

# A development of a solar cell problem. Some further description

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*Abstract*— A new concept of a space charge transport between the two electrodes is presented.

It was found that generation processes could be described in terms of a voltage parameter  $V_{10}$  and of a field parameter  $E_{10}$ .

The parameters  $V_{10}$  and  $E_{10}$  correspond to the total kinetic energy of the rotators and oscillators. Using a concept for allowed electron- hole transitions, the effect of the light on the electric field distribution and on the shape of current – voltage characteristic is determined. For a space charge distribution some new singular solutions are obtained. Also, some new negative resistance is determined

*Keywords*- double injection, space charge transport, trapping levels

## I. INTRODUCTION

One of the fundamental problems of a bipolar space charge theory is to find the interior and the contacts of a solid placed between the two electrodes. We will assume that the divergence of the electric field distribution will be defined by the total concentration of carriers. Also, we will suppose that the contact processes have an influence on the shape of the electric field distribution, which corresponds to a current – voltage characteristic. Additionally, we will determine the effect of the light on the space charge density distributions. In this work we will find new current- voltage characteristics of the planar capacitor system.

## II. THE MODEL

The fundamental problem for electric conduction in the metal – solid (insulator or semiconductor) – metal system is to define the internal and boundary conditions describing interactions between the positive and negative charge carriers in a solid. In general, when an external electric field is applied, those interactions correspond to a configuration of atoms in a space. In this paper, we will make the following assumptions:

(I) The system of atoms defining the bulk (insulator or semiconductor) is very chaotic,

(II) The concentration of atoms is possibly maximal and the splitting of the energy states (the Zeemann internal effect) occurs (this property corresponds to the different structural dislocations and to the Frenkl defects caused by pollutants and impurities),

(III) The Stark external effect (linear or nonlinear) is sufficiently weak (this property is characterised by a dielectric constant).

Under of these conditions, we shall consider interactions between atoms placed in a space and we shall describe the behaviour of the electron in a solid. To this end, let us take into account the Helmholtz - Lagrange - D' Alembert principal

in the form  $\delta \int_{\alpha}^{\beta} H(\tau') d\tau' = 0$ , where  $H(\tau')$  is the

Hamilton–Lagrange function (in general,  $H(\tau')$  is an integral functional),  $\tau'$  is the time,  $\alpha$  and  $\beta$  denote the time parameters, which define the boundary problem for the above principal. In what follows, on the basis of the Helmholtz principal and of the (I)-(III) assumptions, we will describe the internal and boundary processes determining a bipolar space charge transport in a solid placed between the two electrodes.

We will suppose that the function  $H(\tau')$  is described in terms of the total kinetic and potential energies of all the free electrons and of all the atoms that form the given material structure such as an insulator or a semiconductor. This physical property denotes that the Hamilton - Lagrange function corresponds to the crystalline lattice vibrations. By the classical mechanics, the energy function  $H$  can be written as follows  $H = W_k - W_p$  and  $W_k \geq 0$ . Here  $W_k$  is the total kinetic energy of all the atoms and of all the free electrons,

$W_p$  is the total potential energy of the given crystalline lattice. The vector velocity of the given atom is the sum of the vibration velocity vector and of the rotational velocity vector. Since, the kinetic energy is expressed by the quadratic power of the velocity, on this basis we ascertain that the total kinetic energy of the given configuration of atoms has the form

$W_k^{at} = W_k^{tot} + W_k^{mix}$ , where the symbols are respectively the

total kinetic energy  $W_k^{tot} = W_k^{rot} + W_k^{osc}$  of the rotators and oscillators as well as the mixed kinetic energy. The boundary conditions and the energy function  $H$  determine the kinetic and potential energy distributions that correspond to a configuration of atoms.

