ON-LINE DETERMINATION OF SYMMETRICAL COMPONENTS BY SAMPLING

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Symmetrical components of currents and voltages are used for the performance analysis of three-phase power systems (positive-sequence component) and for protection. This paper introduces different algorithms permitting to determine in real time the symmetrical components from few sampled triplets of voltages or currents. By using elaborated algorithms both parasitic harmonics and transients can be suppressed. Finally the attention is given to a general approach to the implementation of an algorithm with a pilot multi-processor equipment.

INTRODUCTION

The method of symmetrical components is one of the basic tools of the electrical three-phase power system analysis. Symmetrical components of currents and voltages are used for several purposes. The performance of power systems is to a large degree determined by the positive-sequence components. In unbalanced disturbances the negative- and the zero-sequence components increase strongly. The use of the analogue filters for the detection of the positive-, negative- and zero-sequence components has some disadvantages. The use of digital methods for protecting of power systems is attracting more and more interest /3/. In recent years this trend has been accelerated by the rapid decreasing in prices of microprocessors.

This paper introduces algorithms permitting to determine in real-time the symmetrical components from few sampled triplets of voltages or currents. One of these algorithms is implemented with a pilot multi-processor equipment.
INSTANTANEOUS VALUES OF THE SYMMETRICAL COMPONENTS

The relations between the phase quantities $G_R$, $G_S$, $G_T$ and the symmetrical components $G_0$, $G_1$, $G_2$ are given by

$$
\begin{bmatrix}
G_0 \\
G_1 \\
G_2
\end{bmatrix}
= \frac{1}{3}
\begin{bmatrix}
1 & -1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\begin{bmatrix}
G_R \\
G_S \\
G_T
\end{bmatrix}
$$

(1)

In eqn (1) $a$ represents a phase-rotating operator

$$
a = \exp(j 2\pi/3); \\
a^2 = \exp(j 4\pi/3)
$$

(2)

The time functions of the phase quantities $g_R$, $g_S$, $g_T$ and their corresponding complex phasors are given by

$$
g_R = \hat{g}_R \cos(\omega t + \varphi_R) \\
g_S = \hat{g}_S \cos(\omega t + \varphi_S) \\
g_T = \hat{g}_T \cos(\omega t + \varphi_T)
$$

(3a)

$$
G_R = \hat{g}_R \exp(j\varphi_R) \\
G_S = \hat{g}_S \exp(j\varphi_S) \\
G_T = \hat{g}_T \exp(j\varphi_T)
$$

(3b)

From eqns (3) it follows, that the time function can be found according to

$$
g_i(t) = \text{Re} \left[ G_i \exp(jut) \right] = \frac{1}{2} \left[ G_i \exp(jut) + G_i^* \exp(-jut) \right]
$$

(4)

Applying expressions (3b) to (1) and using next eqn (4) we have found the time representation of the symmetrical components:

$$
3 g_0 = \hat{g}_R \cos(\omega t + \varphi_R) + \hat{g}_S \cos(\omega t + \varphi_S) + \hat{g}_T \cos(\omega t + \varphi_T)
$$

(5a)

$$
3 g_1(2) = \frac{\sqrt{3}}{2} \left[ \hat{g}_S \cos(\omega t + \varphi_S) + \hat{g}_T \cos(\omega t + \varphi_T) \right]
$$

(5b)

$$
(\mp) \frac{\sqrt{3}}{2} \left[ g_S \sin(\omega t + \varphi_S) - \hat{g}_T \sin(\omega t + \varphi_T) \right]
$$

In eqns (5) $\hat{g}_1 \cos(\omega t + \varphi_1)$ represents known sampled values (Fig.1) and $\hat{g}_1 \sin(\omega t + \varphi_1)$ can be calculated from two or more sampled values by different methods /1/.

