Adaptive estimation of wide range frequency changes for power generator protection and control purposes

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Abstract: A method and a detailed adaptive algorithm for wide-range frequency change measurement is presented. The method is based on a two-stage operating principle in which the results of a coarse frequency measurement are used for both orthogonal filter adjustment and modifications to the basic algorithm that is applied to fine frequency estimation. The adaptive features ensure good selectivity of the algorithm (in the sense of higher and subharmonic attenuation) within the range 5 to 80 Hz. Simulation results presented confirm good accuracy of frequency estimation in the presence of a standard noise level in the input signal, as well as satisfactory frequency tracking properties of the algorithm at relatively high frequency-changing rates.

1 Introduction

Most frequency estimating algorithms used in power protection systems are designed to accurately measure small frequency deviations since these may indicate seriously abnormal operating conditions of the power system. Limited to operating in a narrow frequency band, the algorithms are based on selective digital filters [1] which provide significant suppression of sub and higher harmonics in a measured signal, thus enabling reduction of errors in frequency estimation caused by noise. At the same time, due to the limited range of measured frequencies, the errors caused by varying gains of the digital filters either remain at a negligibly low level or can easily be corrected or compensated entirely [2].

For some applications, however, and for power generator protection and control in particular, accurate frequency measurements and tracking within a much wider frequency range (from zero to the nominal value and higher) are required. In such cases the contradictory requirements (selectivity and reduction of errors due to higher harmonics against accuracy and invariance of filter gains) are much more difficult to meet. To solve the problem some adaptive methods are proposed [3, 4] based on the use of the DFT with a variable data window or on the use of a batch of digital filters with flat (nonselective) frequency response within a number of small frequency subintervals.

In this paper a simple and effective method of adaptive frequency estimation is presented. The method employs the algorithm described in [2] whose selective frequency response is adjusted online to the measured frequency. Theoretical analysis of the method presented is illustrated by the results of a simulation study carried out for different scenarios of measured signal-frequency changes, and also in the presence of higher frequency noise at a level typical of that produced by power generators operating in normal and abnormal conditions.

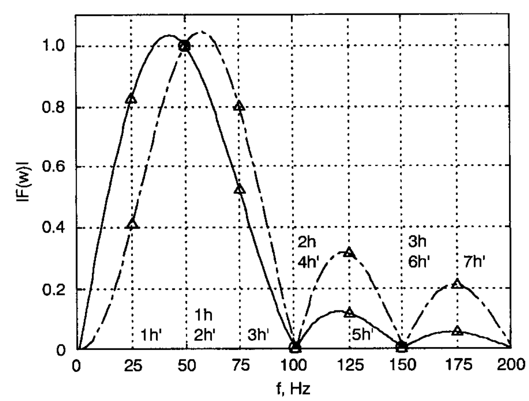

Fig. 1 *Frequency response of digital filters with filter gains for subsequent harmonics of fundamental frequencies 50 Hz and 25 Hz marked*
——— sine type
- - - - cosine type
○ harmonic *h*
△ harmonic *h'*

2 Principle of the method

To show that the standard frequency measuring algorithms are essentially useless for estimation of wide-range frequency changes consider the frequency responses of a typical pair of orthogonal cosine and sine filters shown in Fig. 1. If the filters are designed to measure the nominal frequency (50 Hz, $1h$, Fig. 1) then all higher harmonics ($2h$, $3h$, ..., Fig. 1) will be rejected. Now if the same filters are used to measure frequency 25 Hz (now considered as the fundamental $1h'$, Fig. 1) the filter gains for this frequency will be reduced by almost a factor of two and will change significantly for higher harmonics. The signal of nominal frequency

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(50 Hz, now the second harmonic $2h'$, Fig. 1) is doubled, the third harmonic ($3h'$) remains at the same level as the fundamental and only the fourth and higher even-harmonic signals are rejected. Thus the orthogonal filters considered can hardly be used for large frequency deviation measurements or for frequency-change tracking unless their characteristics are adapted adequately to the measured frequency values.