Generally, for the Helmholtz principal, the boundary conditions are as follows:

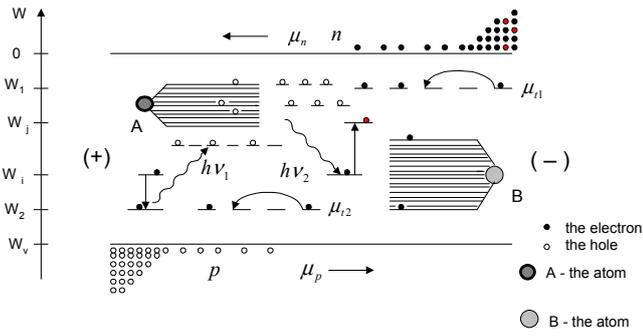


Figure 1. The energy diagram for the internal and boundary processes in a solid placed between the (+) anode and the (-) cathode.  $W$  denotes the total energy of an electron

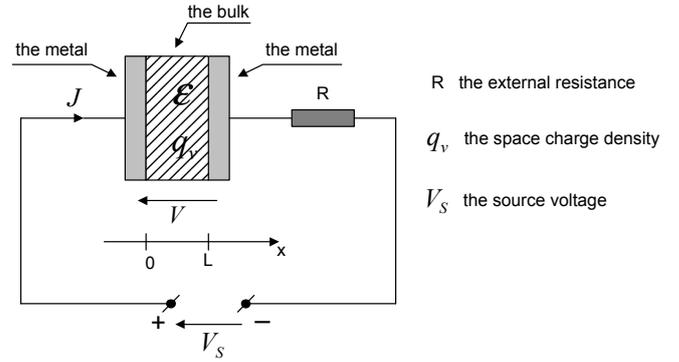


Figure 3. A planar capacitor is connected with an external resistance and a voltage source. Here, (+); (-) denote the terminals of a voltage source

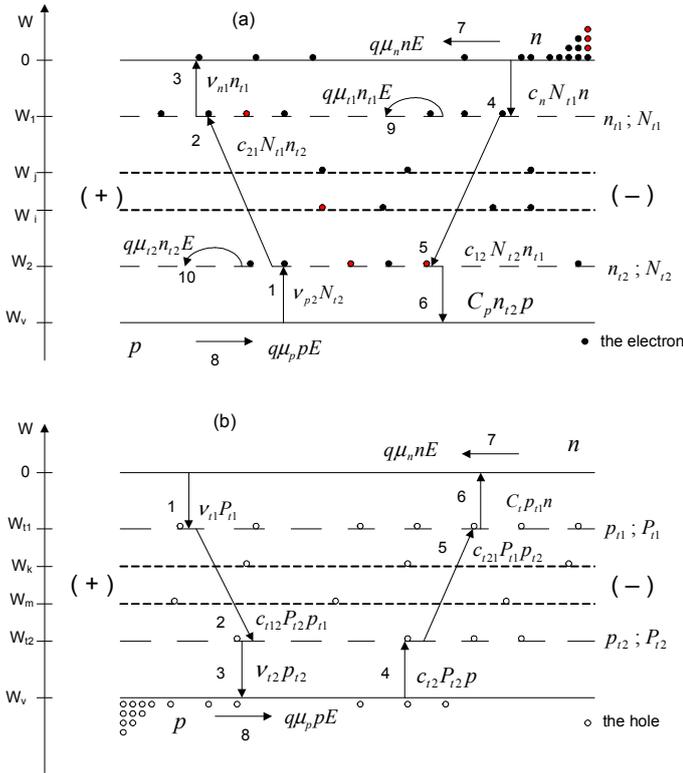


Figure 2. The energy diagram illustrating a model of an electron passage: (a) allowed electron transitions; (b) allowed hole transitions.  $W$  is the total energy of an electron

- (a) The Euler boundary problem expressed by  $\delta\alpha = \delta\beta = 0$ ,
- (aa) the Euler - Jacobi mixed boundary problem is written as  $\delta\alpha = 0$  and  $\delta\beta \neq 0$ ,
- (aaa) the Jacobi boundary problem characterised by  $\delta\alpha \neq 0$  and  $\delta\beta \neq 0$ .

Now, the energy function  $H$  and the boundary problem will be interpreted by quantum - mechanic interactions between atoms as well as between atoms and the free electrons.