Application of the addition theorem

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**Fig. 1** Nomenclature of the sampled values

For the sampling times $t_{k-1}$, $t_k$ (Fig. 1) we have
Applying (6) to (5), we get the SA2-method:

\[ g_{0k} = g_{Rk} + g_{Sk} + g_{Tk} \]  
\[ g_{1(2)k} = g_{Rk} - \frac{1}{2} \left( (\frac{-}{\pi}) \sqrt{3} \cot\Delta \right) g_{Sk} + (\frac{-}{\pi}) \sqrt{3} \cot\Delta \cdot g_{Tk} \]

Application of the first derivative

The orthogonal function \( g, \sin(\omega t + \varphi) \) in eqns (5) can be derived from the first derivative of the time function at the time \( t_k \):

\[ \hat{g} \sin(\omega t_k + \varphi) = \frac{1}{\omega} \frac{dg}{dt} \left[ \hat{g} \cos(\omega t_k + \varphi) \right] = \frac{1}{\omega} g_k' \]

The derivatives can be approximated by finite differentials between two sampling times /2/:

\[ g_k' = \frac{1}{2 \sin\Delta} \omega \]  
\[ g_k' = \frac{1}{2 \sin\Delta} \omega \]

From eqn (9b) with \( \omega t_{k-1} = \omega t_k - \Delta \) we get

\[ \frac{-g_{k-2} + g_{k-1} \cos \Delta + g_k - g_{k+1} \cos \Delta}{2 \sin^2 \Delta} = g_k \]  
\[ \frac{-g_{k-2} \cos \Delta + g_{k-1} + g_k \cos \Delta - g_{k+1}}{2 \sin^2\Delta} = g_{k-1} \]

Applying expressions (9) and (10) to (5) we get formulas (called the SD4-method)

\[ g_{0k} = \frac{1}{2 \sin^2 \Delta} \left[ g_{Rk} - g_{Rk-2} + g_{Sk} - g_{Sk-2} + g_{Tk} - g_{Tk-2} \right. \]

\[ \left. -(g_{Rk+1} - g_{Rk-1} + g_{Sk+1} - g_{Sk-1} + g_{Tk+1} - g_{Tk-1}) \cos \Delta \right] \]

\[ g_{1(2)k} = \frac{1}{2 \sin^2 \Delta} \left[ g_{Rk} - g_{Rk-2} = (g_{Rk+1} - g_{Rk-1}) \cos \Delta \right. \]

\[ \left. - \frac{1}{2} \left( g_{Sk} - g_{Sk-2} + g_{Sk+1} - g_{Sk-1} \right) \cos \Delta \frac{-}{(\pi)} \frac{2}{3} \right] \]

\[ \left. - \frac{1}{2} \left( g_{Tk} - g_{Tk-2} + g_{Tk+1} - g_{Tk-1} \right) \cos \Delta \frac{-}{(\pi)} \frac{2}{3} \right] \]

\[ -(g_{Tk+1} - g_{Tk-1}) \cos \Delta \frac{-}{(\pi)} \frac{2}{3} \]
COMPLEX PHASORS OF THE SYMMETRICAL COMPONENTS

From the complex phasors of the phase quantities

Applying (1) we can calculate the phasors of the symmetrical components from the phasor of the phase quantities. The complex phasors of the phase quantities are calculated from the sampled values by the least square errors method. For N sample window the expressions for symmetrical components become (SQN-method)

\[ 3 \mathbf{G}_0^r = \frac{1}{AC-B^2} \left[ A(F_{Rc} + F_{Sc} + F_{Tc}) - B(F_{Rs} + F_{Ss} + F_{Ts}) \right] \]  
(12a)

\[ 3 \mathbf{G}_0^i = \frac{1}{AC-B^2} \left[ B(F_{Rc} + F_{Sc} + F_{Tc}) - (F_{Rs} + F_{Ss} + F_{Ts})C \right] \]  
(12b)

\[ 3 \mathbf{G}_1(2)^r = \frac{1}{AC-B^2} \left\{ AF_{Rc} - BF_{Rs} - \frac{1}{2} \left[ F_{Sc}(A^\perp \sqrt{3} B) - F_{Ss}(B^\perp \sqrt{3} C) \right. \right. \]  
\[ \left. \left. + F_{Tc}(A^\perp \sqrt{3} C) - F_{Ts}(B^\perp \sqrt{3} C) \right\} \right\} \]  
(12c)

