# Instantaneous Frequency Calculation Using Wigner Distribution

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***Abstract*-This paper introduces two-dimensional methods of observation and diagnosis of non-stationary signals in electrical engineering as well as its further application in instantaneous frequency calculation. The investigations are carried out on two levels: firstly, two-dimensional representations are obtained applying Wigner Distribution, then local frequency moments are calculated to achieve one-dimensional characteristics of instantaneous frequency. It is shown that such characteristic preserves information about the nonstationarity in time domain and can be used for detection and duration of transient states.**

## I. INTRODUCTION

Representation of signals in time and frequency domain has been of interest in signal processing areas for many years, especially taking in the limelight time-varying non-stationary signals. The standard method for study time-varying signals is short-time Fourier transform (STFT). The spectrogram utilizes a short-time window whose length is chosen so that over the length of the window signal is stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the centre of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the nonstationarities occurring during this interval will be smeared in time and frequency [5],[6]. This inherent relationship between time and frequency resolution becomes more important when one is dealing with signals whose frequency content is changing rapidly. A time-frequency characterization of signals that would overcome above drawback became a major goal for signal processing areas [1],[2],[3],[4]. One of the mathematical proposition that represents above goal is Wigner Distribution (WD).

Observation of the signal in joint time-frequency planes is especially dedicated to signals which parameters change in time. One of the prominent example can be investigation of signal with frequency modulation. This work is aimed at application of time-frequency analysis in point of tracking the parameter which is called instantaneous frequency. In order to do this some definition of local frequency moments of the Wigner Distribution are introduced which stay in connection with characteristic of instantaneous frequency [1],[4]. Delivered derivation is supported by simulations concentrated on detection and duration of transient states.

## II. MATHEMATICAL BACKGROUND

The concept of the Wigner distribution could be consider as Fourier transform of instantaneous autocorrelation function [1]:

$$\mathrm{WD}_x(t,\omega)=\int_{-\infty}^{+\infty}\mathrm{x}\left(t+\frac{\tau}{2}\right)\mathrm{x}^*\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}\omega\tau}\mathrm{d}\tau \qquad (1)$$

Local frequency moments of Wigner Distribution are determined by considering WD as a function of frequency for fixed time. Normalized first local frequency moment is than given by [1]:

$$\overline{\Omega^1_{WD_x}}(t)=\frac{\int_{-\infty}^{+\infty}\omega\mathrm{WD_x}(t;\omega)\mathrm{d}\omega}{\int_{-\infty}^{+\infty}\mathrm{WD_x}(t;\omega)\mathrm{d}\omega} \qquad (2)$$