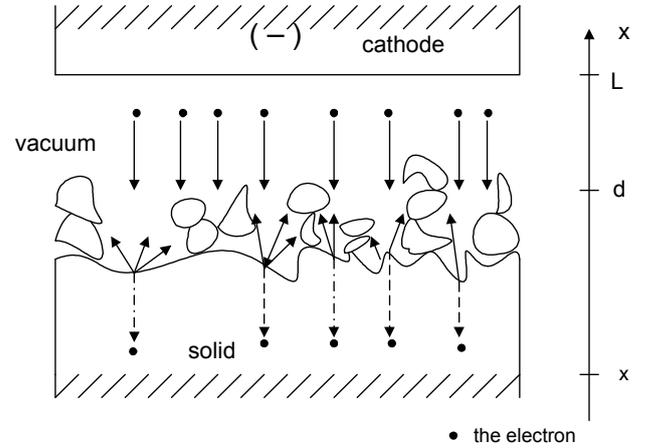


Figure 4. An electron transport between the cathode and a solid plane makes a model of a secondary electron emission.

Thus, according to the quantum mechanics, the total energy of the rotator  $W_{rot}$  and the total energy of the linear oscillator  $W_{osc}$  depend on the natural number in the form

$$W_{rot} = \frac{\hbar^2 \ell(\ell+1)}{2I} \text{ and } W_{osc} = h\nu_{osc} \left(k + \frac{1}{2}\right)$$

for  $\ell, k = 0, 1, 2, 3, \dots$ , where  $\hbar$  is the Planck constant,  $I$  is the moment of inertia,  $\nu_{osc}$  is the frequency and  $\ell$  is the orbital quantum number. Interactions between the given atom and all the adjacent atoms are expressed by the change  $\Delta\ell = \pm 1$  and  $\Delta k = \pm 1$  ( $\Delta$  denotes the difference). Here,  $\Delta\ell = +1$  and  $\Delta k = +1$  denote that a portion of energy of a photon is absorbed by the rotator and by the oscillator. Similarly,  $\Delta\ell = -1$  and  $\Delta k = -1$  denote that the portion of photon energy is emitted by the rotator and by the oscillator, respectively. When portion of energy is given by a photon (for example, the phonon is either bombarded by an incident photon or a photon emitted by an adjacent phonon) to the given phonon (the rotator and the oscillator together), then

interactions between the rotator and oscillator can occur. For the given phonon, the oscillator can supply photon energy to the rotator of the same phonon (that is  $\Delta k = -1$  and  $\Delta \ell = +1$ ).

Similarly, for the given phonon, interactions between atoms may be characterised by  $\Delta k = +1$  and  $\Delta \ell = -1$ . This kind of energy interactions between atoms of the given phonon corresponds to the mixed kinetic energy  $W_k^{mix}$ . When a portion of energy is emitted by the given phonon, then this energy may either be absorbed by some adjacent phonons or by all the adjacent phonons. This kind of energy interactions between atoms corresponds to the potential energy  $W_p$  of the given configuration of atoms. The energy states occupied by the rotators and by the oscillators correspond to the total kinetic energies  $W_k^{rot}$  and  $W_k^{osc}$  (respectively). In other words, interactions between the rotators and oscillators determine a propagation of the disturbance of the given system of atoms in a solid. Also, in the given crystalline lattice, energy interactions between atoms and the free electrons occur. For the orbital electrons in the given atom, we will suppose that the microscopic regions exist in which the total energy (the sum of the positive kinetic energy of an electron and the negative potential energy of the electric field of the nuclei of the given atom and of all the adjacent atoms) of the electrons is  $\geq 0$  (the microscopic regions in which the electrons become free). When the energy states occupied by the phonons are sufficiently high (the system of atoms is chaotic), it is possible for the free electrons to fall into the microscopic region with the negative potential energy of the electric field of the positive nuclei. These electrons pass from the higher positive energy levels to the lower negative energy (trapping) levels. Here, emitting a portion of energy, the electrons lose a portion of the total energy. Such the energy can either be absorbed by the some adjacent phonons or by all the adjacent phonons. Consequently, such the electron can pass from the lower trapping level to the higher trapping level. In this case, also, a propagation of the disturbance of the given composition of atoms occurs. Such interactions are described in terms of the potential mechanical energy  $W_p$ . In what follows, we will assume that the Zeemann internal effect (the splitting of the total energy of an orbital electron) occurs. For an orbital electron in the given atom, we will assume that the total energy (the sum of the positive kinetic energy and the negative potential energy of the electric field of the positive nucleus) is negative and that the zero reference level is at a finite distance from the nucleus. These properties are shown in Fig.1. In the case of an isolated atom the total energy of an orbital electron is negative for any distance from the nucleus. When an orbital electron absorbs a portion of the kinetic energy of a photon or a phonon, the total energy of the electron increases. The inverse case is when an orbital electron can lose a portion of the total energy. This is caused by the Coulomb force interaction between the orbital electron and the positive nucleus. With such the physical assumptions we will

