CLASSIFICATION OF ELECTRIC SIGNALS BASED ON TIME–FREQUENCY SIGNAL DECOMPOSITION

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ABSTRACT

A classification procedure based on time-frequency decomposition of the signal is presented. Parametric spectral ESPRIT method is used for estimation of relevant parameters of signal components and specific areas of the time-frequency plane are chosen, where the signal is expected to show most characteristic patterns. Classification is based on time-domain correlation of reconstructed signals. It is applied to event classification of non-stationary electric signals obtained from a simulated power converter.

Index Terms— Signal analysis, time-frequency analysis, pattern classification, correlation

1. INTRODUCTION

In the latest years, pattern recognition, data mining, decision-making and networking were incorporated as new technologies for automatic classification. This entire advancement aims at processing raw data and extracting information to obtain knowledge in order to solve problems with less or without human action. The decomposition of a band-limited one-dimensional time-domain signal into two-dimensional time-frequency domain can reveal more details of the signal and help to improve the classification performance or pattern recognition [2]. One of many automatic classification techniques, based on correlation [5], is adopted in this paper for events classification in electric power systems.

The proposed classifier makes use of available *a priori* knowledge about the signal, in many ways; it uses the knowledge about the main characteristics, such as: number of expected components, parameters of frequency bands which contain most useful information and the time interval where the most significant changes occur. In pre-processing stage, many ”regions of interest” in the time-frequency plane are defined in order to enhance the classification performance. In order to evaluate the performance of presented pre-processing approach, a simple time-domain correlation is chosen as a classifier, since complicated classification technique can obscure the influence of improperly chosen pre-processing and make the fair comparison impossible [1].

The paper is composed as follows: After the description of applied correlation technique based on time-frequency decomposition of the signal, the ESPRIT method is shortly recalled. Next part presents numerical validation of the proposed classification scheme with results and discussion.

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2. CLASSIFICATION PROCEDURE

The main goal is to design a classification scheme which, using 2-D time-frequency parametric representation of a signal, performs better than a straightforward correlation-based classifier of time-domain waveforms.

It is assumed that the transformation to 2-D time-frequency domain allows revealing more details of the signal and therefore improving the accuracy of pattern matching. Additionally, it helps to reveal the correct pattern buried in noise (disturbances) by exposing the important characteristics of analyzed signal. The transformation to time-frequency domain allows easily using the *a priori* knowledge: only selected areas of the time-frequency plane can be used for the correlation-based classification. In previous work of the author [8] the moments computed from time-frequency representation of the signal were selected as features. This approach obviously destroys a part of useful information, so a complicated neural classifier is needed to obtain an acceptable performance. The correlation scheme developed here improves the performance in the case of matching pattern and decreases the false classification rate in the case of non-matching patterns when using the maximum of available information and at the same time enhancing ”meaningful” parts of the signal.

Waveforms encountered in power systems have usually quite well known structure (line spectra, low noise levels, composed of usually well separated harmonics, etc.), so it is straightforward to select the frequency band where the signal of interest shows most characteristic patterns. Similarly, the time point of the occurrence of a specific pattern can be either determined as the starting point of an event (e.g. beginning of a short-circuit) or using other techniques (e.g. change-point detection algorithms, wavelets) [10]. In this way a rectangular area or multiple areas on the time-frequency plane can be determined where the correlation based pattern recognition algorithm can show possibly best performance. Simplified scheme of this procedure is shown in Figure 1. After the TF transformation of signal and pattern, a specific area of the TF plane is selected. Then, the inverse transformation (or an approximate reconstruction, e.g. in the sense of equal energy of the original and reconstructed signal of the time-domain signal from its calculated parameters) allows usual correlation of time-domain signals and patterns. Similar approach was presented in [12], although applied to different problems and using other transformations.