Adaptation of the orthogonal filter characteristics and the frequency measuring algorithm underlie the method presented. The basic idea of the method is illustrated in Fig. 2. First, the respective narrow frequency interval is determined within which the frequency of the measured signal is located by a coarse frequency measurement. During the coarse measurement the period of the measured signal is evaluated in terms of the number of samples, with an accuracy (or resolution) that depends on the sampling rate used. According to the result obtained the orthogonal filters are adjusted and their data windows modified to ensure optimal filtration of the measured signal. The fine frequency measurement is carried out by the basic algorithm whose properties are also adapted according to the coarse measurement results.

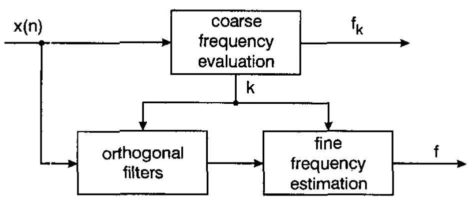

Fig.2 *Block diagram of adaptive frequency estimation algorithm*

2.1 Coarse frequency evaluation

For a measured signal described as

$$x(n) = X\cos(n\omega T + \varphi) \tag{1}$$

where ω is angular frequency and T is the sampling period, consider the function

$$h_{k,m}(\omega) = x(n-k)x(n-m) - x(n)x(n-k-m) \tag{2}$$

which after substituting eqn. 1 into eqn. 2 can be expressed as

$$h_{k,m}(\omega) = X^2 \sin(k\omega T)\sin(m\omega T) \tag{3}$$

Doubling the delay k in eqn. 2 (i.e. taking $2k$ instead of k) the similar form function $h_{2k,m}(\omega)$ can be derived which, if related to eqn. 3, results in the expression

$$\frac{h_{2k,m}(\omega)}{h_{k,m}(\omega)} = \frac{\sin 2k\omega T}{\sin k\omega T} = 2\cos k\omega T \tag{4}$$

(assuming that $\sin(k\omega T)\sin(m\omega T) \neq 0$) which is independent of the signal magnitude X. Expressing the left-hand side of eqn. 4 in terms of eqn. 2 the following relationship is obtained:

$$\cos k\omega T = \frac{1}{2}\frac{x(n-2k)x(n-m) - x(n)x(n-2k-m)}{x(n-k)x(n-m) - x(n)x(n-k-m)} \tag{5}$$

The exact value of frequency can be calculated from eqn. 5 by use of the arccos function. In coarse frequency estimation eqn. 5 is used to determine the approximate values of frequencies that correspond to the values of k satisfying the condition

$$\cos k\omega T = C \tag{6}$$

where C is a certain constant. The best sensitivity of estimation is obtained for $C = 0$ and then

$$\omega = \frac{\pi}{2kT} = \frac{\pi f_s}{2k} = \omega_k \tag{7}$$

where f_s is the sampling frequency and ω_k is the discrete value of angular frequency. The sample discrete values of estimated frequencies which correspond to some selected values of k and calculated for two different values of f_s are shown in Table 1. In practice, the values of measured frequency can rarely be equal to f_k. In most cases they are located within the interval $[f_k, f_{k+1}]$. Assuming that the actual angular frequency is equal to $\omega_k + \Delta\omega_k$ and the condition of eqn. 7 is satisfied, the following relationship can be derived:

$$\cos[(\omega_k + \Delta\omega_k)kT] = -\sin kT\Delta\omega_k \cong -kT(\omega - \omega_k) = -kT\omega + \frac{\pi}{2} \tag{8}$$

where $\Delta\omega_k$ represents the deviation of actual angular frequency ω from the nearest value of ω_k. This function is positive for $\omega < \omega_k$ and negative for $\omega > \omega_k$. Thus the criterion of sign can be used for coarse frequency estimation. The mechanism of such an estimation is illustrated in Fig. 3. If the initial value of frequency is