determine some internal and boundary processes (which are shown in Fig. 1). First let us return to an isolated atom A. Here, let us take into account the two valence electrons of the isolated atom A. These electrons occupy the  $W_A$  energy state with the spin number  $s = \pm 1/2$ . When this atom is packed into a solid (that is, the A atom presents a pollutant or an impurity in Fig.1), these electrons can occupy the higher energy state. Such the property can be explained by the splitting of the  $W_A$  energy state.

In other words, this is caused by the different internal interactions (for example, the Zeemann internal effect) between the A atom and many adjacent atoms. Next, when a portion of the kinetic energy is given to the two valence electrons, these electrons can become free. The two empty energy states (which are left by the electrons) represent the two holes. In Fig. 1 some internal and boundary processes are presented when a solid is placed between the anode and the cathode. Here the pollutants or the impurities are symbolised by the A or B atoms. Between the zero and valence levels many energy states (the trapping states) are available for the holes and for the electrons. When an orbital electron absorbs a photon, this electron can pass from the valence level to the zero level via trapping levels. In Fig. 1 this is symbolised by the photon energy  $h\nu_2$ . The inverse case is when an orbital electron can pass from the higher trapping level to the lower trapping level and an energy portion  $h\nu_1$  is emitted. Here,  $h$  is the Planck constant and  $\nu_1, \nu_2$  denote the photon frequencies. Analogously, we are known as the allowed transitions for the trapped holes. Such the allowed electron – hole transitions are called carrier generation – recombination processes. When an external electric field supplies the different kinetic energy to the two adjacent atoms (in Fig. 1 this property is characterised by the mobilities  $\mu_{t1}$  and  $\mu_{t2}$ ), the trapped electron can pass from trap to trap in the given trapping level. When the external electric field is applied the electron can pass from the cathode into a solid. Moreover, this electron can become free and have the mobility  $\mu_n$ . Similarly, the hole injection occurs when the valence electron can pass from the bulk into the anode and the valence state is empty. When the external electric field gives a portion of kinetic energy to the valence electron of an adjacent atom, this electron can pass from the atom to the given atom and can fill the empty state on the valence level. Such the transport is characterised by the mobility  $\mu_p$  in Fig. 1.

For our considerations, the trapping levels will be grouped into the four permissible energy levels. With this assumption, the so-called effective parameters such as the frequency parameters  $c_{21}$  and  $c_{t12}$  as well as the recombination parameters  $c_{12}$  and  $c_{t21}$  will be used. For the trapped electrons, the concentrations of traps in the first and second trapping level will be represented by  $N_{t1}$  and  $N_{t2}$ ,

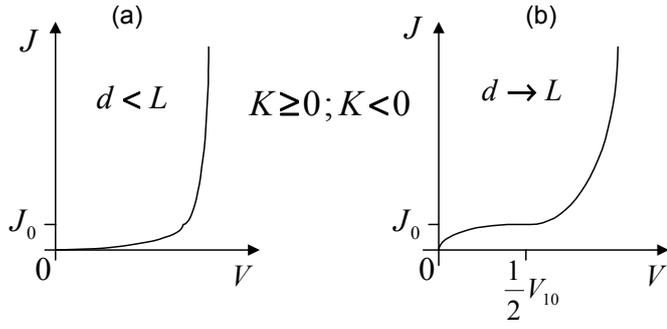


Figure 5. The curves correspond to the general integral for  $K \geq 0$  or  $K < 0$  : (a) the effect of a secondary electron emission is important; (b) a secondary electron emission is weak.