2.1. Correlation of signal and pattern

We assume a band-limited and time-limited signal $s(t)$ and pattern $p(t)$, with its time-frequency representation, as follows [12]:

$$\mathrm{TF}\{p(t,f)\} \equiv 0 \;\forall\; \{t \in [t_1, t_2], f \in [f_1, f_2]\} \qquad (1)$$

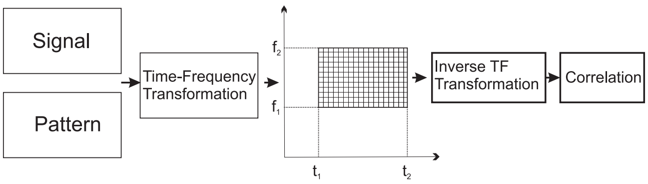

Figure 1. Scheme of correlation-based classification based on TF transformation.

where $[t_1, t_2]$ and $[f_1, f_2]$ define supports in time and frequency domains, respectively.

Any finite and band-limited signal $s(t) \subseteq [t_1, t_2]$ can be decomposed as follows, when using its time-frequency representation $\mathrm{TF}\{s(t, f)\}$.

$$\mathrm{TF}\{s(t, f)\} = \mathrm{TF}\{s_1(t, f)\} \cup \mathrm{TF}\{s_2(t, f)\} \quad (2)$$

where $\mathrm{TF}\{s_1(t, f)\} = \mathrm{TF}\{s(t, f)\}$ and $\mathrm{TF}\{s_2(t, f)\} = \mathrm{TF}\{s(t, f)\} \cap \mathrm{TF}\{s_1(t, f)\}$.

Such decomposition assumes that $s_1(t)$ is the part of signal $s(t)$ which has the same support in time and frequency as pattern $p(t)$ has and $s_2(t)$ represents the remaining part of the signal $s(t)$.

If we assume that both signals and the pattern have their respective inverse time-frequency transforms, then

$$s(t) = s_1(t) + s_2(t) \quad (3)$$

For any band-limited and finite signal $s(t)$ and pattern $p(t)$, which can be decomposed into $s(t) = s_1(t) + s_2(t)$ the following condition is fulfilled:

$$\max\left[|\mathrm{R}(s_1(t), p(t))|\right] > \max\left[|\mathrm{R}(s_2(t), p(t))|\right] \quad (4)$$

Equation 4 is a consequence of the assumptions that the signal $s_1(t)$ is similar to the pattern $p(t)$ and has the same localization in time-frequency plane and $s_2(t)$ lies outside the area in the time-frequency plane where the pattern $p(t)$ is localized.

From the above assumption it follows [12], that:

$$\max\left[|\mathrm{R}(s_1(t), p(t))|\right] > \max\left[|\mathrm{R}(s_2(t), p(t))|\right] \quad (5)$$

The normalized correlation of the signal $s(t) = s_1(t) + s_2(t)$ is defined in usual way [5]. From 4 it follows, that:

$$\max\left[|\mathrm{R}(s_1(t), p(t))|\right] > \max\left[|\mathrm{R}(s(t), p(t))|\right] \quad (6)$$

In the case of single pattern in the time-frequency plane, the above considerations show that the presence of the pattern in the signal assures the highest correlation coefficient when correlating pattern and signal.

In the case of patterns $p_{i,j}$ which do have non-disjoint time-frequency representations, such as:

$$\mathrm{TF}\{p_i(t, f)\} \cap \mathrm{TF}\{p_j(t, f)\} \neq \emptyset \quad (7)$$

the problem can arise, namely a high correlation coefficient in the case when the pattern is not present in the signal [12]. In order to avoid such situation, it is necessary to define a *mutually exclusive pattern to any other pattern*. This is quite straightforward when dealing with the representation of signal in the time-frequency plane.

Any pattern $p(t)$ can be represented as a sum of two mutually exclusive patterns, $p_k(t), p_l(t)$, where $p_l(t)$ represents part which is nullified for any disjoint set of patterns (it represents the non-disjoint part of any set of patterns).

$$p(t) = p_k(t) + p_l(t) \quad (8)$$

If the signal $s(t)$ is weakly correlated with the pattern $p(t)$, then the correlation of $s(t)$ with pattern $p_k(t)$ yields in smaller correlation coefficient than in the case of correlation with the signal $s(t)$ with $p(t)$.