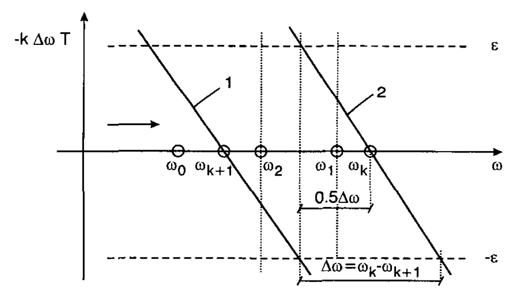

Fig.3 *Illustration of coarse frequency estimation principle*
——— Calculated from eqn. 8 for subsequent delays k and $k + 1$

ω_0 (the starting point of estimation) and the actual frequency to be estimated is ω_1 then k must be decreased until it reaches the value corresponding to ω_k at which the sign of eqn. 8 becomes negative. At this moment k

Table 1: Sample discrete values of frequency obtained for different values of k and different sampling frequencies f_s

a) f_s = 4 kHz															
k	101	100	99	...	41	40	39	...	21	20	19	...	14	13	12
f_k	9.9	10,0	10,1		24,4	25,0	25,6		47,6	50,0	52,6		71,4	76,9	83,3
b) f_s = 10 kHz															
k	251	250	249	...	101	100	99	...	51	50	49	...	31	30	29
f_k	9,96	10,0	10,04		24,7	25,0	25,3		49,02	50,0	51,02		80,6	83,3	86,2

is incremented by 1 and eqn. 8 is positive again. Thus, if the criterion of sign is used, the steady state of the coarse frequency estimation reveals a limit cycle in which estimates oscillate between ω_k and ω_{k+1} in all subsequent sampling instants.

To eliminate this drawback the criterion of value is used instead of sign. According to this criterion, the delay k is kept constant when the measured value of frequency (eqn. 5) is located within $[-\varepsilon, \varepsilon]$ interval, i.e. if the following condition is satisfied:

$$-\varepsilon \leq -kT(\omega - \omega_k) \leq \varepsilon \qquad (9)$$

or, using eqn. 7 in eqn. 9

$$-\varepsilon \leq -\frac{\pi}{2}\frac{(\omega - \omega_k)}{\omega_k} \leq \varepsilon \qquad (10)$$

The optimal value of ε (minimum changes of k, satisfactory resolution of estimation) is obtained when (Fig. 3)

$$\Delta\omega = \frac{\omega_k - \omega_{k+1}}{2} \qquad (11)$$

and then, using eqns. 7, 9 and 10

$$\varepsilon = \frac{\pi}{4(k+1)} \cong \frac{\pi\omega_k}{\omega_s} = \frac{\pi f_k}{f_s} \qquad (12)$$

The value of ε must be calculated for the greatest frequency to be estimated. For example, if the highest frequency to be estimated is 50Hz then for the sampling rate f_s = 4kHz, ε = 0.039.

2.2 Fine frequency estimation

For fine frequency estimation and tracking, an application of the algorithm described in [4] is proposed. The algorithm is based on orthogonal components of the measured signal $x(n)$ obtained by means of a pair of nonrecursive filters F_c, F_s (for instance, of cosine and sine type). The outputs of the filters are described by the equations

$$y_c(n) = |F_c(\omega)|X\cos(n\omega T + \varphi + \alpha(\omega)) \qquad (13a)$$

$$y_s(n) = |F_s(\omega)|X\sin(n\omega T + \varphi + \alpha(\omega)) \qquad (13b)$$

The algorithm uses current and delayed by k and $2k$ samples of orthogonal components according to the relation

$$f = \frac{1}{2\pi kT} \times \arccos\left(0.5\frac{y_s(n)y_c(n-2k) - y_c(n)y_s(n-2k)}{y_s(n)y_c(n-k) - y_c(n)y_s(n-k)}\right) \qquad (14)$$

or (in case when eqn. 7 is satisfied, i.e. for $k = \pi f_s/2\omega_k$) according to the approximate equation

$$f = f_1 + \Delta f \cong f_1\left\{1 - \frac{1}{\pi}\frac{y_s(n)y_c(n-2k) - y_c(n)y_s(n-2k)}{y_s(n)y_c(n-k) - y_c(n)y_s(n-k)}\right\} \qquad (15)$$

