Analysis of non-stationary electric signals using the S-transform

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Abstract

Purpose – The purpose of this paper is to compare the accuracy of tracking the amplitude and frequency changes of non-stationary electric signals.

Design/methodology/approach – Short-time fourier transform (STFT) and S-transform algorithms were applied to analyze non-stationary signals originating from switching of capacitor banks in a power system.

Findings – The S-transform showed possibilities of sharp localization of basic component, and allowed improvement of tracking dynamism the transient components in comparison to STFT.

Practical implications – S-transform is a better tool for the analysis non-stationary waveforms in power systems and its properties can be used for diagnostic and power quality applications.

Originality/value – The dynamic tracking of the changes in time and frequency of real-like signals originating from a power system were investigated in this paper.

Keywords Electric power transmission, Fourier transforms, Electric power systems

Paper type Research paper

Introduction

The term of non-stationarity in signal processing is often used to describe a process in which the spectrum is changing with time. In the case of signals encountered in electric power systems non-stationarity is more often the rule than exception.

Contemporary power systems, which contain a considerable number of non-linear loads, require advanced methods of spectral analysis for their investigation and control. Fourier-based spectral methods are useful in stationary signal analysis but insufficient in the case of numerous real-life problems when signal content changes in time. Non-stationary signals are usually analyzed with short-time fourier transform (STFT). The method presented in this paper employs recently introduced S-transform which is an important development of STFT with improved properties.

In the electrical engineering literature the S-transform is not often considered. Lee and Dash (2003) proposes a neural network-based classification of power quality disturbances with S-transform as pre-processing stage. In Reddy *et al.* (2004) the comparison between S-transform and wavelets was done on the examples of basic power quality disturbances' simulations. Sikorski and Wozniak (2006) showed a comprehensive evaluation of many time-frequency transforms, concluding the superiority of S-transform in many investigated problems.

Presented results show that S-transform allows tracking changes in amplitude and frequency with better precision than STFT. Possible applications in diagnosis and power quality evaluation are also targeted.

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Short-time fourier transform and S-transform

The STFT is the classical method of time-frequency analysis. Investigated signal $x(t)$ is multiplied with an analysis window $\gamma(\tau - t)$, and subsequently, the fourier transform of the windowed signal is calculated (Pinnegar and Mansinha, 2003; Mertins, 1999; Quian and Chen, 1996):

$$\mathrm{STFT}(\tau, f) = \int_{-\infty}^{+\infty} x(t)\gamma(\tau - t)e^{-i2\pi ft}\,\mathrm{d}t \tag{1}$$

where: t and f denote time and frequency, and τ gives the position of γ on the time-axis.

The action of this window is to localize the spectrum in time, and therefore the resulting spectrum is the "local spectrum." This localizing window moves on the time axis to produce local spectra for the entire range of time. Although this approach gives time localization of the spectrum, the STFT is not always a sufficient solution. The width of analysis window is fixed, which causes fixed time-frequency resolution for all spectral components (low- and high-frequency components). This is a fundamental limitation of STFT. Non-stationary signals characterized by wide range of frequency spectrum with transient and sub-harmonic components are difficult to analyze with STFT (Allen and Mills, 2004).

As the STFT does not track the signal dynamics properly due to above limitations a new S-transform was introduced in (Stockwell *et al.*, 1996). The S-transform is a time-frequency spectral localization method, similar to the STFT and continuous wavelets. The S-transform is conceptually a hybrid of STFT and wavelet analysis, bridges the gap between them, containing elements of both but having its own characteristic properties. Just like the STFT, the S-transform uses a window to localize the complex Fourier sinusoid, but unlike the STFT, the width and height of the window scale with frequency in analogy with wavelets.

S-transform employs a moving and scalable localizing Gaussian window. It combines a frequency dependent resolution with simultaneous localizing the real and imaginary spectra. The basis functions for the S-transform are Gaussian modulated cosinusoids whose width varies inversely with the frequency.