$$\max\left[|\mathrm{R}(s(t), p(t))|\right] > \max\left[|\mathrm{R}(s(t), p_k(t))|\right] \quad (9)$$

From (9), assuming the mutual exclusivity of the patterns $p_k(t)$ and $p_l(t)$ (8), the following can be concluded:

$$\left|\int_{-\infty}^{\infty} s(t)p_l(t+\tau)dt\right| > \left|\int_{-\infty}^{\infty} s(t)p_k(t+\tau)dt\right| \quad (10)$$

and

$$\sqrt{\int_{-\infty}^{\infty} \left[p_k^2(t) + 2p_k(t)p_l(t) + p_l^2(t)\right] dt} \approx \sqrt{\int_{-\infty}^{\infty} p_k^2(t)dt} \quad (11)$$

Finally, from equations (7)-(11), it follows that the following holds:

$$\max\left[|\mathrm{R}(s(t), p(t))|\right] > \max\left[|\mathrm{R}(s(t), p_k(t))|\right] \quad (12)$$

The considerations presented above show that transformation of the signal to time-frequency domain, selection of particular areas in time-frequency plane (mutually exclusive areas), subsequent calculation of parameters of the signal and pattern inside the pre-selected "areas of interest" leads to *increase* of the maximum correlation coefficient of the correlated signal and pattern (when signal and pattern are similar) and to *decrease* of the maximum correlation coefficient when both signal and pattern are dissimilar.

3. ESPRIT

The original ESPRIT (Estimation of Signal Parameter via Rotational Invariance Technique) was described by Paulraj, Roy and Kailath and later developed, for example, in [11]. It is based on a naturally existing shift invariance between the discrete time series which leads to rotational invariance between the corresponding signal subspaces. The shift invariance is illustrated below.

After the eigendecomposition of the autocorrelation matrix as:

$$\mathbf{R}_x = \mathbf{U}^{*T}\mathbf{\Lambda}\mathbf{U} \quad (13)$$

it is possible to partition a matrix by using special *selector matrices* which select the first and the last $(M-1)$ columns of a $(M \times M)$ matrix, respectively:

$$\begin{aligned} \mathbf{\Gamma}_1 &= [\mathbf{I}_{M-1}|\mathbf{0}_{(M-1)\times 1}]_{(M-1)\times M} \\ \mathbf{\Gamma}_2 &= [\mathbf{0}_{(M-1)\times 1}|\mathbf{I}_{M-1}]_{(M-1)\times M} \end{aligned} \quad (14)$$

By using of matrices $\mathbf{\Gamma}$ two subspaces are defined, spanned by two subsets of eigenvectors as follows:

$$\begin{aligned} \mathbf{S}_1 &= \mathbf{\Gamma}_1\mathbf{U} \\ \mathbf{S}_2 &= \mathbf{\Gamma}_2\mathbf{U} \end{aligned} \quad (15)$$

For the matrices defined as $\mathbf{S}_1$ and $\mathbf{S}_2$ in (15), for every $\omega_k; k \in \mathbf{N}$, representing different frequency components, and matrix $\mathbf{\Phi}$, defined as:

$$\mathbf{\Phi} = \begin{bmatrix} e^{j\omega_1} & 0 & \cdots & 0 \\ 0 & e^{j\omega_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\omega_k} \end{bmatrix} \quad (16)$$

the following relation can be proven [3]:

$$[\mathbf{\Gamma}_1\mathbf{U}]\mathbf{\Phi} = \mathbf{\Gamma}_2\mathbf{U} \tag{17}$$

The matrix $\mathbf{\Phi}$ contains all information about frequency components. In order to extract this information, it is necessary to solve (17) for $\mathbf{\Phi}$. By using a unitary matrix (denoted as $\mathbf{T}$), the following equations can be derived:

$$\begin{aligned} \mathbf{\Gamma}_1(\mathbf{UT})\mathbf{\Phi} &= \mathbf{\Gamma}_2(\mathbf{UT}) \\ \mathbf{\Gamma}_1\mathbf{U}\underbrace{(\mathbf{T\Phi T}^{*\mathbf{T}})}_{\text{eig. of } \mathbf{\Phi}} &= \mathbf{\Gamma}_2\mathbf{U} \end{aligned} \tag{18}$$

The only interesting subspace here is the *signal subspace*, spanned by signal eigenvectors $\mathbf{U}_s$. Usually it is assumed that these eigenvectors correspond to the largest eigenvalues of the correlation matrix and $\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K]$. ESPRIT algorithm determines the frequencies $e^{j\omega_K}$ as the eigenvalues of the matrix $\mathbf{\Phi}$.