The distinguishing feature of the algorithm is that the results of frequency estimation depend neither on the measured signal magnitude nor on the sampling rate applied. Moreover, if the measured signal does not contain higher harmonics, the variable filter gains do not impair estimation accuracy over a very wide range of frequency change. Unfortunately, the latter property does not hold for higher harmonics in the input signal. However, fairly good accuracy of estimation (no worse than 0.01Hz at the standard level of noise) is obtained within the narrow frequency interval of ± 2Hz around the nominal value.

Remarkable extension of the accurate frequency estimation range of noisy signals has been obtained by use of the adaptive procedure presented. The value of the delay k in eqns. 14 and 15 is always set to equal a quarter of the coarsely estimated fundamental frequency period of the currently measured signal. The data window length of the orthogonal filters is extended (or compressed) to the value of $4k$ (full cycle of the currently measured signal). The filter coefficients (and the filter frequency response) are also modified so that they always cover the full cycle of sine/cosine functions. To avoid additional computation the filter's coefficient sets for the expected range of frequency changes (different filter window lengths) may be calculated offline and stored in the memory of the device (protective relay, control unit).

By comparison with the adaptive algorithm described in [5] the method presented seems to be simpler and more convenient in implementation since neither filter gain correction nor approximation of the measured signal derivative are required.

3 Simulation results

Operation of the algorithm was tested by use of the EMTP software package. The frequency of the measured signal (eqn. 1) was modulated in the following way:

a) step change from the nominal value of 50 to 40Hz

b) linear change from 5 to 80Hz with constant change rates of 20Hz/s and 1Hz/s

In all cases the simulation tests were carried out for input signals that were purely sinusoidal, and distorted by harmonics whose p.u. values were

— 2nd harmonic, 0.05p.u.

— 3rd harmonic, 0.15p.u.

— 4th harmonic, 0.05p.u.

The input signals were sampled at f_s = 4kHz (80 samples of signal in one period of the nominal 50Hz). The relative frequency deviation ε at which the value of delay k is changed was assumed to be equal to 0.04.

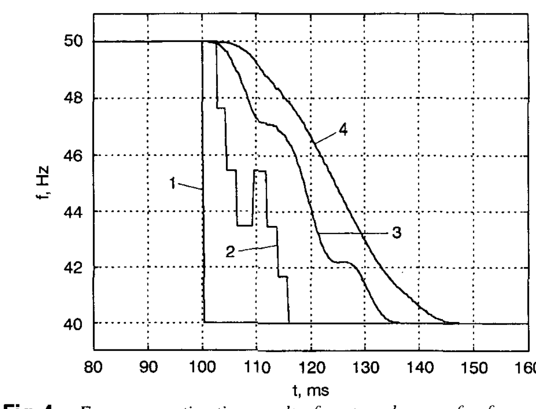

Fig. 4 *Frequency estimation results for step change of reference frequency from 50 to 40Hz with sinusoidal input signal*
1 reference frequency; 2 coarse estimate; 3 fine estimate; 4 fine estimate after half-cycle averaging

In Fig. 4 the coarse and fine frequency estimates (curves 2 and 3, respectively) obtained for the step frequency change of a purely sinusoidal input signal are shown. The coarse estimate reaches the steady-state value (which in this case coincides with the fine estimation result) in 15ms. At moments when the instantaneous frequency deviation exceeds the [$-\varepsilon$, $+\varepsilon$] interval (Fig. 5) the delay k is changed and both filter data windows and parameters and the fine frequency estimation algorithm (sample delays) are modified. The output of the fine estimation block monotonically reaches the steady-state value in approx. 35ms (the additional 20ms is introduced by full-cycle filtration). A smoother transition (curve 4) is possible if an extra half-cycle averaging filter is used at the output.