The S-transform, introduced by Stockwell *et al.* (1996) is defined by the general equation:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t)g(\tau - t, f)e^{-i2\pi ft}\,\mathrm{d}t \tag{2}$$

where $g(\tau, f)$ is a window function. Window function is a modulated Gaussian function, expressed by:

$$g(\tau, f) = \frac{|f|}{\sqrt{2\pi}}\, e^{-(t^2/2\sigma^2)} \tag{3}$$

where σ is defined as:

$$\sigma = \frac{1}{|f|} \tag{4}$$

Finally, the general formula in equation (2) takes the form:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t)\frac{|f|}{\sqrt{2\pi}}\, e^{-((\tau - t)^2 f^2/2)} e^{-i2\pi ft}\mathrm{d}t \tag{5}$$

where f represents frequency and t, τ represent time.

The above derivation of the definition of S-transform originates from the STFT idea. However, the S-transform is related to the wavelet transform, as well.

Continuous wavelet transform $W(\tau, d)$ of the function $h(t)$ is defined by:

$$W(\tau, d) = \int_{-\infty}^{\infty} h(t) w(t - \tau, d) \mathrm{d}t \tag{6}$$

As previously stated, the S-transform can be also presented as continuous wavelets transform using a specific mother wavelet multiplied by a phase coefficient:

$$S(\tau, f) = e^{i2\pi f \tau} W(\tau, d) \mathrm{d}t \tag{7}$$

The mother wavelet $\psi(t, d)$ is defined as:

$$\psi(t, f) = \frac{|f|}{\sqrt{2\pi}} e^{-(t^2 f^2/2)} e^{-i2\pi ft} \tag{8}$$

where the coefficient d represents the inverse of the frequency.

Taking into account the above considerations, we can obtain the general formula of the S-transform as in equation (2).

The equation (2) assumes that the width of the window function is inversely proportional to the local frequency. We can also adjust the resolution of the S-transform by introducing a modified coefficient σ:

$$\sigma = \frac{k}{\sqrt{|f|}} \tag{9}$$

Then, the equation (5) takes the form:

$$S(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{k\sqrt{2\pi}} e^{-((\tau - t)^2 f^2/2k^2)} e^{-i2\pi ft} \mathrm{d}t \tag{10}$$

where the coefficient k controls the resolution in such way that if $k > 1$ the resolution in frequency increases and when $k < 1$ the resolution in time increases.

The S-transform is a linear transform of the signal $h(t)$. The additive noise can be modeled as follows: $h_{\text{noisy}}(t) = h(t) + \eta(t)$. The S-transform of a noisy signal represents a sum of transformed signal and transformed noise, as follows:

$$S\{h_{\text{noisy}}(t)\} = S\{h(t)\} + S\{\eta(t)\} \tag{11}$$

The S-transform output is a complex matrix, where the rows correspond to frequencies and the columns to time. Each column thus represents a "local spectrum" for that point in time. Since $S(\tau, f)$ is complex valued, in practice, usually the module $|S(\tau, f)|$ is plotted and this gives time-frequency S spectrum. The S-transform outperforms the STFT in that it has a better resolution in phase space (i.e. a more narrow time window for higher frequencies), giving a fundamentally more sound time frequency representation (Lee and Dash, 2003).

Investigations

The investigated signal is characteristic for current waveform during subsequent switching of two capacitor banks (Figure 1). One-phase diagram of simulated circuit is shown in Figure 2.

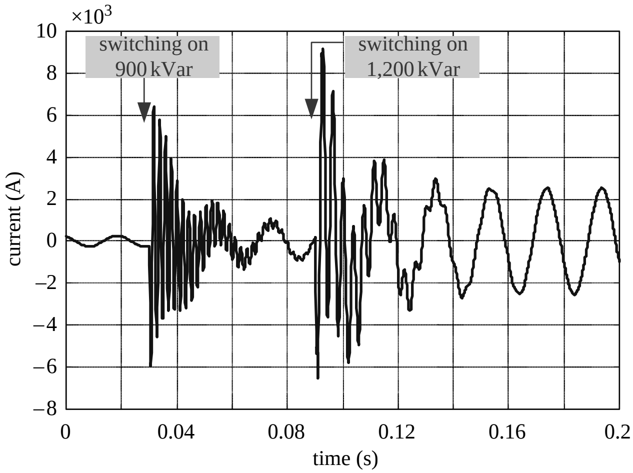

Figure 1.
Current signal measured in the simulated distribution system from Figure 1

