Analysis of Sub-Harmonics in Power Systems

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Abstract—**With a wide range of power electronics-related applications in power systems, harmonic currents are increasing at an alarming rate which has greatly deteriorated the power quality in electrical power networks. Moreover, some of electronic controlled equipments used in power systems, such as power converters, produce sub-harmonics, a type of waveform distortion, which can severely degrade the power system performance. Therefore, they must be closely monitored. Moreover, Fast Fourier Transform cannot accurately analyze waveforms containing sub-harmonics because the synchronization of the sampling procedure to sub-harmonics is practically infeasible. The detection of sub-harmonics requires an approach different from that used for harmonics analysis. In most analysis methods the voltage waveform is expected to be a pure sinusoid with a given frequency and amplitude. Standard tools of harmonic analysis based on the Fourier transform assume that only harmonics are present in the investigated signal and the periodicity intervals are fixed, while periodicity intervals in the presence of inter-harmonics and sub-harmonics can be variable and very long. A novel approach to analysis of non-stationary signals, based on the "subspace" methods, is proposed. "Root-Music" harmonic retrieval method is an example of high-resolution eigenstructure-based methods**

Keywords:- Discrete Fourier Transform, dc arc furnaces frequency measurement, harmonic analysis, power system, subspace methods

I. INTRODUCTION

The quality of voltage waveforms is nowadays an issue of the utmost importance for power utilities, electric energy consumers and also for the manufactures of electric and electronic equipment. The voltage waveform is expected to be a pure sinusoidal with a given frequency and amplitude. Frequency power converters and arc furnaces generate a wide spectrum of harmonic components which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power system. In some cases, large converters systems generate not only characteristic harmonics typical for the ideal converter operation, but also considerable amount of non characteristic harmonics and inter-harmonics as well as sub-harmonics which may strongly deteriorate the quality of the power supply voltage [1,2,3,4].

Sub-harmonics are defined as non-integer harmonics with the frequency below the main fundamental under consideration. The estimation of the spectral components is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms.

Sub-harmonics are especially troublesome for detection and analysis.

There are many different approaches for measuring harmonics, like FFT, application of adaptive filters, artificial neural networks, SVD, higher-order spectra, etc [1,2,6,7,8]. Most of them operate adequately only in the narrow range of frequencies and at moderate noise levels.

Currently employed methods usually assume that only harmonics are present in the waveform and the periodicity intervals are fixed, while periodicity intervals in the presence of inter-harmonics and sub-harmonics are variable and sometimes very long.

In this paper the parameters of signal components are estimated using the root-Music method.

The parametric methods of spectrum estimation are based on the linear algebraic concepts of subspaces and so have been called "subspace methods" [5]. Their resolution is theoretically independent of the SNR. The model of the signal in this case is a sum of random sinusoids in the back-ground of noise of a known covariance function.

Representation of signals in time and frequency domain has been of interest in signal processing areas for many years, especially taking in the limelight time-varying non-stationary signals. The standard method for study time-varying signals is short-time Fourier transform (STFT) that is based on the assumption that for a short-time basis signal can be considered as stationary. The spectrogram utilizes a short-time window whose length is chosen so that over the length of the window signal is stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the canter of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the non-stationary phenomena occurring during this interval will be smeared in time and frequency. This inherent relationship between time and frequency resolution becomes more important when one is dealing with signals whose frequency content is changing. In the paper the time frequency characteristics of signals have been calculated applying sliding sampling windows.

To investigate the methods several experiments were performed using non-stationary signals in a supply system of a dc arc furnace. For comparison, similar experiments were repeated using the FFT.

II. ARC FURNACE

A typical dc arc furnace plant is shown in Fig. 1. It consists of a dc arc connected to a medium voltage ac busbar with two parallel rectifiers that are fed by transformer secondary winding with Δ and Y connection, respectively.

The medium voltage busbar is connected to the high voltage busbar with a HV/MV transformer whose windings are Y-Δ connected. The power of the furnace is 70 MW. The other parameters: Transformer - 80 MVA, 220kV/21kV; Transformer - 87 MVA, 21kV/0.638kV/0.638kV.