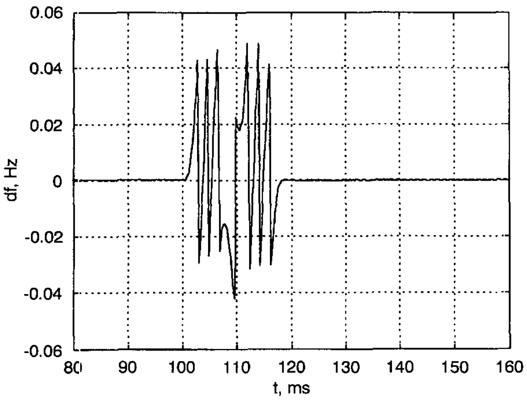

Fig. 5 *Frequency deviation results for step change of reference frequency from 50 to 40Hz with sinusoidal input signal*

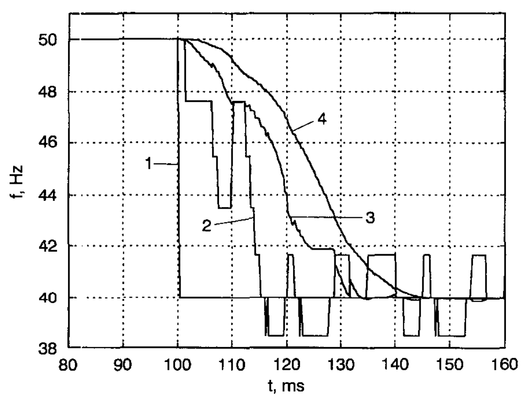

Fig. 6 *Frequency estimation results for step change of reference frequency from 50 to 40Hz with input signal distorted by higher harmonics*
Curves 1–4 as in Fig. 4

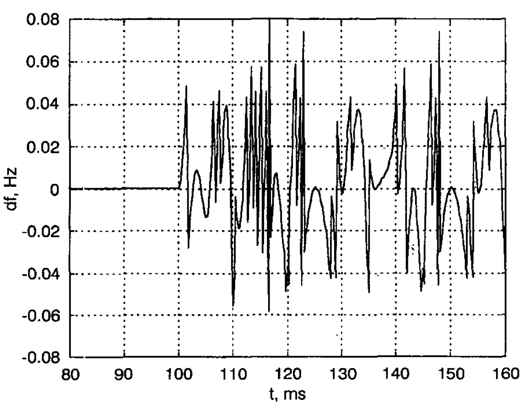

Fig. 7 *Frequency deviation results for step change of reference frequency from 50 to 40Hz with input signal distorted by higher harmonics*

Figs. 6 and 7 illustrate the algorithm operation for the step frequency change of the input signal that is distorted by harmonics. Now the switching of k is more frequent and despite the filter's adaptation, the higher harmonics are not rejected entirely. The resulting steady-state error of estimation is at the level of 0.2Hz and can be reduced to a value of 0.05Hz by additional averaging filtration of the output.

The frequency-tracking properties of the algorithm for an undistorted input signal that changes at the rate of 20Hz/s are shown in Figs. 8–10. The frequency changes are tracked with a time delay approximately equal to one period of the currently measured frequency (i.e. 100ms for f = 10Hz, 20ms for f = 50Hz, and so on). Enlarged fragments of Fig. 8, shown in Figs. 9 and 10, illustrate the mechanism of the coarse and fine estimation. Small oscillations in the output (curve 3; due to the continuously increasing frequency the filters never operate over the full data window) can be removed by simple averaging of results (curve 4). The presence of higher harmonics in the input signal (Figs. 11–13) practically does not affect the accuracy of estimation.