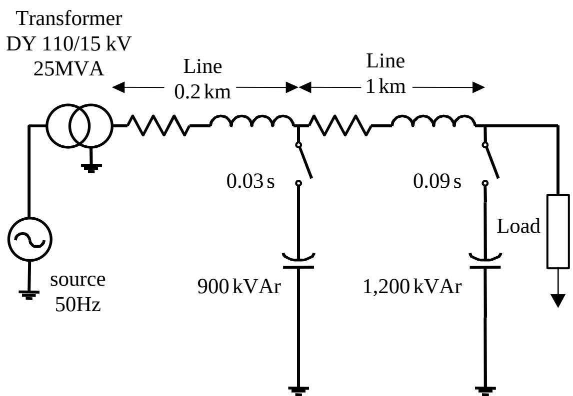

Figure 2.
One-phase diagram of simulated distribution system

The first capacitor bank (900 kVar), installed 0.2 km from the station (110/15 kV, 25 MVA), was switched on at time 0.03 s. Second bank (1,200 kVar), installed 1.2 km from the station, was switched on at time 0.09 s. Measured signal consist of basic harmonic 50 Hz and transient components 270, 475 Hz, appearing after switching operations. The signal was investigated using the STFT and S-transforms (Figure 3).

Classical spectrogram allowed to detect basic component 50 Hz and transient components 270, 475 Hz, appearing after switching operations. In order to preserve sufficient time resolution the Hamming window with width equal to two periods of basic component was applied as in (Sikorski and Wozniak, 2006).

Applying S-transform with moving scalable Gaussian window gives possibilities to adapt time-frequency resolution to analyzed phenomena. Figure 3 shows time-frequency plane with detected basic component 50 Hz and two transient components 475 and 270 Hz using S-transform (a) and STFT (b).

Figure 4 shows comparison of STFT and S-transform of 475 Hz transient component, obtained as a cross-section of Figure 3(a) this frequency.