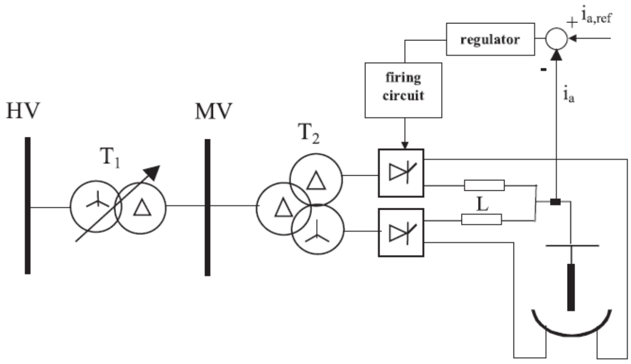

Figure 1. Scheme of the simulated DC arc furnace plant.

In the dc arc furnaces, the presence of the ac/dc static converter and the random motion of the electric arc, whose non linear and time-varying nature is known, are responsible for dangerous perturbations, in particular waveform distortions and voltage fluctuations.

In particular, the behavior of the DC arc can lead to the aperiodicity of AC electrical voltages and currents at MV busbar. The phenomena involved in the arc behavior are very complex. It has been shown that the DC arc voltage waveform has the aperiodic and irregular behavior that characterizes every chaotic phenomenon.

III. ROOT-MUSIC METHOD

A general model of the signal which is considered for estimation of the sinusoidal components within the signal is:

$$\mathbf{x} = \sum_{i=1}^{p} A_i \mathbf{s}_i + \eta \ ; \quad A_i = |A_i| e^{j\phi_i} \tag{1}$$

where $\mathbf{s}_i = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(M-1)\omega_i} \end{bmatrix}^T$; N - number of the signal vector components, η -noise.

The MUSIC method [5] involves projection of this vector (signal vector) onto the entire noise subspace.

If the noise is white, the correlation matrix of $\mathbf{x}$ can be expressed as:

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathrm{E}\left\{ A_i A_i^* \right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} \tag{2}$$

After the calculation of eigenvector and eigenvalues of the correlation matrix, it can be observed that M-p smallest eigenvalues of the correlation matrix (matrix dimension $M>p+1$) correspond to the noise subspace and p largest (all greater than σ_0^2 - noise variance) correspond to the signal subspace.

Matrices of eigenvectors can be divided into signal and noise matrices:

$$\mathbf{E}_{signal} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_p \end{bmatrix} \tag{3}$$

$$\mathbf{E}_{noise} = \begin{bmatrix} \mathbf{e}_{p+1} & \mathbf{e}_{p+2} & \cdots & \mathbf{e}_N \end{bmatrix} \tag{4}$$

and, similarly, two matrices of eigenvalues $\Lambda_{signal}, \Lambda_{noise}$ can be build:

It is possible then to write $\mathbf{R}_x$ as:

$$\mathbf{R}_x = \mathbf{E}_{signal} \Lambda_{signal} \mathbf{E}_{signal}^{*T} + \mathbf{E}_{noise} \Lambda_{noise} \mathbf{E}_{noise}^{*T} \tag{5}$$

MUSIC method uses only the noise subspace for the estimation of frequencies of sinusoidal components $\mathbf{E}_{noise}$ can be used to form a projection matrix $\mathbf{P}_X$ for the noise subspace

$$\mathbf{P}_{noise} = \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} = \mathbf{I} - \mathbf{P}_{signal} \tag{6}$$

The squared magnitude of the projection of $\mathbf{w}$ (an auxiliary vector defined similarly to $\mathbf{s}$ - $\mathbf{w} = \begin{bmatrix} 1 & e^{j\omega_i} & \cdots & e^{j(N-1)\omega_i} \end{bmatrix}^T$) onto the noise subspace is given by:

$$\begin{aligned} \mathbf{w}^{*T} \mathbf{P}_{noise} \mathbf{w} &= \mathbf{w}^{*T} \mathbf{E}_{noise} \mathbf{E}_{noise}^{*T} \mathbf{w} \\ &= \sum_{i=p+1}^{N} E_i\left(e^{j\omega}\right) E_i^*\left(e^{j\omega}\right) \end{aligned} \tag{7}$$

The MUSIC pseudo-spectrum is defined as:

$$\hat{P}\left(e^{j\omega}\right) = \left. \frac{1}{\sum_{i=p+1}^{N} E_i(z) E_i^*\left(1/z^*\right)} \right|_{z = e^{j\omega}} \tag{8}$$

and it exhibits sharp peaks at the signal frequencies where $z = e^{j\omega}; (i = 1, 2, ..., M)$