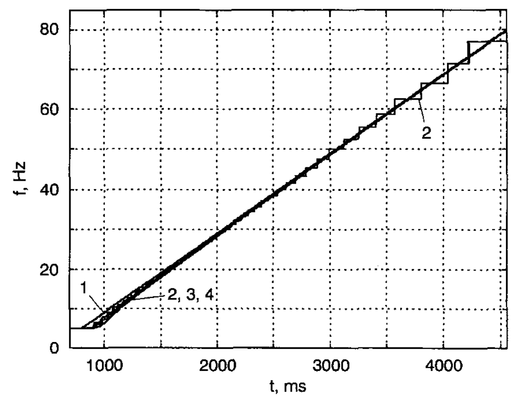

Fig. 8 *Results of frequency tracking for reference frequency changing from 5 to 80Hz at 20Hz/s for undistorted input signal*
Curves 1–4 as in Fig. 4

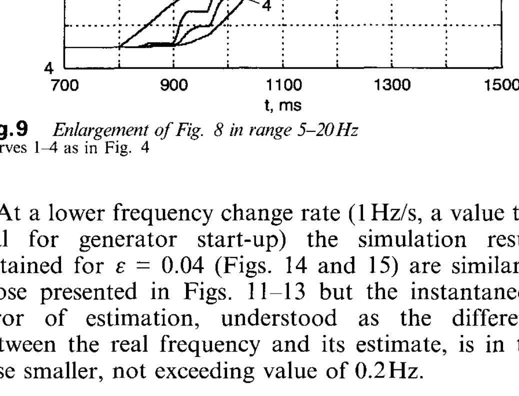

Fig. 9 *Enlargement of Fig. 8 in range 5–20Hz*
Curves 1–4 as in Fig. 4

At a lower frequency change rate (1Hz/s, a value typical for generator start-up) the simulation results obtained for ε = 0.04 (Figs. 14 and 15) are similar to those presented in Figs. 11–13 but the instantaneous error of estimation, understood as the difference between the real frequency and its estimate, is in this case smaller, not exceeding value of 0.2Hz.

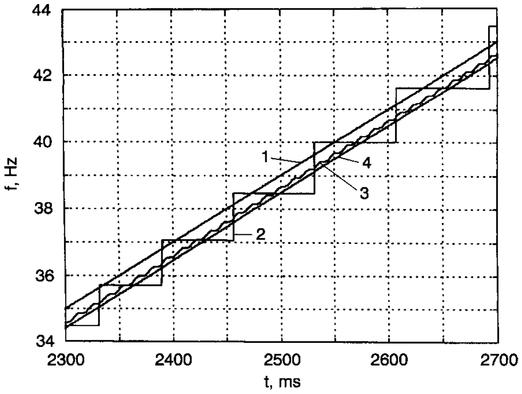

Fig. 10 *Enlargement of Fig. 8 in range 34–44 Hz*
Curves 1–4 as in Fig. 4

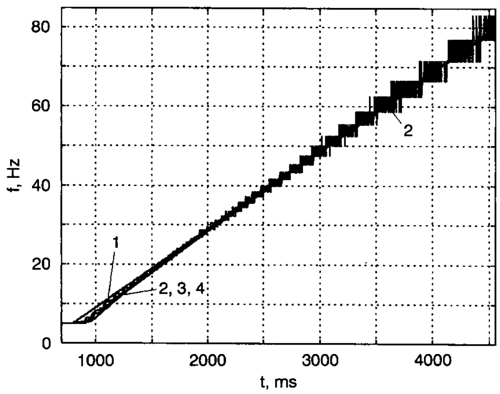

Fig. 11 *Results of frequency tracking for reference frequency changing from 5 to 80 Hz at 20 Hz/s for distorted input signal*
Curves 1–4 as in Fig. 4