\[ 3 \mathbf{G}_1(2)^i = \frac{1}{AC-B^2} \left\{ BF_{Rc} - CF_{Rs} - \frac{1}{2} \left[ F_{Sc}(B^\perp \sqrt{3} A) - F_{Ss}(C^\perp \sqrt{3} B) \right. \right. \]  
\[ \left. \left. + F_{Tc}(B^\perp \sqrt{3} A) - F_{Ts}(C^\perp \sqrt{3} B) \right\} \right\} \]  
(12d)

where \( A = \sum_{n=0}^{N-1} \sin^2(n\omega At) \); \( B = \sum_{n=0}^{N-1} \sin(n\omega At)\cos(n\omega At) \);

\( C = \sum_{n=0}^{N-1} \cos^2(n\omega At) \); \( F_s = \sum_{n=0}^{N-1} g_n \sin(n\omega At) \); \( F_C = \sum_{n=0}^{N-1} g_n \cos(n\omega At) \)

If the sampled data window is equal 1 cycle of the basic component we get \( A = C = \frac{N}{2} \); \( B = 0 \).

If we have to estimate the symmetrical components after an event (e.g. occurrence of a fault), we can calculate the first estimated values from 2 sampled triplets (form. 7 or 12, N=2). After every next new sample we can increase the number N of sampled triplets, which we take into calculation, until the sample data window is equal to 1 cycle of basic component. On this way we get the best possible estimation of the symmetrical component in the course of time.

From the space-phasor

The relation between the time-functions of the symmetrical components and the time functions of the phase quantities is given by the following formulas

\[ 3 \mathbf{g}_0 = g_R + g_S + g_T \]  
(13a)

\[ 3 \mathbf{g}_1 = g_R + ag_S + \overline{a}g_T \]  
(13b)

\[ 3 \mathbf{g}_2 = g_R + \overline{a}g_S + ag_T \]  
(13c)
is the so-called space-phasor.

For the real quantities \( g_R \), \( g_S \) and \( g_T \): \( g_1 = \frac{g_R}{2} \)

The complex phasor of the positive-sequence component can be calculated as follows:

\[
G_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} g_1 \exp(-j\omega t) d(\omega t) = \frac{1}{\pi} \int_{-\pi}^{\pi} (g_r + jg_i) \exp(-j\omega t) d(\omega t)
\]

\[
= \frac{1}{\pi} \sum_{n=0}^{N-1} \left( g_{rn} + jg_{in} \right) \frac{2n+1}{\Delta} \exp(-j\omega t) d(\omega t)
\]

\[
- \frac{2}{\pi} \sum_{n=0}^{N-1} (g_{rn} + jg_{in}) \exp(-jn\omega t)
\]

where \( g_r = \frac{1}{3} \left[ g_R - \frac{1}{2} (g_S + g_T) \right] \); \( g_i = \frac{\sqrt{3}}{6} (g_S - g_T) \)

On this way we get the equations, which are similar to eqns (12) for 1 cycle with exception, that instead of \( \frac{1}{N} \) we get \( \frac{\sin \frac{N}{N}}{N} \).

**FREQUENCY CHARACTERISTIC**

For the examination of filter properties of designed method the frequency characteristics have been investigated. For each formular and each phase the transfer function \( H(\omega) \) from \( g_{out} = H(\omega)g_{in} \)

has been analysed.

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**Fig. 2** Frequency characteristics of the SA2-method

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**Fig. 3** Frequency characteristics of the SD4-method
Fig. 4 Frequency characteristics of the SQ6-method, $f_A = 300$ Hz

Fig. 5 Frequency characteristics of the SQ12-method, $f_A = 600$ Hz

Fig. 2 shows the frequency characteristics for the SA2-method (formulae 7b), which have no convenient filter properties. More suitable is the SD4-method (formulae 11b)(Fig. 3). By this method the harmonics $v = 0,5 \frac{f_A}{f}, (k=0,1,2,\ldots)$ are filtered out. The best properties shows the SQ12-method with the sampling window equal to 1 cycle (formulae 12). For the sampling frequency $f_A = 600$ Hz all harmonics with the exception of $12 k + 1 (k=0,1,2,\ldots)$ are filtered out (Fig. 5). The algorithms could be completed by digital recursive filters which operate with a higher sampling frequency than the algorithms /1/.

