Adaptive Measurement of Power System
Currents, Voltages and Impedances
in Off-nominal Frequency Conditions

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Abstract

In the paper the problem of basic power system electric quantities (current, voltage magnitudes, power and impedance) measurement in off-nominal frequency conditions is considered under assumption that the quantities are calculated from orthogonal components of input current and voltage phasors which are obtained by use of the nonrecursive digital filters. Adaptive algorithms which make the measurement insensitive to wide range frequency changes of the input signals are presented. The adaptive features of the algorithms are obtained by use of the frequency measuring algorithm and modification of orthogonal filters coefficients and data window length. Sample results of simulation studies which show the algorithm performance are presented and discussed in the paper.

Keywords: Digital protection, Filtering, Adaptation

1. Introduction

Most of the digital measurement algorithms used for power system relaying purposes is designed to operate at fixed nominal frequency of input signals since frequency deviations in normal power system operating conditions are very small allowing to get proper accuracy of estimation. There are situations, however, when protections have to operate in serious off-nominal frequency conditions (e.g. during start up of a generator) and if their measuring algorithms are not designed properly, they have sometimes to be interlocked [1] thus impairing protection system reliability. Similarly the relaying systems based on fault impedance estimation methods may maloperate if a fault occurs during off-nominal frequency conditions.

In digital protection systems the signals which the decision whether to trip the faulty power line is based on are usually determined from orthogonal components of current and voltage phasors obtained by use of nonrecursive digital filters. Selective frequency response of the filters and resulting selective features of estimators make them inadequate for application in off-nominal frequency conditions. Contradictory requirements to be selective (suppression of noise) and unselective (to have all-pass unity frequency response insensitive to frequency changes) call for adaptive features of the estimators. The idea of adaptive estimation has already been suggested in some papers [1-4].

In this paper a new approach to the problem of power system electric quantities measurement in off-nominal frequency conditions is presented. Adaptive features of the measurement are obtained by use of a frequency estimation procedure which determines the input signal frequency coarsely. According to the frequency measurement results the orthogonal filters are tuned up to the new frequency band by modification of their data window length and coefficients. The proposed method of adaptive measurement is simple and effective in use.

2. The method development

2.1 The measuring algorithms

For measurement of power system voltage and current magnitudes \(|U|, |I|\) as well as impedance \(Z\) and resistance \(R\), the following standard algorithms are usually used:

\[ |U|^2 = u_c(n) + u_s(n) \]  \hspace{1cm} (1)
\[ |I|^2 = i_c(n) + i_s(n) \]  \hspace{1cm} (2)
\[ |Z|^2 = \frac{u_c^2(n) + u_s^2(n)}{i_c(n) + i_s(n)} \]  \hspace{1cm} (3)
\[ R = \frac{u_c(n)i_c(n) + u_s(n)i_s(n)}{i_c(n) + i_s(n)} \]  \hspace{1cm} (4)
in which \( i_c, i_s, u_c, u_s \) are primary current and voltage phasors at the orthogonal filter inputs. These inputs are related to the respective filter (for instance full-wave Fourier cosine and sine ones). outputs \( i^e(n), u^e(n) \) by filter frequency responses, i.e.:

\[
\begin{align*}
  u^e_c(n) &= g^e_c(\omega)u_c(n) \\
  u^e_s(n) &= g^e_s(\omega)u_s(n)
\end{align*}
\]

with the same type of relation for the current signals, in which: \( g^e_c(\omega) \) and \( g^e_s(\omega) \) are the filters normalized gains (see Fig. 1) defined as:

\[
\begin{align*}
  g^e_c(\omega) &= \left| \frac{F^e_c(\omega)}{F^e_c(\omega_1)} \right| \\
  g^e_s(\omega) &= \left| \frac{F^e_s(\omega)}{F^e_s(\omega_1)} \right|
\end{align*}
\]

where: \( \left| F^e_c(\omega_1) \right|, \left| F^e_s(\omega_1) \right| \) are the filter gains at the filter design frequency \( \omega_1 \), usually equal to the nominal (50/60 Hz) fundamental frequency of power system and \( \omega \) is the actual input signal frequency.

Substituting (5) into (1-4) the frequency dependent relationships are obtained which show how measurement of respective quantities are affected by off-nominal frequency conditions. In case, for instance, of impedance measurement the respective relationship takes the form:

\[
|Z|^2 = \frac{g^e_c^{-1}(\omega)u^e_c(n)^2 + g^e_s^{-1}(\omega)u^e_s(n)^2}{g^e_c^{-1}(\omega)i^e_c(n)^2 + g^e_s^{-1}(\omega)i^e_s(n)^2}
\]

which shows that if \( g^e_c(\omega) \neq g^e_s(\omega) \neq 1 \) for \( \omega \neq \omega_1 \), see Fig. 1) the essential errors will appear in measurement of impedance (and the other quantities also).