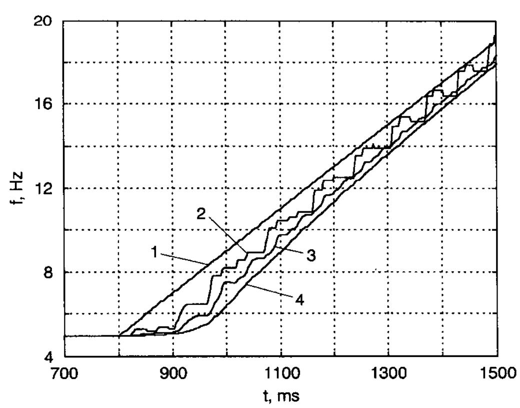

Fig. 12 *Enlargement of Fig. 11 in range 5–20 Hz*
Curves 1–4 as in Fig. 4

4 Conclusions

The results of simulation tests lead to the following conclusions:

- frequency estimation errors do not exceed 0.05 Hz even if the measured input signal is distorted by higher harmonics
- in the case of step changes of measured frequency the steady-state results of estimation are obtained after 1.0–1.5 cycles of input signal

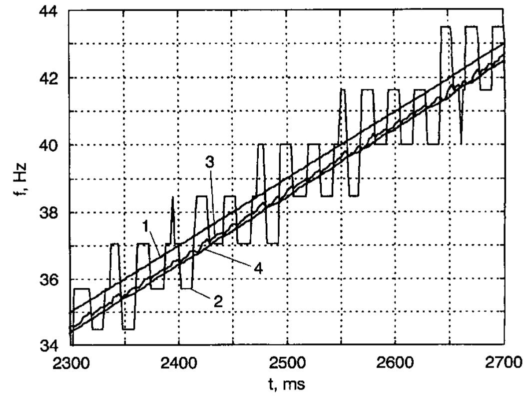

Fig. 13 *Enlargement of Fig. 11 in range 34–44 Hz*
Curves 1–4 as in Fig. 4

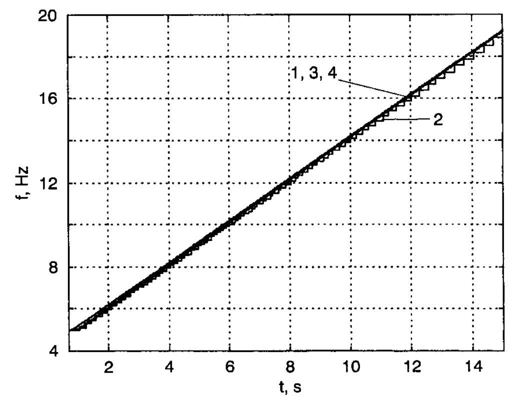

Fig. 14 *Results of frequency tracking for reference frequency changing at 1 Hz/s for distorted input signal in frequency range from 5 to 20 Hz*
Curves 1–4 as in Fig. 4

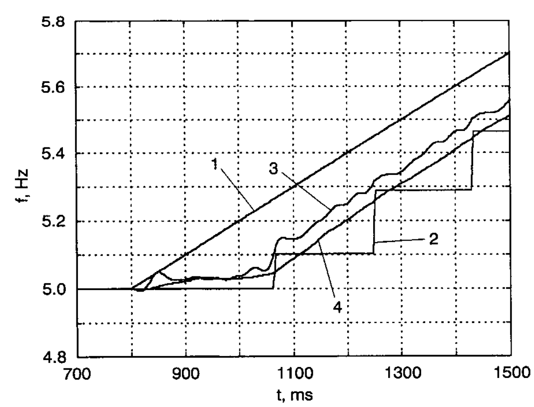

Fig. 15 *Enlargement of Fig. 14 in neighbourhood of 5 Hz*
Curves 1–4 as in Fig. 4

- transient properties of the algorithm are sufficient for tracking fast changes of power system frequency
- the total delay of frequency tracking does not exceed one cycle of instantaneous frequency value within the whole range of frequency changes.

Simulation tests also confirmed the distinguishing (as compared with other methods) properties of the algorithm, i.e. insensitivity to filter gains and input-signal amplitude variations combined with good immunity to higher harmonic noise. Thus the method seems to show promise for application in power generator protection and control systems. In our opinion the adaptive mechanism described can also be successfully used for other purposes such as adaptive estimation of power system impedance or current and voltage magnitudes.

5 References

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