In theory, the equation (17) is satisfied exactly. In practice, matrices $\mathbf{S}_1$ and $\mathbf{S}_2$ are derived from an estimated correlation matrix, so this equation does not hold exactly. It can be solved for $\mathbf{\Phi}$ using Total Least Squares approach.

The ESPRIT method is suitable for stationary signals where an entire block of data is used for estimation of parameters. However, when the signal is non-stationary, this method can be modified as block-based (sliding temporal window approach [9]).

4. CLASSIFICATION OF EVENTS

4.1. Introduction

The problem of classification, using the method presented above of signals obtained from the industrial power frequency converters is considered in this section.

Object of the signal classification can be control or optimization of the modern frequency power converters, which generate a wide spectrum of harmonic components. Especially, the task of fault detection is difficult. In large converter systems, which generate not only characteristic harmonics typical for the ideal converter operation, but also a considerable amount of non-characteristic harmonics, the task of fault detection is even more difficult [4, 9].

The characteristics of the signal can be better analyzed and understood if the correct representation is chosen. In case of the heavily distorted signals, which contents change with time, it can be expected that the time and frequency characteristics are the most important. The parametric time-frequency transformation can provide advantages when analyzing non-stationary signals due to its better temporal resolution, excellent performance in the presence of noise, and no phase dependence than classical Fourier-based spectra. In the case of time-frequency representation of a signal it is possible to study simultaneously the time and frequency characteristics of the signal with best possible resolution than non-parametric time-frequency transformations. The signal classification is the assignment of the time-series to a specific class with given characteristics.

4.2. Numerical simulations

The signals under investigation are short-circuit currents obtained from a typical 3 kVA-PWM converter simulated with the Power System Blockset of MATLAB. Simulation system contains inverter and asynchronous machine models, as well as fault simulation circuit.

The signal (example in Figure 2 for a given short-circuit resistance) is then transformed to its time-frequency representation using parametric ESPRIT method with the help of temporal sliding window as shown in Figure 3.

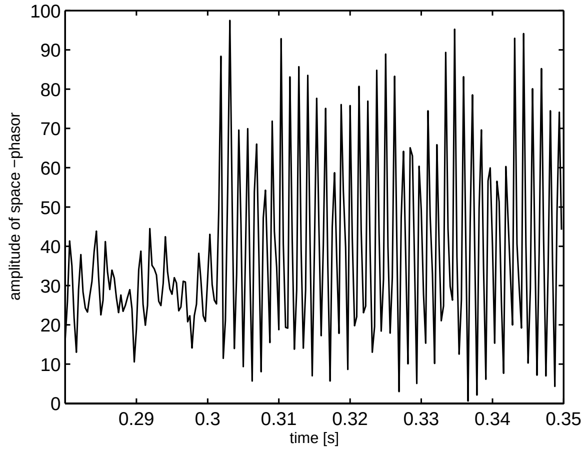

Figure 2. Investigated signal - inverter output current.

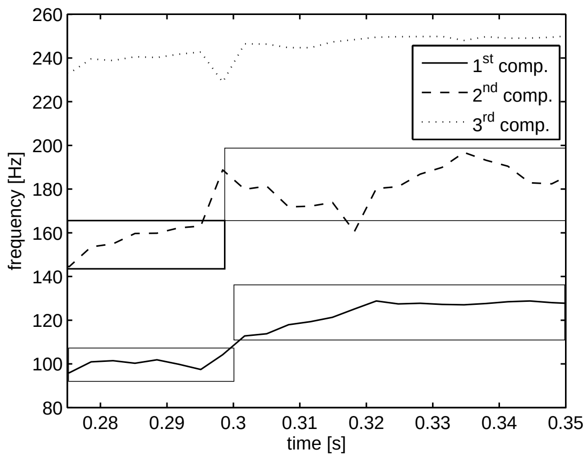

Figure 3. Time-frequency parametric representation (ESPRIT-based) of the signal with selected regions.