In the paper a new class of algorithms is proposed whose derivation and particular properties of are presented in [2]. The basic algorithms are listed below:

\[
|U|^2 = \frac{1}{\sin(ka_0T)} u_c(n-k)u_c(n) - u_s(n-k)u_s(n)
\]

with \( |X|^2 \) being calculated in the same way, and

\[
|Z|^2 = \frac{u_c(n-k)u_s(n-k) - u_s(n-k)u_c(n)}{i_c(n-k)i_s(n) - i_s(n-k)i_c(n)}
\]

\[
R = \frac{i_c(n-k)u_s(n) - u_s(n-k)i_c(n)}{i_c(n-k)i_c(n) - i_s(n-k)i_s(n)}
\]

\[
(X)^2 = (Z)^2 - (R)^2
\]

in which \( k \) is an arbitrary delay with an optimal value being equal to \( N/4 \) (a quarter of filters data window length).

An interesting property of the algorithms (9-11) is that they enable to make the impedance \( Z \), as well as resistance \( R \) and reactance \( X \) measurement totally independent of input signal frequency. Substitution of (5) into (9-10) results in cancellation of filter gains so that in case of \( R \) measurement, for instance:

\[
R = \frac{g^e_c^{-1}(\omega)g^e_s^{-1}(\omega)\left[i^e_c(n-k)u^e_s(n) - u^e_s(n-k)i^e_c(n)\right]}{g^e_c^{-1}(\omega)g^e_s^{-1}(\omega)\left[i^e_c(n-k)i^e_s(n) - i^e_s(n-k)i^e_c(n)\right]}
\]

so: \( R = R^F, Z = Z^F \) and \( X = X^F \)

for any frequency. Unfortunately this property does not hold in case of \( U \) and \( I \) measurements. Also the measurement of impedance and resistance may become frequency dependent if orthogonal components of currents and voltages in (9,10) are linear combinations of primary power system voltages and currents (like in case of symmetrical components calculation [6]). Thus the effective application of algorithms (8-10) to measurement of these quantities in off-nominal conditions calls for adaptation of filters and algorithm which can be achieved by:

- estimation of the actual frequency of input signals,
- modification of the orthogonal filters i.e. their data windows and frequency responses to the actual frequency,
- correction of coefficients \( \sin(ka_0T) \) and delays in algorithm (8).

### 2.2 Adaptive solution

In order to remove or minimize estimation errors it is necessary to adapt the orthogonal filters to the actual frequency of the signals. It is realized by the following
procedure:
- determine the number of samples in one period of input signals \( N_n \),
- set the filter data window length to \( N_m \) and modify filter coefficients (i.e. the filter frequency response).

Thus the procedure of adaptive measurement can be represented by the block diagram which is shown in Fig.2. Each of phase voltages and currents is applied to a pair of orthogonal filters. The filters are modified according to actual frequency of signals. Outputs of the filters are orthogonal components of phase voltages and currents used for further measurements.

2.3 Coarse frequency estimation

There is a large number of methods which can be applied to coarse frequency estimation \([1,5,6,7,8]\). One of them which was developed by the authors \([5]\) the main idea of which is presented in the paper.

According to the method the following function (written here for an arbitrary input voltage signal) is calculated \([2,5]\):

\[
\cos(k\omega_n T) = 0.5 \frac{U(n-2k)U(n-d)-U(n)U(n-2k-d)}{U(n-k)U(n-d)-U(n)U(n-k-d)} \quad (13)
\]

where: \( k = N_m/4 \) and \( N_m \) is the number of samples in one cycle of the coarsely estimated frequency \( \omega_n \), \( d \) is a certain arbitrary time delay \((d>0)\).

It can be shown that the function (13) is proportional to the frequency deviation, i.e.: \( \cos(k\omega_n T) \approx -kT\omega_n \). The value of \( k \) is then updated according to the procedure:

\[
\begin{align*}
& \begin{cases}
- kT\omega_n < -\delta \\
- kT\omega_n > \delta
\end{cases} & \text{then} & \begin{cases}
\text{increment } k \text{ by } 1 \\
\text{decrement } k \text{ by } 1
\end{cases} (14)
\end{align*}
\]

where \( \delta \) is a discrimination threshold assumed for the expected range of frequency changes.