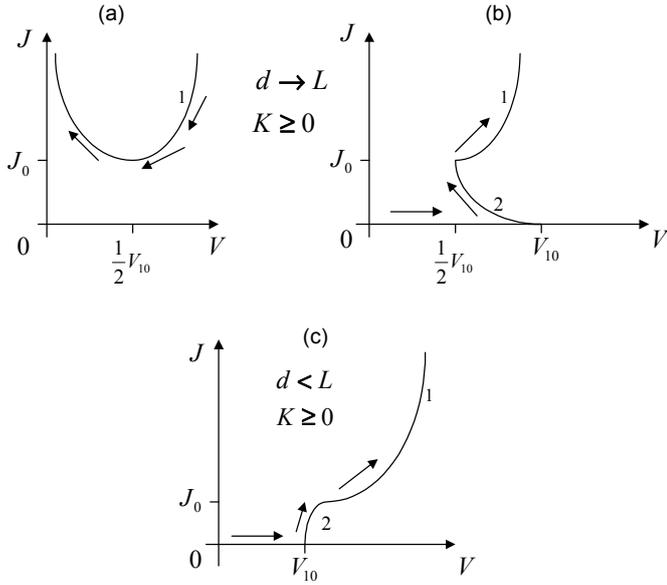


Figure 6. The curves correspond to the general integral for  $K \geq 0$  : (a) a boundary function  $f_0$  is decreasing for  $2V < V_{10}$  or  $f_0$  is increasing for  $2V \geq V_{10}$  as well as a secondary electron emission is weak; (b)  $f_0$  is increasing and  $f_0(0) = 0$  as well as a secondary electron emission is weak; (c) a secondary electron emission is important and  $f_0$  is increasing and  $f_0(0) = 0$ . Here, 1- is determined by the electric field  $E(x, K, E(0))$ , 2- corresponds to the singular solutions..

respectively. Analogously, for the trapped holes, the concentrations of traps in the first and second trapping level will be equal to  $P_{t1}$  and  $P_{t2}$ , respectively. The system of atoms will be treated as an unlimited reservoir of traps, that is  $P_{t1} \gg p_{t1}$ ;  $P_{t2} \gg p_{t2}$ ;  $N_{t1} \gg n_{t1}$  and  $N_{t2} \gg n_{t2}$ . Here, a solid is characterised by the following functions and parameters such as:

(a)  $n_{t1}$ ,  $n_{t2}$ ,  $p_{t1}$ ,  $p_{t2}$  are the concentrations of the trapped holes and electrons, respectively

(b)  $\mu_{t1}$  and  $\mu_{t2}$  are the mobilities of trapped electrons

(c)  $\nu_{p2}$ ,  $\nu_{n1}$ ,  $\nu_{t1}$ ,  $\nu_{t2}$  denote the frequency parameters

(d)  $c_n$ ,  $C_p$ ,  $C_t$ ,  $c_{t2}$  denote the recombination parameters

All the parameters are shown in Fig. 2. The metal-solid-metal system will be represented by a planar capacitor system with the anode  $x = 0$  and the cathode  $x = L$  (the system is shown in Fig. 3). Also,  $L$  denotes the distance between the electrodes. Moreover, we will assume that the diffusion current is negligible [1-6]. For a solid, we will assume that the polarisation effect is characterised by the dielectric constant  $\epsilon$ . Additionally, the carrier mobilities are independent of the electric field intensity  $E$  [7 - 9]. For the  $x = L$  contact, we will consider a special case of electron emission from the cathode into a solid (this property is shown in Fig. 4). Here, for a solid, we assume that the contact structure is strongly nonhomogeneous. On these conditions a secondary electron emission occurs [10-14]. This property will be represented by a vacuum capacitor with the contact voltage  $V_2$  and a distance parameter denoted by  $L - d$  [15]. A vacuum space is described by the following functions such as