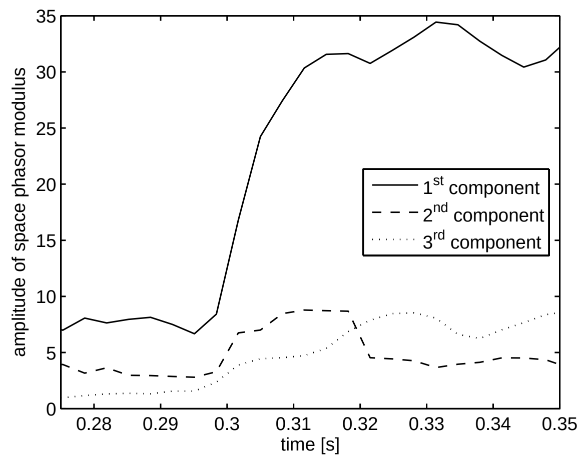

Figure 4. Corresponding amplitudes of components as in Fig. 3.

Table 1. Average of the highest correlation coefficients 1000 trials using ESPRIT, STFT and time-domain correlation.

Correlation coefficient	Classification		
	ESPRIT	STFT	time-domain
Pattern present	0.64	0.55	0.32
No pattern	0.11	0.21	0.29

Taken the representation of the waveform in time-frequency plane, as the next step, the areas in this plane can be chosen, either manually (based on observation) or automatically (based on some optimization algorithm, which e.g. minimizes the classification error). Optimization approach is not developed in this paper.

In Figure 3 the time-frequency representation (ESPRIT-based) of the modulus of the space-phasor of inverter output currents is presented. Three components are shown here and the fundamental component is removed for clarity. Selected areas for subsequent reconstruction are outlined as rectangular areas in time-frequency plane and summarized below:

- time interval: 0.27-0.3 s; freq. band: 92-108 Hz,
- time interval: 0.27-0.3 s; freq. band: 143-165 Hz,
- time interval: 0.3-0.35 s; freq. band: 112-138 Hz,
- time interval: 0.3-0.35 s; freq. band: 165-200 Hz.

It follows the classification procedure. As already shown in Figure 1, the parameters of the signal and pattern are extracted from their time-frequency representations, by taking only these parts of signal which are contained within the selected "regions of interest" (as shown in Figure 3). Extracted parameters (components' frequencies, amplitudes, duration in time, etc.) allow "reconstitution" (incomplete reconstruction) of preprocessed signals and patterns. The procedure is then followed by computation of classical, time-domain correlation sequence. The result of classification depends on the highest value of the correlation coefficient which show, to some extent, the degree of similarity between signal under classification and previously selected pattern.

Result of application of described classification scheme are presented in Table 1. One thousand waveforms were simulated using different drive parameters (parameters of LC filter, value of short-circuit resistance, value of the shaft mechanical torque applied to the asynchronous machine in order to validate this classification approach.

From the analysis of Table 1 it can be noted that the use of high-resolution ESPRIT method and selection of areas of obtained time-frequency representation allows highest sensitivity of detection of a pattern (here: short-circuit waveform) hidden in the current waveform at the converter output (precisely: the signal is composed of all three currents in the form of space–phasor [6]). For comparison, the STFT signal representation was also applied in this classification scheme. Classical time-domain correlation is almost useless for this classification task.

5. CONCLUSIONS

A method of classification of signals was presented, based on the time-frequency parametric representation and automatic signal classification with the help of a standard correlation technique. The investigations proved the validity of the proposed approach. The method's applications is limited to signals characterized by line spectra and it was adjusted to signals encountered in electrical power systems. It allows further improvements which can additionally increase its performance which can include the design of the classification system with many classes, optimized and/or automatic choice of specific areas in the time-frequency plane, etc.

6. REFERENCES

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