According to the result of the coarse frequency estimation the orthogonal filters are adjusted and their data windows modified to ensure adequate gains and the optimal filtration of the measured signal. The delay \( k \) in (8-10) is set to the value determined by the coarse frequency estimation procedure (14) while the data window length of the orthogonal filters is always extended (or compressed) to the value of \( N_m = 4k \) (full cycle of the actual frequency).

2.4 Evaluation of adaptive orthogonal filter errors

Having the value of \( N_m \) determined the outputs of modified orthogonal filters can be calculated from the filter equations which in case of cosine and sine filters are given by:

\[
y_c(n) = \sum_{k=0}^{N_m/2} w_c(k)y(n-k) \\
y_s(n) = \sum_{k=0}^{N_m/2} w_s(k)y(n-k)
\]

where:

\[
w_c(k) = \cos[2\pi(k + 0.5)/N_m] \\
w_s(k) = \sin[2\pi(k + 0.5)/N_m]
\]

As the frequency \( \omega_n \) corresponding to \( N_m \) is determined by the relation:

\[
\omega_n = \omega_s / N_m
\]

where \( \omega_s \) is the sampling frequency, then, for \( \omega_n \) being now a new filter design frequency \((\omega_1 = \omega_n)\), the respective filter gains at \( \omega_n \) are:

\[
F_c(\omega_n) = F_s(\omega_n) = N_m / 2
\]

Since the frequency is measured coarsely \( \omega_n \) represents the approximate value of actual frequency \( \omega \) (\( \omega_n = a\omega \) only in particular cases when \( a = a_0 \) with \( a \) being any integer greater than 2). As a result the normalized filter gains (6) are only approximately equal to one with errors \( \epsilon_c, \epsilon_s \) which can be defined as:

![Fig.2. Block diagram of proposed adaptive procedure](image-url)
The errors of filter gain settings versus sampling frequency.

\[ E_c = g_c (\omega_m + \Delta \omega) - 1 = \frac{2}{N_m} F_c (\omega_m + \Delta \omega) - 1 \]  
(21)

\[ E_s = g_s (\omega_m + \Delta \omega) - 1 = \frac{2}{N_m} F_s (\omega_m + \Delta \omega) - 1 \]  
(22)

To calculate \( E_c \) and \( E_s \), the frequency responses of cosine and sine filters can be used and the resulting equations have the form:

\[ E_c = \frac{2 \sin(\alpha) \cos(\beta)}{\sin(\gamma)} - 1 \]  
(23)

\[ E_s = \frac{2 \sin(\beta) \cos(\alpha)}{\sin(\gamma)} - 1 \]  
(24)

If the filters are tuned to a certain design frequency \( \omega_m \) with data window being set to \( N_m = 4k \) then, since the smallest possible change of \( k \) in the coarse frequency estimator is equal to \( \pm 1 \), then the new lower (with respect to \( \omega_m \)) design frequency \( \omega_m^- \) and the higher one \( \omega_m^+ \), to which the filters can be tuned are:

\[ \omega_m^- = \frac{\omega_m}{4(k+1)} = \frac{\omega_m}{N_m + 4} \]  
(25)

\[ \omega_m^+ = \frac{\omega_m}{4(k-1)} = \frac{\omega_m}{N_m - 4} \]  
(26)

Thus assuming that the normalized filter gain errors \( E_{c-}, E_{s-} \) and \( E_{c+}, E_{s+} \) are calculated from (23) and (24) for the center frequencies from the intervals \( [\omega_m^-, \omega_m] \) and \( [\omega_m, \omega_m^+] \), respectively, the values of \( \alpha, \beta \) and \( \gamma \) can be calculated from the following relationships:

\[ \alpha = \pi \left( \frac{N_m + 4}{N_m} \right) \]  
\[ \beta = \frac{\pi}{N_m} \]  
\[ \gamma = \frac{2\pi}{N_m} \]  
(27)

The plots of the errors \( E_{c-}, E_{s-} \) and \( E_{c+}, E_{s+} \) versus sampling frequency \( f_s = \omega_m / 2\pi \) and calculated for \( f_m = 50 \text{Hz} \) are shown in Fig.3. The error decrease when sampling frequency increases and reach the values of one percent for \( f_s \) equal approximately to 5 kHz. There is another way of measurement accuracy improvement which can be used at the cost of increased computational burden. Instead of coarse the fine frequency measurement procedure can be applied [2,5] and, according to the results obtained, the actual normalized gains (17,18) and (6) of filters can be determined.

