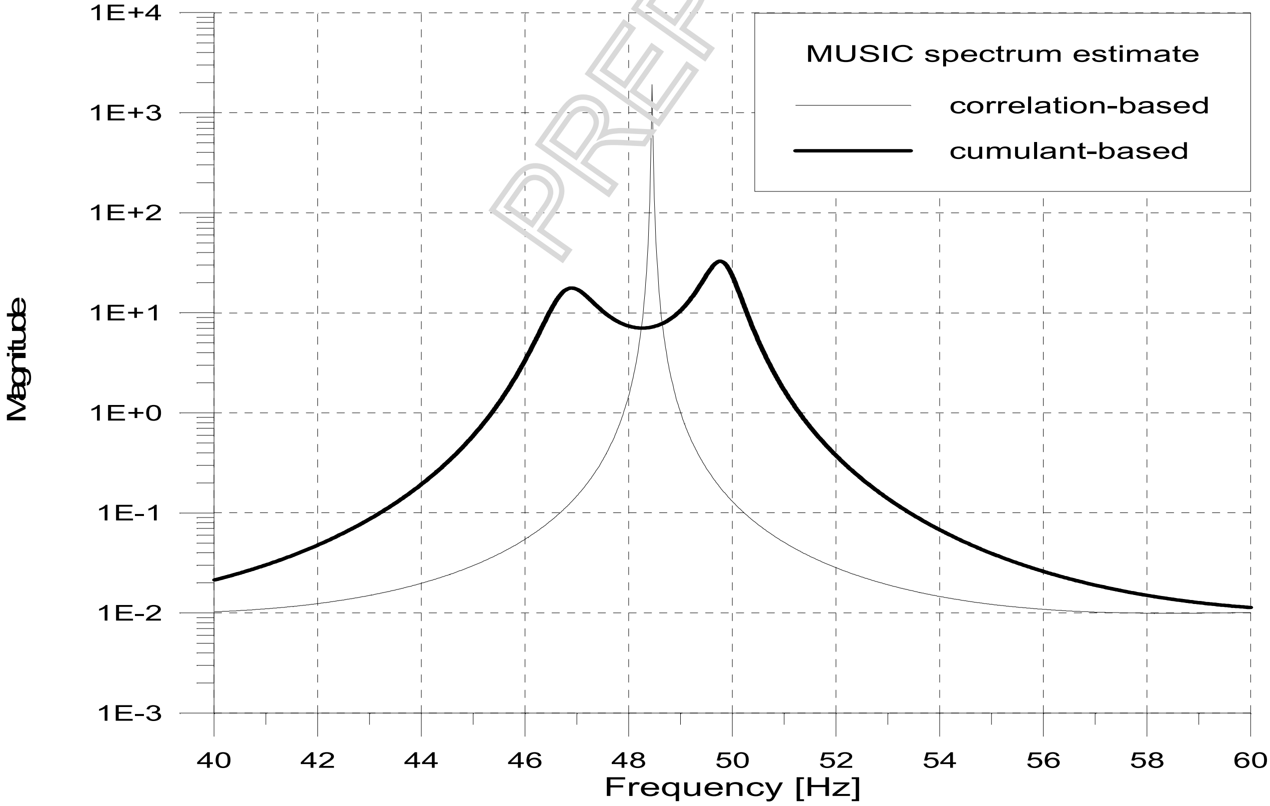
$$C_{4,x} = -\tfrac{1}{8}\sum_{k=1}^{p}\alpha_k^4\left[\cos\omega_k\left(\tau_1-\tau_2-\tau_3\right)+\cos\omega_k\left(\tau_2-\tau_3-\tau_1\right)+\right.$$
$$\left.+\cos\omega_k\left(\tau_2-\tau_3-\tau_1\right)\right]$$
$$C_{2,x} = \tfrac{1}{2}\sum_{k=1}^{p}\alpha_k^2\cos(\omega_k\tau)$$

If $x(n)$ is a sum of p real-valued sinusoids, then the diagonal slice of the fourth-order cumulant retains all of the pertinent information about the number of harmonics, their amplitudes and frequencies

$$C_{4,x}(\tau) = -\tfrac{3}{8}\sum_{k=1}^{p}\alpha_k^4\cos(\omega_k\tau)$$



MUSIC spectrum estimates in noiseless case.



MUSIC spectrum estimates in coloured noise case (SNR=0 dB).