APPLICATIONS

The introduced algorithms permit to determine different operation values and to solve various tasks in the field of network protection, for example the measuring of load asymmetry, the differential protection of a transmission line or an exact fault determination. If the measured variables are much smaller than the disturbance variables, it is especially important to adjust the sampling frequency to such a degree that it becomes an integral multiple of the actual power frequency. The necessary determination of frequency can be executed in case of faults too, by means of the positive-sequence component of voltage or of current. When comparing the phasors of selected components the data transfer in the differential protection of a transmission line becomes more simple and more reliable. The measured values of a station can be joined to further protective functions, for example for distance protection. The nature of a fault can clearly be concluded from the component values. According to this switching measures are started selective to the phases.
Dimensioning

For network frequency processes the sampling frequency should be about 1 kHz to obtain a genuine transfer of fundamental oscillation and of low range harmonics. Undesired frequency components can be diluted by filter algorithms which are either integrated in the transformation algorithms or which must be passed through additionally. Harmonic vibrations of a higher range can effectively be damped by the \( \sin \frac{x}{x} \) valuation, with \( x = \frac{\pi f}{f_A} \) of the step sampling. It is assumed that the A/D-converters are suitably scaled and that the word length is thus adapted to the mathematical operations that no informations can be lost. The word length of the output values can be estimated from the demand that any change in the steady values of the metrical components can just be detected for every sampling step.

\[
2^{n-1}(1-\cos \frac{\pi f}{f_A}) \geq 1
\]  

(15)

If input and output values are of the same magnitude, for example in the determination of the positive-sequence component, a word length of 8 bit will be sufficient. Short-circuit currents whose values may range between 0.8 to 10 times the value of the rated current may require a word length of 12 bit.

Negative-sequence voltages are occurring in rating, covering some percents of the positive-sequence voltage. Detection can then be realized by means of a 16-bit-word length or by the compensation of the positive-sequence component. For that purpose the value of the positive-sequence component, which is to be expected from the next sampling step is determined by a parallel processor and will be subtracted from the measured values, even before the A/D-conversion.

Hardware

Multiprocessor equipments are recommended for realisation. They proved to be suitable for both the parallel determination of different parameters, such as positive-, negative-, zero -sequence components and frequency, and for the increase of capacity and speed of the processor. By means of a suitable splitting of the algorithms more complex arithmetical processors can mostly be avoided.

Fig. 6 Multiprocessor system
Figure 6 shows the configuration of four 8-bit processors 8085, which we are at present using for fundamental investigations. The processor system can be extended at will, as all the processors are basing on identical double-Europe-plates, one half of which is connected to a local bus, the other half to a common bus. Communication is effected via the common memory. The access to the bus is transmitted according to the polling method via decentralized arbiters. Thus each processor has equal access possibilities. The transmission of the access will only take place if the bus is released by the processor. The system is very flexible as there exists no fixed coordination of the connection and the assignment of the buses remains free. This requires however a careful and critical distribution of the tasks with special regard to timing, as there is no master processor available.

CONCLUSIONS

The symmetrical components originally determined for phasors only are now including time values. Transformation can thus suitably be connected with the sampling of measured triplets as the valuation is done by means of angle theorems or derivatives. A further approach can be realized via space phasors. Stationary values can be determined from at least two triplets of values. If several triplets are applied, and especially if the quantity of same includes a half or a whole period of the network frequency, harmonics can very effectively be filtered out. The filtering efficiency can considerably be improved by either joining digital filters or by integrating them into the algorithms. A suitable data window presumed, the components can practically be determined in real time, which will open up new aspects for measuring technique and for network protection, all the more if several components can be worked together. Multi-processors with a decentral organisation are suitable for a modular arrangement.

REFERENCES

/1/ Hosemann, G., Lobos, T.: "Ermittlung der symmetrischen Komponenten durch Abtastalgorithmen", accepted for publication in Archiv für Elektrotechnik, West-Germany.

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