3. Samples of simulation study results

The proposed adaptive method was used for measurement of current, voltage and impedance symmetrical components in faulty power system. Extensive simulation tests have been carried out for input signals generated with use of EMTP and MATLAB.
programs. The three-phase voltage and current signals (at sampling rate of 4 kHz) were delivered to the model of the measurement block at the output of which the values of symmetrical components magnitudes as well as impedance components were obtained. The six pairs of nonrecursive full-cycle cosine and sine filters were used to obtain the orthogonal components of phase voltages and currents. The filter parameters (window lengths, gain values) as well as parameters of signal delaying in measurement equations were changed according to the coarse frequency estimate. The latter one was calculated on the basis of a selected voltage signal.

In Fig.4 the measurement results of symmetrical sequence voltages magnitude obtained by use of the algorithm (8) are shown. When the frequency adaptation procedure is switched off (Fig.4a) correct magnitude values (1.0 p.u.) are obtained only for signals having the nominal frequency of 50 Hz. When the adaptation procedure is active (Fig.4b) the values of estimated magnitude are still perfect for nominal frequency of 50 Hz and almost accurate for the other frequencies.

In Fig.5 the example results of symmetrical resistance components are presented. As it was mentioned earlier

![Fig.5. Frequency properties of zero, positive and negative sequence resistance measurement. a) without adaptation, b) with adaptation; measuring algorithm (10)](image)

(see discussion of eqn.(12)) the full frequency independence of measurement without adaptation can be obtained for zero sequence resistance ($R_0$) only (Fig.5a) while the positive and negative sequence resistance measurement change remarkably. When the adaptation procedure is used (Fig.5b) a wide range frequency independence of all symmetrical resistance components is obtained. Similar results are achieved in case of symmetrical impedance and reactance components measurement.

Transient properties of the algorithms considered are illustrated by Fig.6 in which the results of zero sequence voltage magnitude calculation obtained by use of the estimators (1) and (8) are shown. When the frequency adaptation procedure is switched off (Fig.6a) correct magnitude values (1.0 p.u.) are obtained only for signals having the nominal frequency of 50 Hz (curves 1 and 2). The transient state observed for first 20-25 ms is an effect of measurement initialisation. In case when the signal frequency differs from 50 Hz both algorithms deliver false values of measured magnitude with an error fluctuating or constant (curves 3, 4) depending on the applied estimator. When the adaptation procedure is
active (Fig. 6b) the values of estimated magnitude are still satisfactory for nominal frequency and almost accurate for other frequencies. Curves 3 and 4 obtained for \( f = 30 \) Hz reveal longer transient state of estimation resulting from the fact that the filter window lengths have been adequately changed to match the full cycle of currently estimated signal frequency.

The impact of higher harmonics which may appear in the input signal on accuracy and dynamics of zero sequence impedance measurement is shown in Fig. 7. In case of undistorted signal (Fig. 7a) the algorithm (10) ensures accurate resistance estimation in relatively short time (25 ms), also when the adaptation procedure is inactive. Performance of the standard algorithm (4) is rather poor (+22% measurement error). In case of contaminated input signal (the 2nd, 3rd and 4th harmonic of 0.15 p.u. magnitude each) the algorithm (10) requires adaptation (Fig. 7b, curves 2, 3) which reduces the steady state error to the value of 1.5% at the cost of extended transient state (approx. 75 ms).

4. Conclusions

In the paper an independent of frequency method of power system currents, voltages and impedance is presented. The method is based on algorithms of increased immunity to input signals frequency changes and employs an adaptive technique which enables on-line modification of digital filters frequency response and parameters of the algorithms used. Remarkably high insensitivity of measurement to frequency changes can be obtained by use of algorithms (9-11) for calculation of impedance components. The presented results of simulation analysis with use of EMTP and MATLAB software packages confirmed good accuracy of symmetrical components estimation which does not exceed 1.5% over 10 Hz - 90 Hz frequency range at the 5 kHz sampling rate of input signals. The transient performance of the algorithms presented is also satisfactory (1.5 cycle response time to a step change of frequency) thus making them suitable for use in transient off-nominal conditions encountered in power system.

5. References


Fig. 7. Time response of zero sequence resistance estimator in case of sudden frequency drop (50 to 40 Hz at \( t = 100 \) ms) and input signals: a) undistorted, b) distorted by noise; estimators: 1 - eqn.(4), 2 - eqn.(10), 3 - eqn.(10) with adaptation