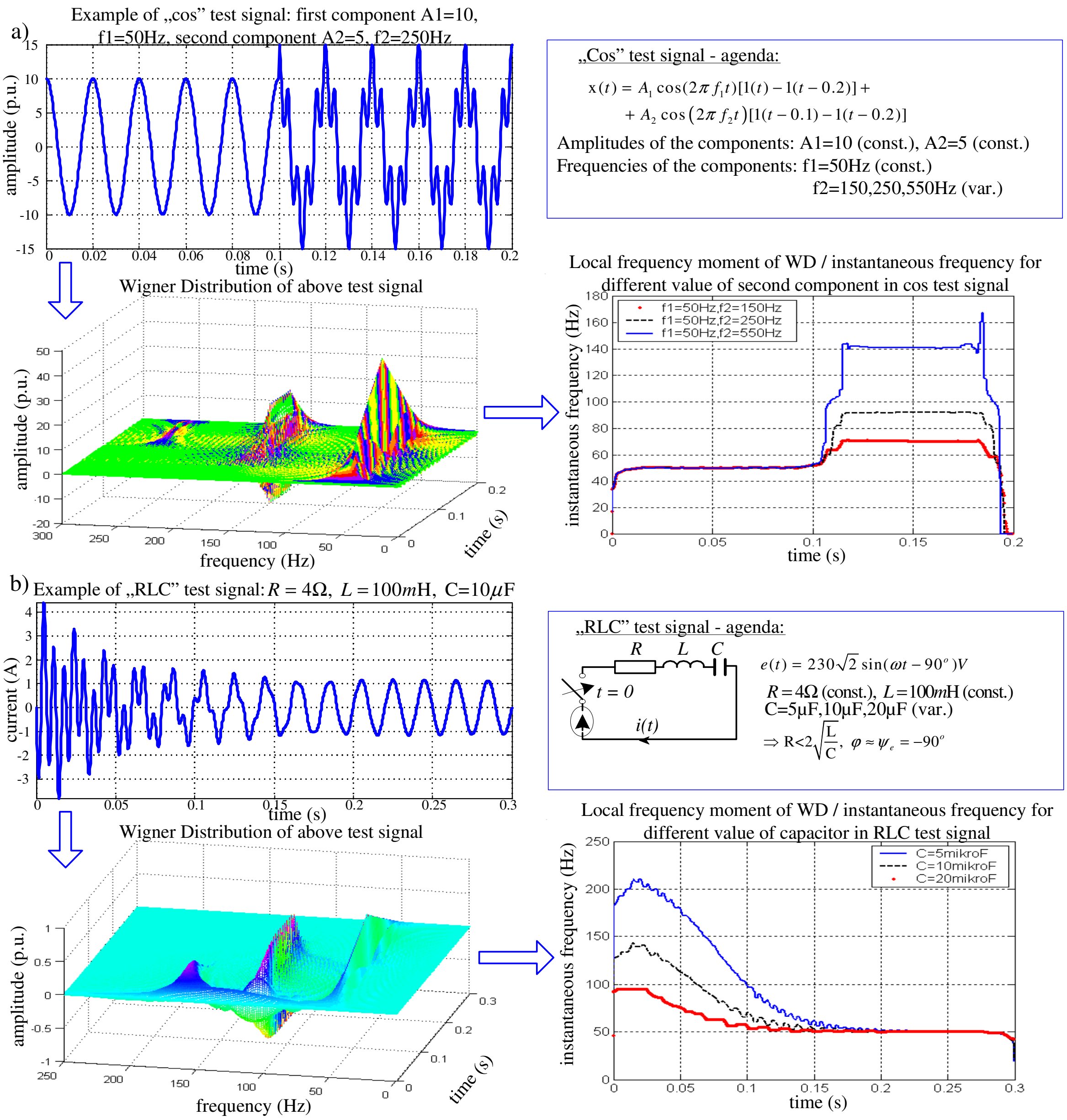
For complex signal $\mathrm{x}(t)=|\mathrm{x}(t)|\mathrm{e}^{\mathrm{j}\psi(t)}$ above equation is associated with instantaneous frequency *f*(t) defined as derivative of the phase [1],[4],[6]:

$$f(t)=\frac{1}{2\pi}\frac{d\varphi(t)}{dt}=\frac{1}{2\pi}\overline{\Omega^1_{WD_x}}(t) \qquad (3)$$

Thus, calculation of normalized first local frequency moment of WD can be then interpreted as a function which describes position of central point of the instantaneous spectrum for fixed time. It is worth emphasizing that following this interpretation, information about the frequency structure is lost, however information about the transient events is still preserved.

## III. SIMULATION AND CALCULATION

The character of Wigner distribution and its normalized local frequency moments has been tested for two simulated signals: sum of two cosine functions and transient signal in RLC branch. To avoid influences in instantaneous frequency characteristic median filter was additionally applied. When small order of the filter is used no influence on dynamic of curve is achieved. Observing Fig. 1a we can detect presence of higher component when the direction of the curve is moved forward higher frequencies region. Fig. 1b depicts shifting the position of central point of the instantaneous spectrum forwards input components as a results of decaying transient component. The duration time of the transient state can be also clearly characterized, however detailed information about the frequency or amplitude of the components is hidden.




Rys. 1. Particular steps of calculation of instantaneous frequency for "cos" (a) and "RLC" (b) test signals when Wigner Distribution is proposed

## IV. Summary

This paper is aimed at complex analysis of transient states using time-frequency analysis and its further application. Proposed alternative approach to classical spectrogram is the Wigner Distribution. Except obtained time-frequency view of investigated nonstationarity, we can recalculated obtained representation in order to achieve some additional parameters. Presented example concerns calculation of instantaneous frequency. It has been shown that normalized first order local frequency moment of Wigner Distribution leads to instantaneous frequency, which illustrates how the centre point of spectrum change in time. This characteristic can be applied in detection and duration time of transient state.

## References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