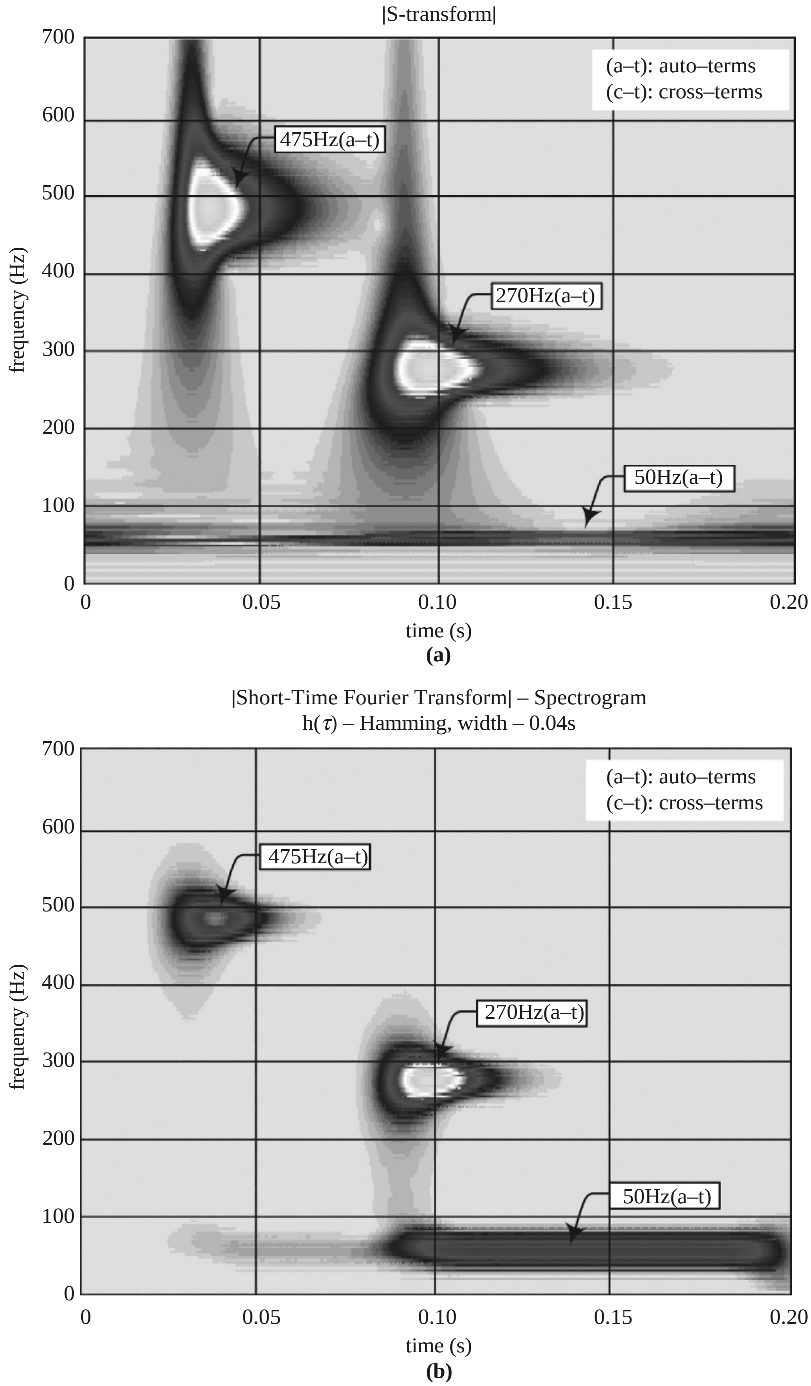

Figure 3.
Time-varying spectrum obtained using S-transform (a) and STFT (b)

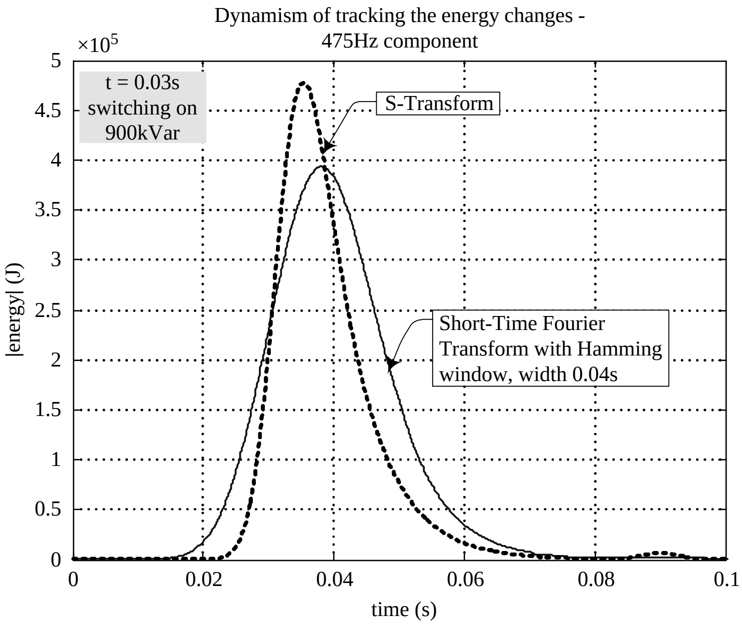

Figure 4.
Comparison of STFT and S-transform of 475 Hz transient component

Conclusions

In performed experiments the S-transform showed possibilities of sharp localization of basic component, and allowed improvement of tracking dynamism of the transient components in comparison to STFT.

It was necessary to adjust the *k* coefficient to obtain the optimal representation for a given problem and adjust in this way the multi-resolution property of the S-transform. The same advantageous tracking capability showed the S-transform when analyzing the time-varying multi-component signal.

Performed experiments allow concluding that the S-transform is a better tool for the analysis non-stationary waveforms in power systems and its properties can be used for diagnostic and power quality applications.

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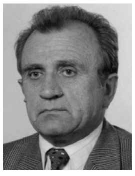

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