After the calculation of the frequencies, the powers of each component can be estimated from the eigenvalues and eigenvectors of the correlation matrix, using the relation

$$\mathbf{e}_i^{*T} \mathbf{R}_x \mathbf{e}_i = \lambda_i \tag{9}$$

by substituting

$$\mathbf{R}_x = \sum_{i=1}^{p} \mathrm{E}\left\{ A_i A_i^* \right\} \mathbf{s}_i \mathbf{s}_i^T + \sigma_0^2 \mathbf{I} = \sum_{i=1}^{M} P_i s_i s_i^{*T} + \sigma_0^2 \mathbf{I} \,, \tag{10}$$

P_i-power of the component. The resulting equations can be solved for P_i.

IV. Investigations

The voltage and current waveforms analyzed in this paper have been recorded at the MV busbar in the supply system of a typical DC arc furnace plant as shown in Fig.1. The investigations have been carried out using the Fourier and root-Music methods. The signals have been sampled with the frequency of 5000 Hz. Fig. 2 shows the current. The amplitudes of the frequency component are relative to the maximum value in the spectrum.

For analysis of the sub-harmonics (frequencies under 50 Hz) a low-pass (40 Hz) Butterworth IIR filter of the 8. order has been applied. The investigations show certain fluctuations of frequencies. The non-stationarity of the sub-harmonics component is especially visible on the Fig. 3. The fundamental component (50) Hz has a constant frequency and the frequency of the sub-harmonic fluctuates, because of the non-stationarity of the arc.

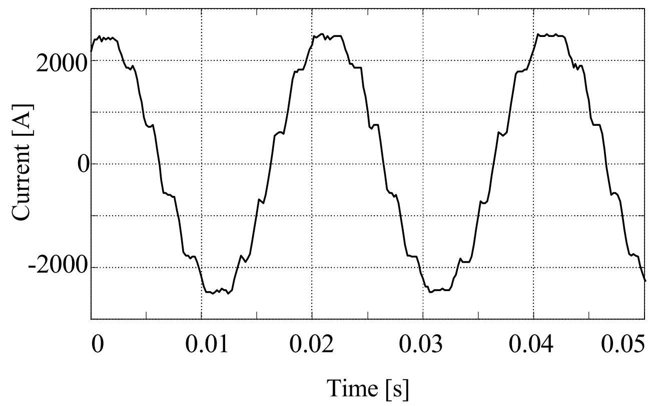

Figure 2. Current waveform in the supply system of the furnace.

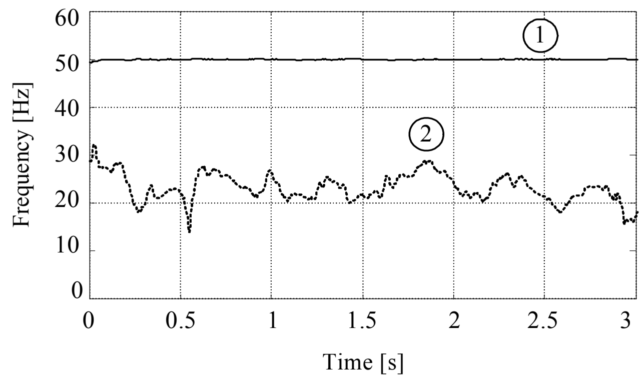

Figure 3. Time frequency characteristics of the two current components in the frequency band 10-60 Hz, estimated by root-MUSIC method.

V. Conclusions

It has been shown that high-resolution spectrum estimation methods, such as root-MUSIC method could be effectively used for parameter estimation of distorted, non stationary signals which contain sub-harmonics. The accuracy of the estimation depends on the signal distortion, the sampling window and the number of samples taken into the estimation process.

The proposed methods were investigated under different conditions and found to be variable and efficient tools for detection of all higher harmonics existing in a signal. They also make it possible the estimation of inter-harmonics and the calculation of the time-frequency characteristics of signal components.

When using both the high-resolution method like root-MUSIC the estimation accuracy in most cases is better than when using the Fourier algorithm. Application of the proposed advanced methods makes it possible the estimation the changes in time the parameters of signal components, even in situations when the frequencies of the components differ insignificantly. In the time-frequency characteristics of signal components fluctuations of frequency characteristics is visible. Especially when comparing the frequency of the main component and the sub-harmonic. However, the computation of the methods is more complex than FFT.

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