(a)  $E_0$  is the electric field intensity

(b)  $n_0$  is the electron concentration

(c)  $\vartheta$  is the effective velocity of an electron

In our considerations, for the planar capacitor system, the basic equations such as the Gauss equation, the continuity equation, the generation-recombination equations and the field integral are used [16-18]. The applied voltage  $V$  between the electrodes of a planar capacitor is  $V = const. > 0$ . The space charge transport through the system is characterised by a current density-voltage function in the form  $J = J(V)$  or voltage - current density  $V = V(J)$ , where  $J$  is the current density. In order to find these functions, we have to define the boundary conditions describing the mechanisms of carrier injection from the anode  $x = 0$  and the cathode  $x = L$  into the bulk.

### III. DISCUSSION and CONCLUSIONS

In the above, we have determined the divergence of the electric field distribution. For the bulk, when carrier generation processes are dominant, the electric field intensity satisfies a homogeneous equation. Upon these conditions there exist two singular particular solutions  $E_1(x, E(0))$  and  $E_2(x, E(d))$  as well as the general integral  $E(x, K, E(0))$  with a constant of integration denoted by  $K$ . The two mechanisms of carrier injection are expressed by boundary functions in the form  $J = f_0[E(0)]$  and  $J = f_L[E(L)]$ . First let us discuss the shape of a current density - voltage characteristic  $J = J(V)$ , which is determined when a

boundary function  $f_0[E(0)]$  is strongly increasing and  $f_0(0) = 0$ . Similarly, the same assumption is for a boundary function  $f_L[E_0(L)]$ . We see that the  $J(V)$  curve is displaced and strongly increasing. For singular solutions, we ascertain that the  $J(V)$  curve can exist only when the external voltage is sufficiently great. In other words, there exists a set of values of applied voltage  $V$  in which the current density  $J$  is not defined. This property denotes that the whole system acts as a solar cell. Also, we ascertain that the  $J(V)$  curve is strongly increasing for  $V > V_{10}$  and  $J(V) \equiv 0$  for  $0 \leq V \leq V_{10}$ , where  $V_{10}$  is a voltage parameter characterising generation processes. Therefore, the system acts as a perfect blocking diode and a voltage stabiliser. Also, this mathematical property corresponds to a solar cell. In this case, at the anode region  $0 \leq x \leq d/2$ , the space charge density is negative. Analogously, for the bulk, at the  $x = d$  contact region, that is  $(d/2) \leq x \leq d$ , the space charge density is positive. Thus, the whole system acts as an n-p-n junction. Moreover, we see that the bulk absorbs a portion of external solar energy in order to form the internal electric field (this physical property is expressed by a constant of integration  $K < 0$ ). Some example is illustrated in Fig. 5a, when a boundary function  $f_0[E(0)]$  is strongly increasing. A function from Fig. 5 is typical for the *SiC* structure. In Fig. 5-6 there is  $J_0 = f_0(E_{10})$  and  $E_{10}$  denotes a field parameter characterising generation processes. In the above we have supposed that the divergence of the electric field can be described in terms of the total energy of an orbital electron. The main idea of a space charge theory is to identify the internal and boundary processes that occur in the metal - solid - metal system. In general, this problem can be solved by the following electric field conditions:

- (a) the transient state of the discharging capacitor, characterised by  $V = 0$
- (b) the transient state or the stationary state of the charging capacitor, described by the time-voltage function  $V(t) = const.$  or  $V(t) = V_m \sin \omega t$  and the parameters  $V_m$  and  $\omega$  are given

(c) the open system, in which the total current density is equal to zero

In the above concept, by making use of the current density – voltage characteristic  $J(V)$ , we have identified the interior and the boundaries together.

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